ALGEBRA

1. Symbolic Toolbox

One of the most powerful tools that can be used in Matlab is the "Symbolic Toolbox". To use Matlab's facility for doing symbolic mathematics, it is necessary to declare the variables to be "symbolic". The best way to do that is to use the **syms** declaration statement:

```
>>syms a b x y;
>>c = 5;
>>E = a*x^2 + b*x + c;
```

The **syms** statement makes all the variables listed with it into symbolic variables and gives each of the variables a value that is equal to its own name. Thus, the value of **a** is **a**, the value of **b** is **b**, etc. The variable **E** is now symbolic because it was assigned to be equal to an expression that contained one or more symbolic variables. Its value is **a*x^2 + b*x + 5.**

For example, suppose that you need factor x^2-3x+2. Note that you must type 3*x for 3x. Then you type:

```
>> syms x
```

By syms you are declaring that x is a variable. Then type

```
>> factor(x^2-3*x+2)
ans =
(x-1)*(x-2)
```

To factor x^2+2x+1, you write:

```
>> syms x
>> factor(x^2+2*x+1)
ans =
(x+1)^2
```

To factor the equation x^2-y^2;

```
>>syms x y
>> factor(x^2-y^2)
ans =
(x-y)*(x+y)
```

Expand command can be used to expand the terms of any power equation. Let's use expand command to expand the following equation $(x^2-y^2)^3$.

```
>> expand((x^2-y^2)^3)
ans =
x^6-3*x^4*y^2+3*x^2*y^4-y^6
```

The simplify command is useful to simplify some equations like.

```
>> simplify((x^3-4*x)/(x^2+2*x))
ans =
x-2
```

2. Solving Equations

2.1 Algebraic Equations

By using Symbolic Toolbox, you can find solutions of algebraic equations with or without using numerical values. If you need to solve equations, you can use the command solve. For example, to find the solution of $x^3+x^2+x+1=0$ you write:

```
>> solve('x^3+x^2+x+1=0')
```

And Matlab give you the answer in the form

```
ans =
[ -1]
[  i]
[ -i]
```

That means the three solutions for the equation are 1, j, and –j.

```
>>x=solve('sin(x)+x=0.1')
x =
5.001042187833512e-2
```

In expressions with more than one variable, we can solve for one or more of the variables in terms of the others. Here we find the roots of the quadratic ax^2+bx+c in x in terms of a, b and c. By default solve sets the given expression equal to zero if an equation is not given.

```
>> x=solve('a*x^2+b*x+c','x')
x =
[ 1/2/a*(-b+(b^2-4*a*c)^(1/2))]
[ 1/2/a*(-b-(b^2-4*a*c)^(1/2))]
```

You can solve an equation in two variables for one of them. For example:

```
>> y=solve('y^2+2*x*y+2*x^2+2*x+1=0', 'y')
y =
[-x+i*(x+1)]
[ -x-i*(x+1)]
```

You can solve more than one equation simultaneously. For example to find the value of x and y from the equations: $5x+10y=46$ and $28x+32y=32$, you write:

```
>> [x,y]=solve('5*x+10*y=46', '28*x+32*y=32')
```

And you get the following result:

```
x =
 -48/5
 y =
 47/5
>> [x,y]=solve('log(x)+x*y=0', 'x*y+5*y=1')
x =
.8631121967939437
y =
.1705578823046945
```

To solve the system $x^2 + x + y^2 = 2$ and $2x-y = 2$. We can type:

```
>> [x,y] = solve( 'x^2+ x+ y^2 = 2', '2*x-y = 2')
```

And get the solutions

```
x =
[ 2/5]
[ 1]
y =
[ -6/5]
[ 0]
```

This means that there are two points which are (2/5, -6/5) and (1, 0).

Now let's find the points of intersection of the circles $x^2+y^2=4$ and $(x-1)^2+(y-1)^2=1$.

```
>>[x,y]=solve('x^2+y^2=4','(x-1)^2+(y-1)^2=1')
x=
[ 5/4-1/4*7^(1/2)]
[5/4+1/4*7^(1/2)]
y=
[ 5/4+1/4*7^(1/2)]
[5/4-1/4*7^(1/2)]
```

In same way if you have more then two equations you can use the same command to solve them for example:

```
[x,y,z]=solve('x+y+z=1','x+2*y-z=3','2*x-2*z=2')
 x =
 1/2
 y =
 1
 z =
 -1/2
```

2.2 DIFFERENTIAL EQUATIONS

2.2.1 First Order Differential Equations

Matlab can solve linear ordinary differential equations with or without initial/boundary conditions. Do not expect Matlab can solve nonlinear ordinary differential equations which typically have no analytical solutions. Higher derivatives can be handled as well. The command for finding the symbolic solution of differential equations is dsolve. For that command, the derivative of the function *y* is represented by Dy. For example, suppose that we want to find the solution of the equation $x y' - y = 1$. We will have:

```
>> dsolve('x*Dy-y=1', 'x')
ans =
-1+x*C1
```

This means that the solution is any function of the form $y = -1 + cx$, where c is any constant. The letter “D” has a special meaning and cannot be used otherwise inside **dsolve**. It means “first derivative of”. The C1 is a constant of integration.

If we have the initial condition $y(1) = 5$, we can get the particular solution on the following way:

```
>>dsolve('Dy+y=cos(t)')
ans =
1/2*cos(t)+1/2*sin(t)+exp(-t)*C1

>> dsolve('x*Dy-y=1', 'y(1)=5', 'x')
ans =
-1+6*x
```

2.2.2 Second Order Differential Equations

The second order linear equations can be solved similarly as the first order differential equations by using dsolve. For the command dsolve, the second derivative of *y* is represented with D2y. The letters “D2” mean second derivative.

For example, the command for solving $y''-3y'+2y = \sin x$.

```
>> dsolve('D2y-3*Dy+2*y=sin(x)', 'x')
ans =
3/10*cos(x)+1/10*sin(x)+C1*exp(x)+C2*exp(2*x)
```

If we have the initial conditions y(0) = 1, y'(0)=-1, we would have:

```
>> dsolve('D2y-3*Dy+2*y=sin(x)', 'y(0)=1', 'Dy(0)=-1', 'x')
ans =
3/10*cos(x)+1/10*sin(x)+5/2*exp(x)-9/5*exp(2*x)
```

Example: d^2y/dx^2 **-2dy/dx -3y=**x^2

```
>> dsolve('D2y - 2*Dy - 3*y=x^2', 'x')
ans =
-14/27+4/9*x-1/3*x^2+C1*exp(3*x)+C2*exp(-x)
```

Example: d^2y/dx^2 **-2dy/dx -3y=**x^2**, with y(0)=0,** and **dy/dx =1 at x=1**

```
>> dsolve('D2y - 2*Dy - 3*y=x^2','y(0)=0, Dy(1)=1','x')
ans =
-1/3*x^2+4/9*x-14/27+1/9*(-11+14*exp(3))/(3*exp(3)+exp(-1))*exp(-x)
+1/27*(33+1 4*exp(-1))/(3*exp(3)+exp(-1))*exp(3*x)
```

2.2.3 Higher Order Differential Equations

Similarity you can use the same way to solve the higher order differential equations.

3. Representing Functions

There is a way to define functions in MATLAB that behave in the usual manner. To represent a function in Matlab, we use “inline” command. For example to declare $f(x)=x^2+3x+1$ you write:

```
>> f=inline('x^2+3*x+1')
f=
Inline function:
f(x) = x^2+3*x+1
```

Therefore to find f(2), to get the answer you write:

```
>> f(2)
ans =
11
```

The function $g(x,y)=x^2-3xy+2$ is defined as follows.

```
>> g=inline('x^2-3*x*y+2')
g =
Inline function:
g(x,y) = x^2-3*x*y+2
```

Now we can evaluate $g(2,3)$ in the usual way.

```
>>g(2,3)
ans =
 -12
```

In some cases, if we need to define function f as a vector. Then we use:

```
>> f = inline(vectorize('x^2+3*x-2'))
```

```
f =
Inline function:
f(x) = x.^2+3.*x-2
```

In this case, we can evaluate a function at more than one point at the same time. For example, to evaluate the above function at 1, 3 and 5 we have:

```
>> f([1 3 5])
ans =
2 16 38
```

4. Differentiation

The Matlab function that performs differentiation is **diff**. These operations show how it works:

```
>> syms x
>>diff(x^2)
ans =
2*x
>>diff(sin(x)^2)
ans =
2*sin(x)*cos(x)
```

For example, let's find the derivative of $f(x)=\sin(e^x)$.

```
>> syms x
>> diff(sin(exp(x)))
```

and get the answer as:

```
ans =
cos(exp(x))*exp(x)
```

Note: Instead of using syms to declare of variables you can use two Quotes ' ' to declare that the variable x is the interested variable in equation; you can use the same example in otherwise

```
>>diff('sin(exp(x))')
ans =
cos(exp(x))*exp(x)
```

The n^{th} derivative of f is in the written in the form *diff(f,n)*. then to find the second derivative we write;

```
>> diff(sin(exp(x)),2)
ans =
-sin(exp(x))*exp(x)^2+cos(exp(x))*exp(x)
```

For example to find the first derivative of *x3+3x2+8x* you simply write:

```
>> syms x
>> diff(x^3+3*x^2+8*x)
ans =
 3*x^2+6*x+8
```

Moreover to get the 3rd derivative, write:

```
>> diff(x^3+3*x^2+8*x ,3)
ans =
6
```

Note: To get higher derivatives, you can write the degree in place of 3.

To compute the partial derivative of an expression with respect to some variable we specify that variable as an additional argument in diff. For example to find the derivative for x in equation $f(x,y)=x^3y^4+y\sin x$.

```
>>syms x y
>> diff(x^3*y^4+y*sin(x),x)
ans =
3*x^2*y^4+y*cos(x)
```

Next we compute diff for y

```
>> diff(x^3*y^4+y*sin(x),y)
ans =
4*x^3*y^3+sin(x)
```

Finally we compute $d^3 fx^3$.

```
>> diff(x^3*y^4+y*sin(x),x,3)
ans =
6*y^4-y*cos(x)
```

5. Integration

By using the Symbolic Toolbox, you can find both definitive and in-definitive integrals of functions. We can use MATLAB for computing both definite and indefinite integrals using the command int. If **f** is a symbolic expression in **x**, then:

int(f) → $\int f(x)dx$

For the indefinite integrals, consider the following example:

```
>> int('x^2')
ans =
1/3*x^3
```

Similarly as for diff command, we do not need the quotes if we declare *x* to be a symbolic variable. Therefore the above command can be re-written in otherwise such as:

```
>> syms x
```

```
>> int(x^2)
ans =
1/3*x^3
```

For example to find the in-definitive integral of $x^3+sin(x)$, you write:

```
>> syms x
>> int(x^3+sin(x))
ans =
1/4*x^4-cos(x)
```

A definite integral can be taken by giving three arguments. The second and third arguments in that case are the first and second limits of integration.

int(f, a, b) → $\int_{x=a}^{x=b} f(x)dx$

For the definitive integrals:

```
>> int(x^2, 0, 1)
ans =
1/3
```

Try these examples,

```
int(x,1,2)
int(x*sin(x),-2,7)
```

Moreover to get definitive integral to ln(x)+1/(x+1) from x=1 to x=2 write, you simply write:

```
>> int('log(x) + 1/(x+1)', 1, 2)
ans =
log(6)-1
```

6. Limits

You can use limit to compute limits. For example, to evaluate the limit when x goes to 2 of the function $(x^2-4)/(x-2)$,we have:

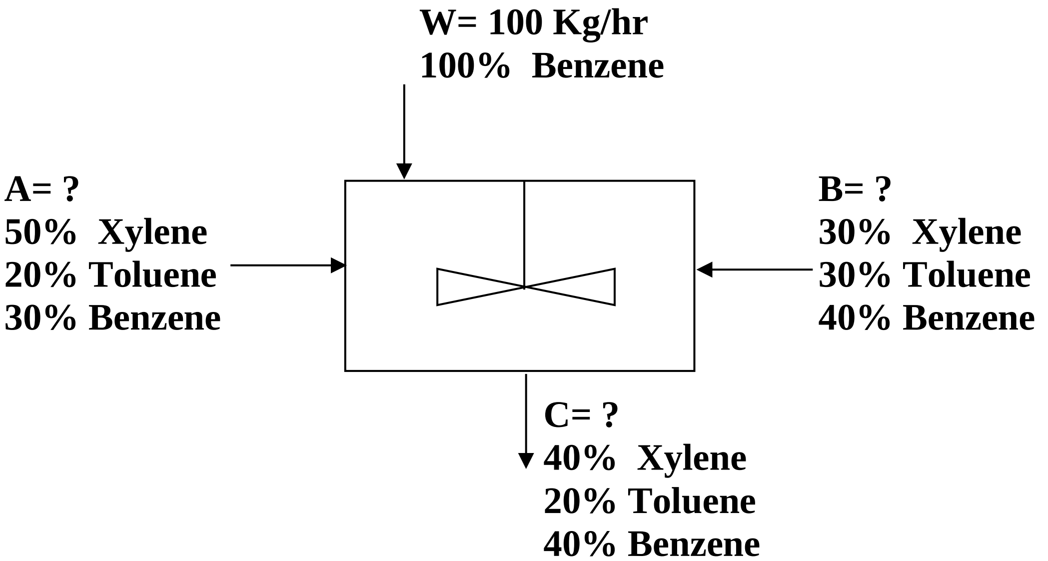
```
>> syms x
>> limit((x^2-4)/(x-2), x, 2)
ans =
4
```

Limits at infinity:

```
>> limit(exp(-x^2-5)+3, x, Inf)
ans =
3
```

Exercise 1:

For the mixer shown below write a code to find the values of streams A, B and C?



Solution: By making component material balance on each component within the mixer you can reach to a system of three equations which can be solve by using the command solve to find the unknowns A, B, C.

Type the following command:

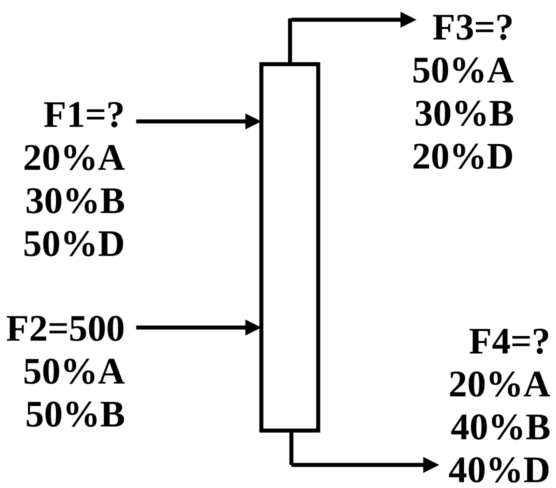
[A,B,C]=solve('.5*A+.3*B=.4*C','.2*A+.3*B=.2*C','.3*A+.4*B+100=.4*C')

The results will be:

```
A =
   600
B =
   200
C =
   900
```

Exercise 2:

For the following distillation column calculate the values of F1, F3 and F4?



Solution:

```
[F1,F3,F4]=solve('.2*F1+250=.5*F3+.2*F4','.3*F1+250=.3*F3+.4*F4','.5*F1=.2*F3+.4*F4')
```

The results will be:

```
F1=
    1000
 F3=
    500
 F4 =
   1000
```

Exercise 3:

Calculate the heat required to increase the temperature of 1 mol of methane from 533.15 K to 873.15 K at a pressure approximately 1 bar. where

$$\frac{Cp}{R} = A + BT + CT^2 + DT^{-2}$$

A=1.702 , B=9.081*10^{-3} , C=-2.164*10^{-6} , D=0 , R=8.314 and

$$Q = n\int_{Tin}^{Tout} Cp\,dT$$

Solution:

```
syms T;
T1=533.15; T2=873.15; n=1;
A=1.702; B=9.081e-3; C=-2.164e-6; R=8.314;
Cp=R*(A+B*T+C*T^2);
Q=single(int(Cp,T1,T2))
```

The results will be:

```
Q =
  1.9778e+004
```

Exercise 4:

Evaluate the following double integral

$$\int_0^{\pi}\int_0^{\sin x}(x^2 + y^2)dy \cdot dx$$

Solution: MATLAB can also do multiple integrals. The following command computes the double integral:

```
syms x y;
int(int(x^2 + y^2, y, 0, sin(x)), 0, pi)
ans =
-32/9+pi^2
```

To convert the way of the result displaying, type the code:

```
single(-32/9+pi^2)
ans =
```

Practice Problems

1) Write required code to solve each of the following:-
   a) Factor $x^3+3x^2y+3xy^2+y^3$.
   b) Simplify $(x^3-8)/(x-2)$.
   c) Expand $(x^2+1)(x-5)(2x+3)$.
   d) Solve $\sin x = 2-x$ for x.
   e) Solve $5x+2y+4z = 8$, $-3x+y+2z = -7$, $2x+y+z = 3$ for x, y and z.
   f) Solve $y^2-5xy-y+6x^2+x = 2$ for x.
   g) Find the first derivative of the function $(\sin x/(\ln(x^2+1))-e^x$ and evaluate it at x=3.
   h) Find the 12th derivative of the function $(x/2+1)^{65}$
   i) Find the first and second partial derivatives for x of the function $e^{x2}\sin xy$

2) Obtain the first and second derivatives of the following functions using MATLAB's symbolic mathematics.
   a) $F(x) = x^5 - 8x^4 + 5x^3 - 7x^2 + 11x - 9$
   b) $F(x) = (x^3 + 3x - 8)(x^2 + 21)$
   c) $F(x) = (3x^3 - 8x^2 + 5x + 9)/(x + 2)$
   d) $F(x) = (x^5 - 3x^4 + 5x^3 + 8x^2 - 13)^2$
   e) $F(x) = (x^2 + 8x - 11)/(x^7 - 7x^6 + 5x^3 + 9x - 17)$

3) Chemical engineer, as well as most other engineers, uses thermodynamics extensively in their work. The following polynomial can be used to relate specific heat of dry air, C_p KJ/(Kg K), to temperature (K):
   $C_p= 0.994 +1.617\times10^{-4}T+9.7215\times10^{-8}T^2 - 9.5838 \times 10^{-11} T^3 + 1.9520 \times 10^{-14} T^4$
   Determine the temperature that corresponds to a specific heat of 1.2 KJ/(Kg K).

4) Evaluate the triple integral:
   $$\int_0^6 \int_0^6 \int_{-4}^4 (x^3 - 2yz)dx \cdot dy \cdot dz$$

5) The average values of a specific heat can be determined by $Cp_{mh} = \frac{\int_{T_1}^{T_2} Cp dT}{T_2 - T_1}$
   Use this relationship to verity the average value of specific heat of dry air in the range from 300 K to 450 K, C_p KJ/(Kg K), to temperature (K):
   $C_p= 0.99403 +1.617\times10^{-4}T+9.7215\times10^{-8}T^2 - 9.5838 \times 10^{-11} T^3 + 1.9520 \times 10^{-14} T^4$