Polynomials in Matlab

The equation $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdot\cdot\cdot + a_1 x + a_0$ is called a polynomial in x. The terms of this polynomial are arranged in descending powers of x while the ai's are called the coefficients of the polynomi0al and are usually real or complex numbers. The degree of the polynomial is n (the highest available power of x).

In Matlab, a polynomial is represented by a vector. To create a polynomial in Matlab, simply enter each coefficient of the polynomial into the vector in descending order (from highest-order to lowest order). Let's say you have the following polynomial:

$y=x^4+3x^3-15x^2-2x+9$

To enter this polynomial into Matlab, just enter it as a vector in the following manner:

y = [1 3 -15 -2 9]

Also to represent the polynomial $y=2x^3 + 2x^2 + 4x + 1$.

y = [2 2 4 1];

Matlab can interpret a vector of length n+1 as an n^{th} order polynomial. Thus, if your polynomial is missing any coefficients, you must enter zeros in the appropriate place in the vector. For example, $p(x) = x^6-2x^4-x^3+2x^2+5x-8$ is a polynomial that is arranged in descending powers of x. The coefficients of the polynomial are 1, 0, −2, −1, 2, 5, and −8. In this case, there is an understood term of x^5. However, Matlab doesn't "understand" that a term is missing unless you explicitly tell it so.

For other example, the polynomial $y=x^4+1$ would be represented in Matlab as:

y = [1 0 0 0 1]

1. Roots

To calculate the roots of a polynomial, enter the coefficients in an array in descending order. Be sure to include zeroes where appropriate.

For example to find the roots of the polynomial $y= x^4 + 6x^3 + 7x^2 - 6x - 8 = 0$ type the command:

```
p = [ 1 6 7 -6 -8 ];
r = roots(p)
```

yields

```
r =
   -4.0000
   -2.0000
   -1.0000
    1.0000
```

Note: The coefficients could be entered directly in the roots command. The same answer as above would be obtained using the following expression.

```
 r = roots([ 1 6 7 -6 -8 ])
r =
   -4.0000
    1.0000
   -2.0000
   -1.0000
```

For example finding the roots of $y=x^4+3x^3-15x^2-2x+9=0$ would be as easy as entering the following command;

```
 r=roots([1 3 -15 -2 9])
r =
   -5.5745
    2.5836
   -0.7951
    0.7860
```

The **roots** command can find imaginary roots.

```
p = [ 1 -6 18 -30 25 ];
r = roots(p)
r =
    1.0000 + 2.0000i
    1.0000 - 2.0000i
    2.0000 + 1.0000i
    2.0000 - 1.0000i
```

It can also find repeated roots. Note the imaginary portion of the repeated roots is displayed as zero.

```
 p = [ 1 7 12 -4 -16 ];
 r = roots(p)
r =
    -4.0000
    -2.0000 + 0.0000i
    -2.0000 - 0.0000i
    1.0000
```

2. PolyVal

You can use polyval and the fitted polynomial p to predict the y value of the data you've fitted for some other x values. The syntax for determining the value of a polynomial $y=x^4 + 6x^3 + 7x^2 - 6x - 8$ at any point is as follows.

```
p = [ 1 6 7 -6 -8 ];
y= polyval(p, 3)
y=
   280
```

Where p is the vector containing the polynomial coefficients, (see above). Similarly, the coefficients can be entered directly in the polyval command.

```
 y = polyval([1 6 7 -6 -8], 3)
y =
   280
```

The polynomial value at multiple points (vector) can be found.

```
 z = [ 3 5 7];
 y = polyval(p,z)
y =
   280   1512   4752
```

3. Polyfit

To determining the coefficients of a polynomial that is the best fit of a given data you can use **polyfit** command. The command is **polyfit(x, y, n),** where x, y are the data vectors and 'n' is the order of the polynomial for which the least-squares fit is desired.

Exercise 1:

Fit x, y vectors to 3 rd order polynomial

```
 x = [ 1.0 1.3 2.4 3.7 3.8 5.1 ];
 y = [ -6.3 -8.7 -5.2 9.5 9.8 43.9 ];
 coeff = polyfit(x,y,3)
coeff =
   0.3124   1.5982   -7.3925   -1.4759
```

After determining the polynomial coefficients, the **polyval** command can be used to predict the values of the dependent variable at each value of the independent variable.

```
ypred = polyval(coeff,x)
ypred =
   -6.9579   -7.6990   -5.6943   8.8733   10.6506   43.8273
```

Its clear that there is a deviation between the actual y points and predicted y points because that the polynomial is best fit to this actual points.

Exercise 2:

Fit the following data describing the accumulation of species A over time to a second order polynomial, and then by using this polynomial, predict the accumulation at 15 hours.

Time (hr)	1	3	5	7	8	10
Mass A acc.	9	55	141	267	345	531

Solution: First, input the data into vectors, let:

```
M = [9, 55, 141, 267, 345, 531];
time = [1, 3, 5, 7, 8, 10];
```

Now fit the data using polyfit

```
coeff = polyfit(time,M,2)
coeff =
    5.0000 3.0000 1.0000
```

So, Mass A = 5*$(\text{time})^2$ + 3 * (time) + 1

Therefore to calculate the mass A at 15 hours

```
MApred = polyval(coeff,15)
MApred =
  1.1710e+003
```

Exercise 3:

Fit the following vapor pressure vs temperature data in fourth order polynomial. Then calculate the vapor pressure when T=100 ^{O}C.

Temp (C)	-36.7	-19.6	-11.5	-2.6	7.6	15.4	26.1	42.2	60.6	80.1
Pre. (kPa)	1	5	10	20	40	60	100	200	400	760

Solution:

```
vp = [ 1, 5, 10, 20, 40, 60, 100, 200, 400, 760];
T = [-36.7, -19.6, -11.5, -2.6, 7.6, 15.4, 26.1, 42.2, 60.6, 80.1];
p=polyfit(T,vp,4)
pre= polyval(p,100)
```

The results will be:

```
p =
    0.0000    0.0004    0.0360    1.6062   24.6788
pre =
  1.3552e+003
```

Exercise 4:

The calculated experimental values for the heat capacity of ammonia are:

T (C)	Cp (cal /g.mol C)
0	8.371
18	8.472
25	8.514
100	9.035
200	9.824
300	10.606
400	11.347

1. Fit the data for the following function

$$Cp = a + bT + CT^2 + DT^3 \qquad \text{Where T is in C}$$

2. Then calculate amount of heat Q required to increase the temperature of 150 mol/hr of ammonia vapor from 0 C to 200 C if you know that:

$$Q = n \int_{Tin}^{Tout} Cpdt$$

Solution:

```
T=[0,18,25,100,200,300,400]
Cp=[8.371, 8.472, 8.514, 9.035, 9.824, 10.606, 11.347]
P=polyfit(T,Cp,3)
n=150;
syms T
Cpf=P(4)+P(3)*T+P(2)*T^2+P(1)*T^3;
Q= n*int(Cpf, 0,200)
  2.7180e+005
```

Practice Problems

1) Find the 3rd order polynomial that satisfies the data of water for saturation temperature and pressure. By using the predicted polynomial compute the saturation pressure at 65 C.

Temp(C)	0	10	20	30	40	50	60	70	80	90	100
Pre. (kPa)	.6108	1.227	2.337	4.241	7.375	12.335	19.92	31.16	47.36	70.11	101.33

2) Write a MATLAB program to fit the following vapor pressure vs. temperature data to calculate the values of constants A and B in following equation.

$$\log(P^0) = A - \frac{B}{T + 273.15}$$

Temp (C)	-36.7	-19.6	-11.5	-2.6	7.6	15.4	26.1	42.2	60.6	80.1
Pre. (kPa)	1	5	10	20	40	60	100	200	400	760

3) The experimental velocity of an incompressible fluid in a pipe of radius 1 m is tabulated as below:

r (m)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
V	1.0	0.99	0.96	0.91	0.84	0.75	0.64	0.51	0.36	0.19	0.0

Where: r is the distance from the centre of the pipe and u is the velocity of the fluid. Write a MATLAB program to fit the experimental velocity to the following function: $u=a+br+cr^2$

4) Fully developed flow moving a 40 cm diameter pipe has the following velocity profile:

Radius r, cm	0.0	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0
Velocity v, m/s	0.914	0.890	0.847	0.795	0.719	0.543	0.427	0.204	0

Find the volumetric flow rate Q using the relationship $Q = \int_0^R 2\pi r v dr$.

Where r is the radial axis of the pipe, R is the radius of the pipe and v is the velocity. Solve the problem using two steps.

(a) Fit a polynomial curve to the velocity data using **polyfit**.

(b) Integrate the equation using **int**.

5) The rate at which a substance passes through a membrane is determined by the diffusivity D,(cm^2/s) of the gas. D varies with the temperature T (K) according to the following law:

$D = D_o \exp(-E/RT)$ Where:

D_o is the pre-exponential factor

E is the activation energy for diffusion

R =1.987 cal/mol K.

Diffusivities of SO_2 in a certain membrane are measured at several temperatures with the data listed below.

$T(K)$	347	374.2	396.2	420.7	447.7	471.2
$D(cm^2/s)\times10^6$	1.34	2.5	4.55	8.52	14.07	19.99

Write a MATLAB program to calculate the values of D_o and E.

6) A constant temperature, pressure-volume thermodynamic process has the following data:

Pressure (kpa)	420	368	333	326	312	242	207
Volume (m^3)	0.5	2	3	6	8	10	11

We know that $W = \int p dV$. Where W is work, p is pressure, and V is volume. Calculate the work required to compression from 5 m^3 to 2 m^3.