MATERIAL AND ENERGY BALANCE

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CHAPTER 1

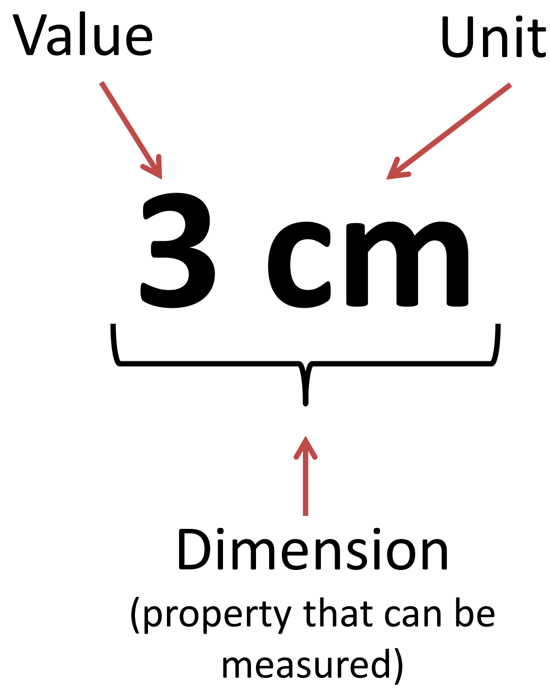
INTRODUCTION TO ENGINEERING CALCULATIONS

Week 1

CONTENT

- UNITS & DIMENSIONS
- CONVERSION OF UNITS
- SYSTEMS OF UNITS
- FORCE & WEIGHT
- SIGNIFICANT FIGURES
- MEAN, VARIABLE & STANDARD DEVIATION
- FITTING A STRAIGHT LINE
- FITTING NONLINEAR LINE

UNITS AND DIMENSIONS



Dimension vs. Dimensionless

Dimension Quantity	Dimensionless Quantity
• Has value & unit. • Eg.: length, volume. (Base units) The values of two or more base unit may be added or subtracted only if their units are the same, e.g.; 5.0 kg + 2.2 kg = 7.2 kg. On the other hand, the values and units *of any base unit* can be combined by multiplication or division, e.g.: $\frac{1500\ \text{km}}{12.5\ \text{h}} = 120\ kmh^{-1}$	• Doesn't has unit. • Eg.: Atomic weight. Ratios of base units. Reynolds number, *Re.* For flow in a pipe, the Reynolds number is given by the equation: $Re = \frac{Du\rho}{\mu}$ where ρ is fluid density, *u* is fluid velocity, D is pipe diameter And μ is fluid viscosity. When the dimensions of these variables are combined, the dimensions of the numerator exactly cancel those of the denominator.

From all the physical variables in the world, the seven quantities listed in Table 2.1 have been chosen by international agreement as a basis for measurement.

Table 2.1 Base quantities

Base quantity	*Dimensional symbol*	*Base SI unit*	*Unit symbol*
Length	L	metre	m
Mass	M	kilogram	kg
Time	T	second	s
Electric current	I	ampere	A
Temperature	Θ	kelvin	K
Amount of substance	N	gram-mole	mol or gmol
Luminous intensity	J	candela	cd

Dimensional Homogeneity in Equations

"EVERY VALID EQUATION MUST BE HOMOGENEOUS: THAT IS, ALL ADDITIVE TERMS ON BOTH SIDES OF THE EQUATION MUST HAVE THE SAME DIMENSIONS."

Dimensional Homogeneity in Equations

- For dimensional homogeneity, the dimensions of terms which are added or subtracted must be the same, and the dimensions of the right-hand side of the equation must be the same as the left-hand side.
- As a simple example, consider the Margules equation for evaluating fluid viscosity from experimental measurements:

$$\mu = \frac{M}{4\pi h\Omega}\left(\frac{1}{R_o^2} - \frac{1}{R_i^2}\right).$$

- The terms and dimensions in this equation are listed in Table 2.3.
- Numbers such as 4 have no dimensions; the symbol π represents the number 3.1415926536 which is also dimensionless.
- A quick check shows this equation is dimensionally homogeneous since both sides of the equation have dimensions L-1MT-1 and all terms added or subtracted have the same dimensions.

Table 2.3 Terms and dimensions of Eq. (2.4)

Term	*Dimensions*	*SI Units*
μ (dynamic viscosity)	$L^{-1}MT^{-1}$	pascal second (Pa s)
M (torque)	L^2MT^{-2}	newton metre (N m)
h (cylinder height)	L	metre (m)
Ω (angular velocity)	T^{-1}	radian per second ($rad\ s^{-1}$)
R_o (outer radius)	L	metre (m)
R_i (inner radius)	L	metre (m)

Example:

$$\mu = \frac{M}{4\pi h\Omega}\left(\frac{1}{R_o^2} - \frac{1}{R_i^2}\right).$$

So, $\frac{M}{LT} = \left(\frac{\frac{ML^2}{T^2}}{\frac{4\pi L}{T}}\right)\left(\frac{1}{L} - \frac{1}{L}\right) = \left(\frac{ML^2}{T^2}\right)\left(\frac{T}{4\pi L}\right)\left(\frac{1}{L}\right) = \frac{M}{LT}$

CONVERSION OF UNITS

***Conversion factor*:**

Used to convert a measured quantity to a different *unit of measure* without changing the relative amount.

Example

- Convert an acceleration of 1 cm/s^2 to its equivalent in km/yr^2.

$$\frac{1\ \text{cm}}{\text{s}^2}\left|\frac{3600^2\ \text{s}^2}{1^2\ \text{h}^2}\right|\frac{24^2\ \text{h}^2}{1^2\ \text{day}^2}\left|\frac{365^2\ \text{day}^2}{1^2\ \text{yr}^2}\right|\frac{1\ \text{m}}{10^2\ \text{cm}}\left|\frac{1\ \text{km}}{10^3\ \text{m}}\right.$$

$$= \frac{(3600 \times 24 \times 365)^2}{10^2 \times 10^3}\frac{\text{km}}{\text{yr}^2} = \boxed{9.95 \times 10^9\ \text{km/yr}^2}$$

- A principle illustrated in this example is that raising a quantity (in particular, a conversion factor) to a power raises its units to the same power. The conversion factor for h^2/day^2 is therefore the square of the factor for h/day:

$$\left(\frac{24\ \text{h}}{1\ \text{day}}\right)^2 = 24^2\ \frac{\text{h}^2}{\text{day}^2}$$

SYSTEMS OF UNITS

SYSTEMS OF UNITS →
- SI (Systeme Internationale d'Unites)
- CGS system
- American engineering system

e.g.: Force →
- $kg \cdot m/s^2$ (SI),
- $g \cdot cm/s^2$ (CGS)
- $lb_m \cdot ft/s^2$ (American engineering).

Table 2.3-1 SI and CGS Units

Base Units		
Quantity	Unit	Symbol
Length	meter (SI)	m
	centimeter (CGS)	cm
Mass	kilogram (SI)	kg
	gram (CGS)	g
Moles	gram-mole	mol or g-mole
Time	second	s
Temperature	kelvin	K
Electric current	ampere	A
Light intensity	candela	cd

Multiple Unit Preferences	
tera (T) = 10^{12}	centi (c) = 10^{-2}
giga (G) = 10^{9}	milli (m) = 10^{-3}
mega (M) = 10^{6}	micro (μ) = 10^{-6}
kilo (k) = 10^{3}	nano (n) = 10^{-9}

Derived Units			
Quantity	Unit	Symbol	Equivalent in Terms of Base Units
Volume	liter	L	0.001 m^3
			1000 cm^3
Force	newton (SI)	N	1 $kg \cdot m/s^2$
	dyne (CGS)		1 $g \cdot cm/s^2$
Pressure	pascal (SI)	Pa	1 N/m^2
Energy, work	joule (SI)	J	1 $N \cdot m$ = 1 $kg \cdot m^2/s^2$
	erg (CGS)		1 $dyne \cdot cm$ = 1 $g \cdot cm^2/s^2$
	gram-calorie	cal	4.184 J = 4.184 $kg \cdot m^2/s^2$
Power	watt	W	1 J/s = 1 $kg \cdot m^2/s^3$

Example:
Conversion Between Systems of Units

Convert 23 $lb_m \cdot ft/min^2$ to its equivalent in $kg \cdot cm/s^2$.

FORCE AND WEIGHT

Force, F = $1 \text{ newton (N)} \equiv 1 \text{ kg}\cdot\text{m/s}^2$

Example:

How much is the force (in newtons) required to accelerate a mass of 4.00 kg at a rate of 9.00 m/s^2 ?

F = ma = mass x acceleration

$$F = \frac{4.00 \text{ kg}}{} \left| \frac{9.00 \text{ m}}{\text{s}^2} \right| \frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2} = 36.0 \text{ N}$$

Weight = mg = mass x gravitational acceleration

$$g = 9.8066\ \text{m/s}^2$$
$$= 980.66\ \text{cm/s}^2$$
$$= 32.174\ \text{ft/s}^2$$

Example:

Water has a density of 62.4 lb_m/ft^3. How much does 2.000 ft^3 of water weigh (1) at sea level and 45° latitude and (2) in Denver, Colorado, where the altitude is 5374 ft and the gravitational acceleration is 32.139 ft/s^2?

Solution:

Given: $\rho = 62.4\ \text{lb}_m/\text{ft}^3$

$V = 2\ \text{ft}^3$

$\therefore \rho = \dfrac{m}{V}$; $m = \rho V$

$$= \left[62.4\ \frac{\text{lbm}}{\text{ft}^3}\right] \times \left[2\ \text{ft}^3\right] = 124.8\ \text{lb}_m$$

(a) At sea level, $g = 9.81\ \text{m/s}^2$ or $32.174\ \text{ft/s}^2$:

$\therefore\ w = mg$; $m = 124.8\ \text{lbm}$; $1\ \text{lb}_f = 32.174\ \dfrac{\text{lb}_m\text{ft}}{\text{s}^2}$

$$= \left[(124.8\ \text{lb}_m)\left(\frac{1\ \text{lb}_f}{32.174\ \frac{\text{lbm.ft}}{\text{s}^2}}\right)\right] \times \left[32.174\ \frac{\text{ft}}{\text{s}^2}\right]$$

$$= 124.8\ \text{lb}_f$$

(b) At $g = 32.139\ \text{ft/s}^2$:

$\therefore\ w = mg$

$$= \left[(124.8\ \text{lbm})\left(\frac{1\ \text{lb}_f}{32.174\ \frac{\text{lbm ft}}{\text{s}^2}}\right)\right] \times \left[32.139\ \frac{\text{ft}}{\text{s}^2}\right]$$

$$= 124.7\ \text{lb}_f$$

Significant Figures

- *A significant figure* is any digit, 1-9, used to specify a number. Zero may also be a significant figure when it is not used merely to locate the position of the decimal point.

- For example, the numbers 6304, 0.004321, 43.55 and 8.063×10^{10} each contain four significant figures.

- For the number 1200, however, there is no way of knowing whether or not the two zeros are significant figures; a direct statement or an alternative way of expressing the number is needed.

- For example, 1.2×10^3 has two significant figures, while 1.200×10^3 has four.

A number is rounded to n significant figures using the following rules:

- If the number in the (n + 1)th position is less than 5, discard all figures to the right of the nth place.
- If the number in the (n + 1)th position is greater than 5, discard all figures to the right of the nth place, and increase the nth digit by 1.
- If the number in the (n + 1)th position is exactly 5, discard all figures to the right of the nth place, and increase the nth digit by 1.

For example, when rounding off to four significant figures:

1.426348 becomes 1.426;

1.426748 becomes 1.427;

1.4265 becomes 1.427.

- It is good practice during calculations to carry along one or two extra significant figures for combination during arithmetic operations; final rounding-off should be done only at the end.
- After **multiplication or division**, the number of significant figures in the result should equal the **smallest number of significant figures** of any of the quantities involved in the calculation. For example:

$$(6.681 \times 10^{-2})(5.4 \times 10^{9}) = 3.608 \times 10^{8} \rightarrow 3.6 \times 10^{8}$$

2 SF

$$\frac{6.16}{0.054677} = 112.6616310 \rightarrow 113.$$

3 SF

- For **addition and subtraction**, look at the position of the last significant figure in each number relative to the decimal point. The position of the last significant figure in the result should be the **same as that most to the left**, as illustrated below:

$$24.335 + 3.90 + 0.00987 = 28.24487 \rightarrow 28.24$$

2 SF relative to the decimal point

$$121.808 - 112.87634 = 8.93166 \rightarrow 8.932.$$

3 SF relative to the decimal point

Mean, Variance, and Standard Deviation

Mean:

Technically, the mean (denoted μ), can be viewed as the most common value (the *outcome*) you would expect from a measurement (the *event*) performed repeatedly. It has the same units as each individual measurement value. For variable x measured n times, the *arithmetic mean* is calculated as follows:

Sum of n values

$$\bar{x} = \text{mean value of } x = \frac{\sum^{n} x}{n} = \frac{x_1 + x_2 + x_3 + \ldots x_n}{n}$$

Standard deviation:

The standard deviation (denoted σ) also provides a measure of the spread of repeated measurements either side of the mean. An advantage of the standard deviation over the variance is that its units are the same as those of the measurement. The standard deviation also allows you to determine how many significant figures are appropriate when reporting a mean value. Standard deviation σ is calculated as follows:

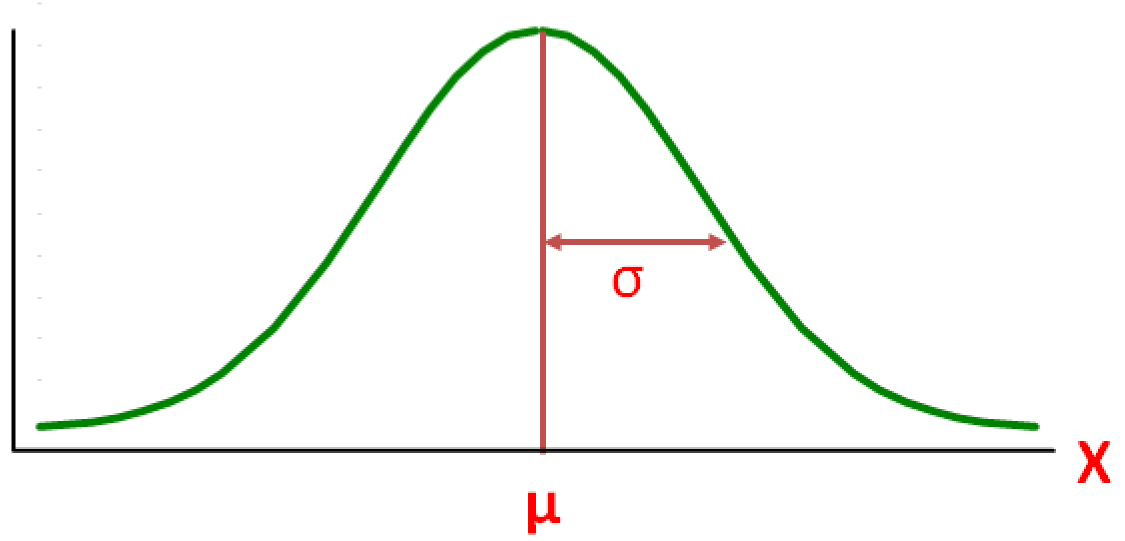
$$\sigma = \sqrt{\frac{\sum^{n}(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2 + \ldots (x_n-\bar{x})^2}{n-1}}.$$

Variance:

The variance (denoted σ^2) represents the spread (the *dispersion*) of the repeated measurements either side of the mean. As the notation implies, the units of the variance are the square of the units of the mean value. The greater the variance, the greater the probability that any given measurement will have a value noticeably different from the mean.

The Population Mean and Standard Deviation



Exercise

- Compute the mean, standard deviation, and variance for the following data:
  - 1 2 3 3 4 8 10

Mean = $\frac{\sum^{n} x}{n}$

= $\frac{1+2+3+3+4+8+10}{7}$

= 4.428571

Standard deviation = $\sqrt{\frac{\sum^{n}(x-\bar{x})^2}{n-1}}$

= $\sqrt{\frac{(1-4.429571)^2+(2-4.429571)^2+(3-4.429571)^2+(3-4.429571)^2+(4-4.429571)^2+(8-4.429571)^2+(10-4.429571)^2}{7-1}}$

= 3.309438

Variance = σ^2

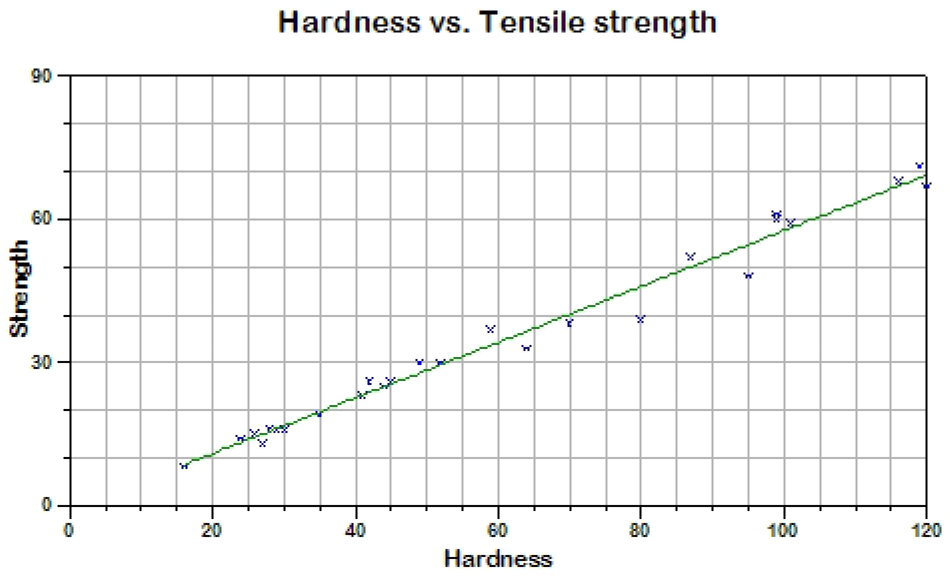
= $(3.309438)^2$

= 10.95238

Goodness of Fit: Least-Squares Analysis

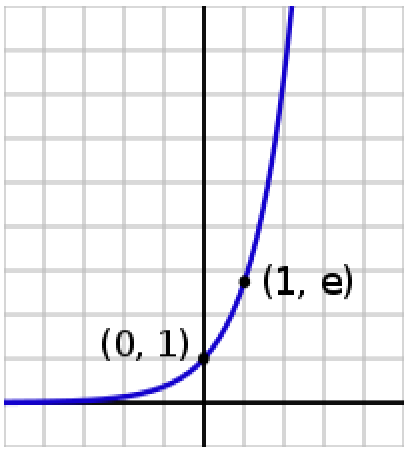
- A popular technique for locating the line or curve which minimises the residuals is *least-squares analysis.*

Fitting a Straight Line

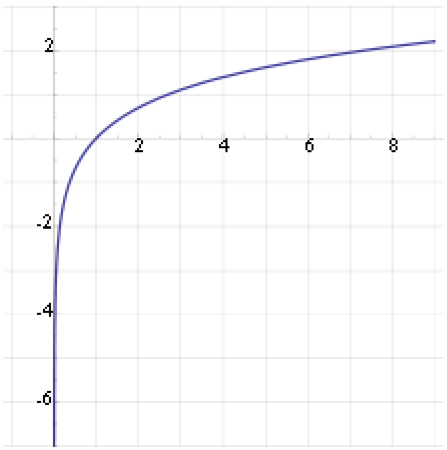


- Regression line:
  - Y = mx+b

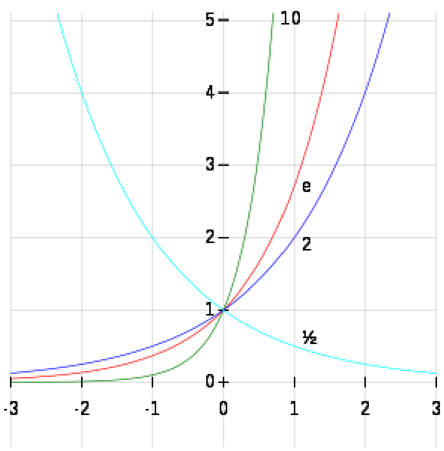
Fitting Nonlinear Line



$y = e^x$

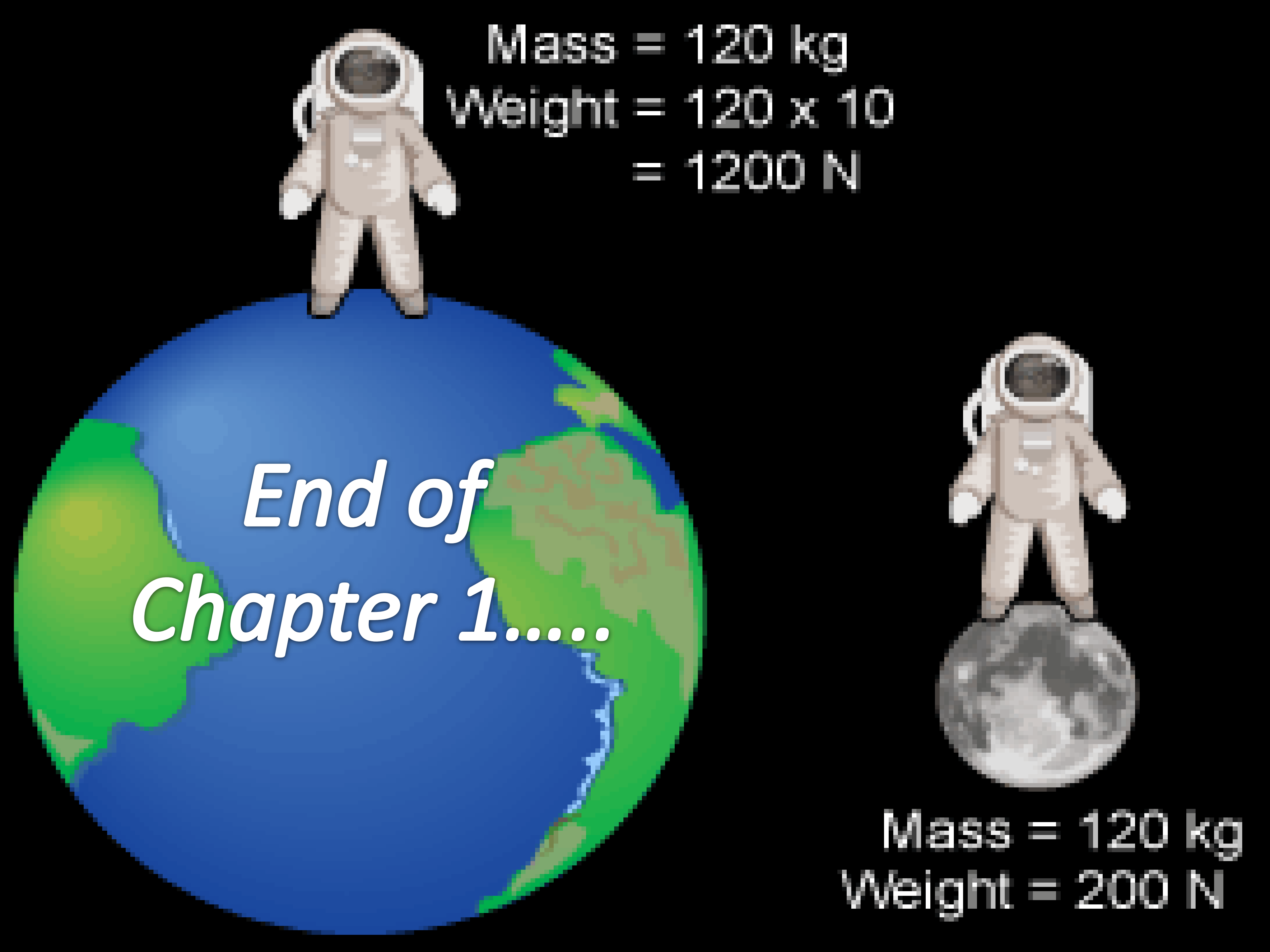


$y = C \log (x)$



$y = b^x$

base 10 (green),base *e* (red), base 2 (blue), and base ½ (cyan)


Mass = 120 kg
Weight = 120 x 10
= 1200 N
End of Chapter 1.....
Mass = 120 kg
Weight = 200 N

EXERCISES

1.Using the table of conversion factors, convert:

a)760miles/h to m/s

b)921 kg/m^3 to lb_m/ft^3

c)5.37X103 kJ/min to hp

2. Calculate

a)The weight in lbf of a 25.0-lbm object

b)The mass in kg of an object that weighs 25 newtons