

# الجامعة التكنولوجية

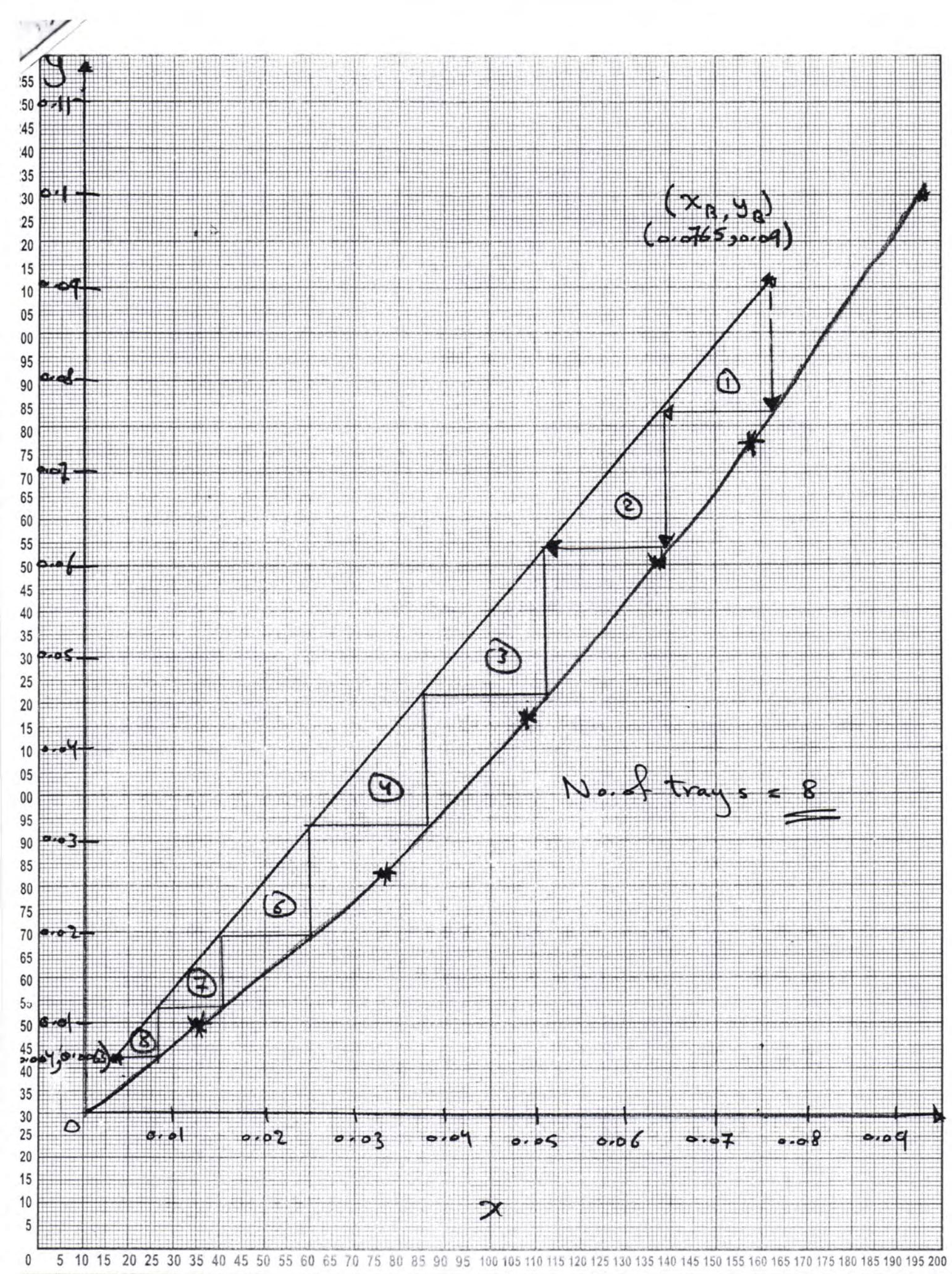
قسم الهندسة الكيميائية

المرحلة الثالثة

انتقال كتلة

أ.م.د. عامر عزيز





(b) to solve analytically :-

$$N = \frac{\log \left\{ \left[ \frac{y_B - \alpha x_T}{y_T - \alpha x_T} \right] \left[ 1 - \frac{1}{A} \right] + \frac{1}{A} \right\}}{\log A}$$

$$A = \frac{L}{\alpha G} \quad \rightarrow \alpha = \text{slope} \approx 1.1$$

$$A = \frac{150}{1.1 * 130} = 1.049$$

$$N = \frac{\log \left[ \frac{0.09 - 1.1 * 0.004}{0.0063 - 1.1 * 0.004} \right] \left[ 1 - \frac{1}{1.049} \right] + \frac{1}{1.049}}{\log 1.049}$$

$$N = \frac{\log 3.2}{\log 1.049} \rightarrow N \approx 25$$

$\therefore \text{real No. of trays} = \frac{25}{0.45} \approx 56$

(c) Using Kremser - Chart

$$\bar{A} \approx 1.05$$

$$\frac{y_T - \alpha x_T}{y_B - \alpha x_T} = \frac{0.0063 - 1.1 * 0.004}{0.09 - 1.1 * 0.004} = \frac{1.9 * 10^{-3}}{8.5 * 10^{-2}}$$

$$= 0.02$$

From chart find (N) from (1.05) and (0.02)

$$N \approx 25$$

$$\text{Real No. of trays} = \frac{25}{0.45} \approx 56$$

Ex.(2) :- It is required to remove 99% of the solute (C) from a solution of (C) in gas (G) by using a pure solvent liquid (L) in a counter-current Cascade. The feed containing 8% C in the mixture enters the column at the bottom at a rate of 5500 kg/hr. The solvent enters at the top at a rate of 7685 kg/hr.

- Write down the equation of the operating line.
- Determine the number of trays required to perform the separation if the overall tray efficiency 40%.
- The equil. relation  $y = 1.32x$ .

Sol. Feed rate (mixture) = 5500 kg/hr  
 99% to be removed  $\therefore$  remaining (0.01)

$$y_B = 0.08$$

$$y_T = 0.08 * 0.01 = 0.0008$$

$$x_T = 0 \text{ (Free Solvent)}$$

Now making over all M.B to find ( $x_B$ )

$$G(y_B - y_T) = L(x_B - x_T)$$

$$5500(0.08 - 0.0008) = 7685(x_B - 0)$$

$$x_B = 0.056$$

To find operating line

$$G(y - y_T) = L(x - x_T)$$

$$5500(y - 0.0008) = 768 s(x - 0)$$

$$y = 1.39 X + 0.0008 \quad \text{operating line}$$

We have two points

$$(0, 0.0008) \text{ and } (0.056, 0.08)$$

\* To find No. of plates using all previous methods

① Graphical method :-

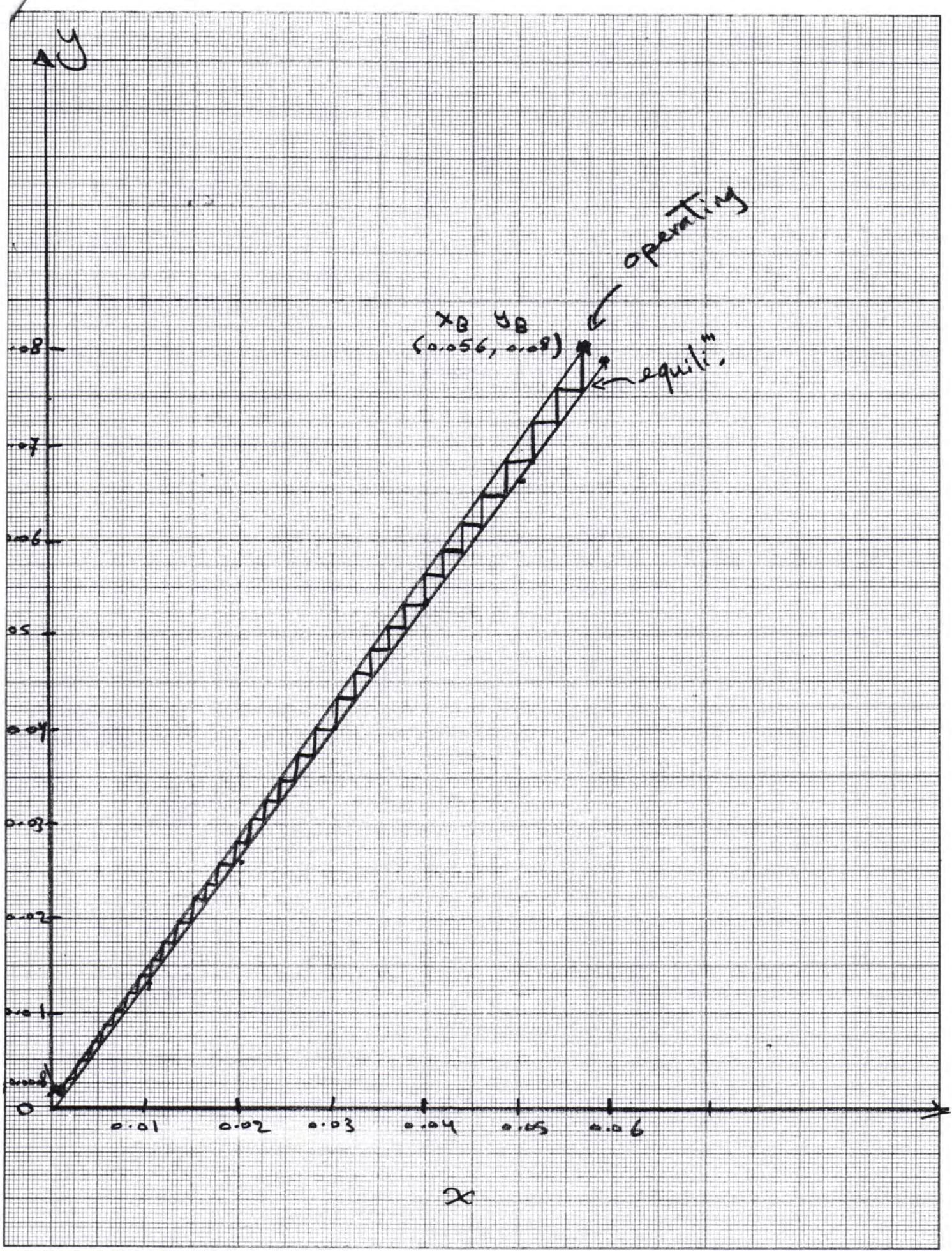
- a - Draw equil<sup>"</sup> curve using  $y = 1.32 X$
- (y) various from (0.0008 to 0.08)
- (X) various from (0, to 0.056)

b - Draw operating line according to relation found

$$\text{Total No. of trays} = 33$$

$$\text{Real No. of trays} = \frac{33}{0.4}$$

$$\approx 83$$



5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145 150 155 160 165 170 175 180 185 190 195 200

② Analytical method :-

$$N = \frac{\log \left[ \frac{y_B - \alpha x_T}{y_T - \alpha x_T} \left( 1 - \frac{1}{A^-} \right) + \left( \frac{1}{A^-} \right) \right]}{\log A^-}$$

$$y_B = 0.08, y_T = 0.0008, x_T = 0, \alpha = 1.32$$

$$A^- = \frac{L}{\alpha \cdot G} = \frac{7685}{(1.32) * 5500} \Rightarrow \bar{A} = 1.058$$

using Kremser eq. for  $\bar{A} \neq 1$

$$N = \frac{\log \left[ \left( \frac{0.08 - 0}{0.0008 - 0} \right) \left( 1 - \frac{1}{1.058} \right) + \frac{1}{1.058} \right]}{\log 1.058}$$

$$N \approx 34$$

$$\text{No. of real plates} = \frac{34}{0.4} = 85$$

③ Using Kremser Chart

Calculate the term for y-axis for absorption

$$\frac{y_T - \alpha x_T}{y_B - \alpha x_T} = \frac{0.0008 - 0}{0.08 - 0} = 0.01$$

$$\text{For } \bar{A} \approx 1.06$$

$$N \approx 35, \text{ No. of real plates} = \frac{35}{0.4} = 88$$

Q<sub>3</sub> :- A sieve tray tower is being designed for a gas absorption process. The entering gas contains 1.8% (molar) of (A) which must be absorbed. The gas should leave the tower containing no more than 0.1% (molar) of (A). The liquid to be used as absorbent initially contains 0.01% of (A). The system obeys Henry's law with  $m = 1.41$ . At the bottom of the tower, the molar liquid-to-gas ratio = 2.115, and at the other extreme is (2.326). For these conditions, it has been seen that  $E_{MGE} = 0.65$ . Find No. of trays?

Sol. :-

$$A_1 = \frac{2.115}{1.41} = 1.5 \quad , \quad A_2 = \frac{2.326}{1.41} = 1.65$$

$$\text{then } A = \sqrt{1.5 * 1.65} = 1.57$$

$$y_B = 0.018 \quad , \quad y_T = 0.001 \quad , \quad x_T = 0.0001$$

analytically :-

$$N = \frac{\log \left[ \frac{y_B - mx_T}{y_T - mx_T} \left( 1 - \frac{1}{A} \right) + \frac{1}{A} \right]}{\log A}$$

$$\therefore N_{\text{theo.}} = 4.64$$

$$E_D = \frac{\ln \left[ 1 + E_{MGE} \left( \frac{1}{A} - 1 \right) \right]}{\ln (1/A)} = 0.596$$

$$\text{and } N_{\text{act.}} = \frac{4.64}{0.596} = 7.79 \approx 8 \text{ tray.}$$

## Packed column

Absorption is frequently conducted in packed column, particularly when:

- (1) The required column diameter is less than 2 ft.
- (2) The pressure drop must be low, as for a vacuum service.
- (3) Corrosion consideration favors the use of ceramic or polymeric material.
- (4) Low liquid holdup is desirable.

The countercurrent packed towers operates in a different manner from plated towers. In packed towers the fluids are in contact continuously in their path through the tower, while in plated towers the fluids are contacted occasionally. So, packed columns are continuous differential contacting devices that do not have the physically distinguishable stages found in tray towers. Thus, packed columns are best analyzed by mass transfer considerations rather than by the equilibrium – stage concept. Nevertheless, in practice packed – towers performance is often analyzed on the basis of equivalent equilibrium stages using a packed height equivalent to a theoretical (equilibrium) plates (stage), called the HETP.

The most important reason to use packed towers in absorption is to provide a large contact area as possible as can. There are many types of packing such as, Rasching rings, Lessing rings, Bert Saddles, Pall ring, and many others, most of these types of packing are made of cheap inert materials such as glass, ceramic, graphite or, plastic, but sometimes it may be made of stainless –steal.

## Calculations of the height of packing (Z)

### For dilute mixtures :

Consider mass transfer occur in the column of cross sectional area equal to ( S ) ( m<sup>2</sup> ), containing packing of specific surface area equal to ( a ), ( m<sup>2</sup> / m<sup>3</sup> ).

Specific surface area = total surface area / volume of the column

Total surface area = surface area of one pack \* number of packing

Volume of column = S \* dZ

Total mass transfer area in height dZ (Interfacial area for transfer) = a . S . dZ

The total no. of moles of (A) transfer per unit area per unit time = N<sub>A</sub>

Total moles of (A) transfer / time = N<sub>A</sub> \* a \* S \* dZ

According to Whitman two-film theory:-

$$N_A = K_{OG} (P_A - P_A^*) \quad \dots\dots\dots(1)$$

$$N_A * a * S * dZ = K_{OG} (P_A - P_A^*) * a * S * dZ \quad \dots\dots\dots(2)$$

The equation is based on gas phase and is used in calculating the height of the packing (Z) are applied only for dilute or weak solution which leads to the assumption that L<sub>m</sub> & G<sub>m</sub> are constant through out the solution. Don't forget that the solutions can be considered dilute if the mole fraction of the solute in the inlet streams (gas & liquid) are less than 0.1 (i.e.  $\leq 10\%$ ). The change in gas composition of (A) through the height (dZ) equal to (dy<sub>A</sub>).

Total no. of moles of (A) transfer / time =  $N_A * a * S * dZ$

$$N_A * a * S * dZ = G_m * dy_A \quad \dots \dots \dots (3)$$

$$G_m * dy_A = K_{OG} (P_A - P_A^*) * a * S * dZ \quad \dots \dots \dots (4)$$

Divide the right hand side of the above equation by  $(P_T / P_T)$ :-

$$G_m * dy_A = K_{OG} (y_A - y_A^*) P_T * a * S * dZ \quad \dots \dots \dots (5)$$

$$\int_0^Z dZ = \left( \frac{G_m}{K_{OG} * P_T * a * S} \right) \int_{Y_{AT}}^{Y_{AB}} \frac{dy_A}{(y_A - y_A^*)}$$

Integration eq.:-

$$Z = \left( \frac{G_m}{K_{OG} * P_T * a * S} \right) * \int_{Y_{AT}}^{Y_{AB}} \frac{dy_A}{(y_A - y_A^*)} \quad \dots \dots \dots (6)$$

$Z = (HTU)_{OG} * (NTU)_{OG}$  [Height of packing for dilute solution for gas phase]

Where:- ( HTU ) = height of transfer unit.

(NTU) = number of transfer unit.

$K_{OG}$  = is constant depends on the physical properties of gas and hydrodynamic properties of fluid.

$G_m$  = constant for dilute solution =  $\left( \frac{G_{top} + G_{Bott.}}{2} \right)$ .

$P_T$  = constant, no friction losses.

$K_{OG}, K_{OL}$  = volumetric overall mass transfer coefficient ( Kmole/ sec. mole fraction.  $m^3$  packing).

H.W :- Derive the (Z) equation for liquid phases:-

$Z = (HTU)_{OL} * (NTU)_{OL}$  [Height of packing for dilute solution for liquid phase].

## Summary:-

### For dilute solutions:-

#### For gases ( Overall Mass Transfer):-

$$Z = (HTU)_{OG} * (NTU)_{OG} \quad [\text{Height of packing for dilute solution for gas phase}]$$

$$(HTU)_{OG} = \left( \frac{G_m}{K_{OG} * P_T * a * S} \right)$$

$$(NTU)_{OG} = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_A^*)}$$

#### For liquids (Overall Mass Transfer):-

$$Z = (HTU)_{OL} * (NTU)_{OL} \quad [\text{Height of packing for dilute solution for liquid phase}]$$

$$(HTU)_{OL} = \left( \frac{L_m}{K_{OL} * C_T * a * S} \right)$$

$$(NTU)_{OL} = \int_{X_{AT}}^{X_{AB}} \frac{dX_A}{(X_A^* - X_A)}$$

Where the integration of  $(NTU)_{OG} = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_A^*)}$  is :-

$$(NTU)_{OG} = \left( \frac{y_B - y_T}{(y - y^*)_{Lm}} \right)$$

$$(y - y^*)_{Lm} = \frac{(y_B - y_{B^*}) - (y_T - y_{T^*})}{\ln \frac{(y_B - y_{B^*})}{(y_T - y_{T^*})}}$$

### For gases (Individual Mass Transfer):-

$Z = (HTU)_G * (NTU)_G$  [Height of packing for dilute solution for gas phase]

$$(HTU)_G = \left( \frac{G_m}{K_G * P_T * a * S} \right)$$

$$(NTU)_G = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_{Ai})}$$

### For liquids (Individual Mass Transfer):-

$Z = (HTU)_L * (NTU)_L$  [Height of packing for dilute solution for liquid phase]

$$(HTU)_L = \left( \frac{L_m}{K_L * C_T * a * S} \right)$$

$$(NTU)_L = \int_{X_{AT}}^{X_{AB}} \frac{dX_A}{(X_{Ai} - X_A)}$$

The integration of  $(NTU)_G = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_{Ai})}$  is :-

$$(NTU)_G = \left( \frac{y_B - y_T}{(y - y_i)_{Lm}} \right) \text{ where } (y - y_i)_{Lm} = \frac{(y_B - y_{Bi}) - (y_T - y_{Ti})}{\ln \frac{(y_B - y_{Bi})}{(y_T - y_{Ti})}}$$

Notice: 1)  $\frac{L_m}{G_m}$  slope of operating line =  $\frac{(y_B - y_T)}{(x_B - x_T)}$  from ( M. B. ).

2)  $m$  = slope of equilibrium line =  $\frac{(y_B^* - y_T^*)}{(x_B - x_T)}$  from ( equilibrium data ).

## Relation between overall and individual mass transfer coefficient

$$\frac{1}{K_{OG}} = \frac{1}{K_G} + \frac{H}{K_L} \quad \dots\dots\dots(1) \quad (\text{in gas phase})$$

$$\frac{1}{K_{OL}} = \frac{1}{K_G} + \frac{1}{H \cdot K_L} \quad \dots\dots\dots(2) \quad (\text{in liquid phase})$$

Multiply equation (1) by  $(G_m / a \cdot s \cdot P_T)$ :-

$$\frac{G_m}{K_{OG} \cdot a \cdot s \cdot P_T} = \frac{G_m}{K_G \cdot a \cdot s \cdot P_T} + \frac{H \cdot G_m}{K_L \cdot a \cdot s \cdot P_T} \quad \dots\dots\dots(3)$$

$$(HTU)_{OG} = (HTU)_G + \left[ \frac{H \cdot G_m}{K_L \cdot a \cdot s \cdot P_T} \right] * \frac{Lm}{Lm} \frac{C_T}{C_T}$$

$$(HTU)_{OG} = (HTU)_G + (HTU)_L \frac{G_m}{Lm} * \frac{H \cdot C_T}{P_T} \quad \dots\dots\dots(4)$$

$$\therefore P_A = H \cdot C_A \rightarrow \frac{P_A}{P_T} = \frac{H \cdot C_A}{P_T} \rightarrow \therefore y_A = \frac{H \cdot C_T}{P_T} \cdot X_A$$

$\therefore y_A = m \cdot X_A \quad \dots\dots\dots$  straight line equation

$$(HTU)_{OG} = (HTU)_G + (HTU)_L \cdot m \cdot \frac{G_m}{Lm} \quad \dots\dots\dots(5) \quad (\text{For gas phase})$$

Same steps can be applied to equation (2) for liquid phase to get :

$$(HTU)_{OL} = (HTU)_L + (HTU)_G \cdot \frac{Lm}{m \cdot G_m} \quad \dots\dots\dots(6) \quad (\text{For liquid phase})$$

Where  $\left(\frac{Lm}{m \cdot G_m}\right)$  is the absorption factor (A).

## How to find the intermediate concentration

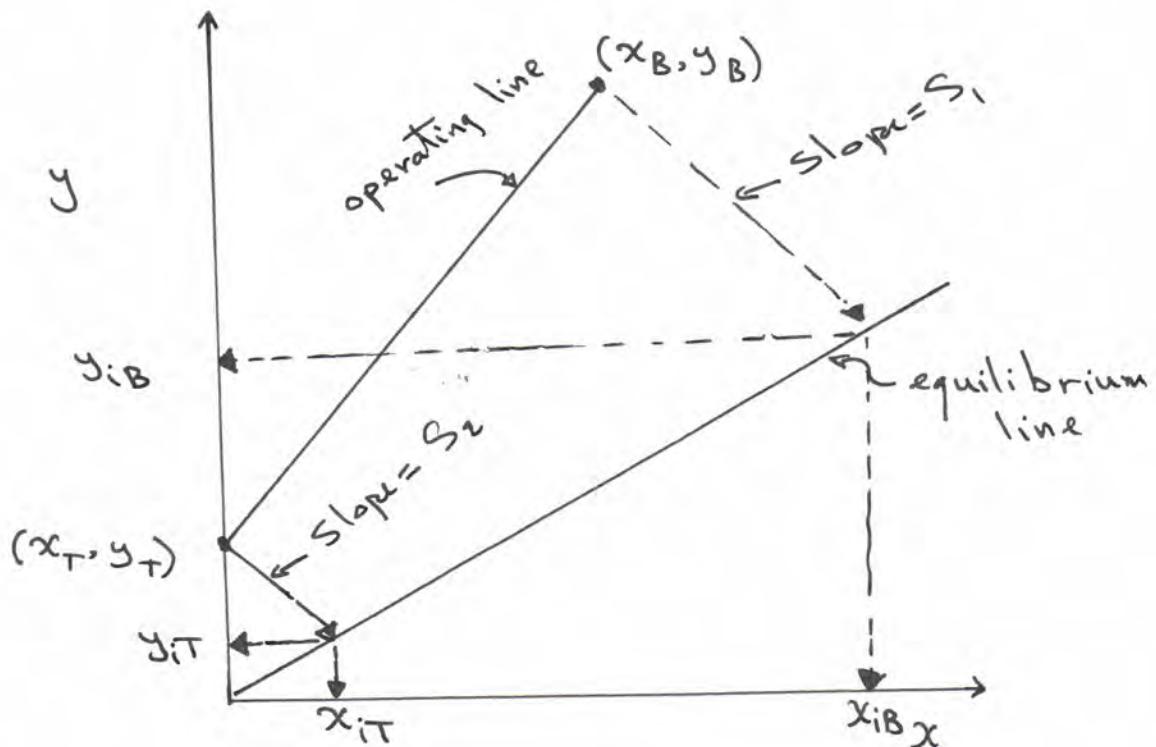
To find  $(x_{iT}, y_{iT})$  and  $(x_{iB}, y_{iB})$ , we must follow these steps:-

- 1- Plot the equilibrium data.
- 2- Plot p ( $x_T, y_T$ ) and q ( $x_B, y_B$ ).
- 3- Plot the line (pq), which is represent the operating line.
- 4- Calculate the slopes which are:

$$S_1 = \frac{-K_x \cdot a / (1 - x_B)}{K_y \cdot a / (1 - y_B)}$$

$$S_2 = \frac{-K_x \cdot a / (1 - x_T)}{K_y \cdot a / (1 - y_T)}$$

- 5- From point p ( $x_T, y_T$ ) and slope  $S_2$ , intersect the equilibrium line which give  $(x_{iT}, y_{iT})$  and from point q ( $x_B, y_B$ ) and slope  $S_1$ , intersect the equilibrium line which give  $(x_{iB}, y_{iB})$ .



## Height Equivalent To A Theoretical Plate ( H. E. T. P )

( H. E. T. P ) : packed height required to achieve the same separation as on the ( n th ) tray.

( H. E. T. P ) is often used to characterize the performance of a packing.

A good packing has a small ( H. E. T. P ).

( H. E. T. P ) range from ( 1 – 3 ).

$$( \text{H. E. T. P} ) = ( \text{HTU}_{\text{OG}} ) * [ \frac{\ln ( m \cdot \frac{\text{Gm}}{\text{Lm}} )}{( m \cdot \frac{\text{Gm}}{\text{Lm}} ) - 1 } ]$$

$$( \text{H. E. T. P} ) * N = Z \quad \rightarrow \quad N = Z / ( \text{H. E. T. P} )$$

( H. E. T. P ) depends on :-

1- Type and size of packing.

2- Gas and liquid flow rates.

3- Transport properties.

4- Equilibrium relation.

5-Uniformity of liquid and gas distribution.

## Minimum Liquid Flow Rate In Adsorption Column

Reducing liquid flow rate  $\rightarrow$  reducing slope ( $L/G$ ) of operating line.

This mean ( $x_B$ ) increase  $\rightarrow$  tall column  $\rightarrow (\infty)$ .

(less driving force, absorption more difficult).

If liquid flow reduced until it reach equilibrium line at ( $R$ ), this give minimum liquid flow and driving force = zero.

So ( $P_T R$ ) give minimum slope or  $(L/G)_{min}$ .

$$(L/G)_{op} = (1.1 - 1.5) * (L/G)_{min}$$

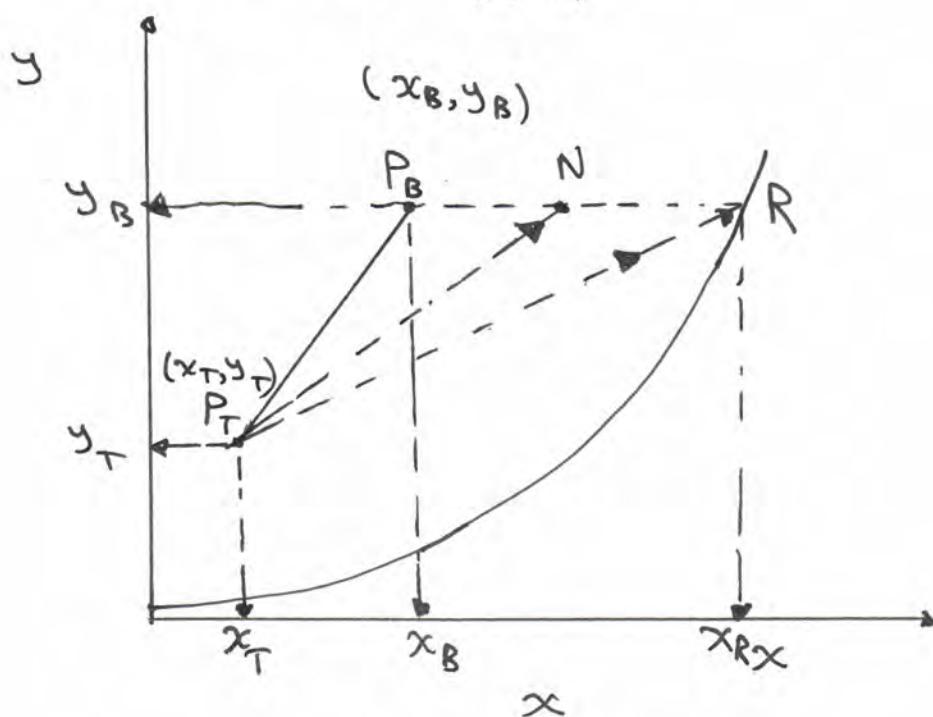
### Steps of calculations:-

1- Find ( $x_B^*$ ) from  $y_B = m \cdot x_B^*$ .

2- Find  $(L/G)_{min} = \frac{(y_B - y_T)}{(x_B^* - x_T)}$ .

3- Find  $(L/G)_{op} = (1.1 - 1.5) * (L/G)_{min}$ .

4- Finally find ( $x_B$ ) from  $(L/G)_{op} = \frac{(y_B - y_T)}{(x_B - x_T)}$ .



Ex.(1) :- Acetone is being absorbed by water in a packed tower having cross-sectional area ( $0.18 \text{ m}^2$ ) at (S.C). The inlet air contains 2.6 mol% acetone and outlet 0.5 mol%. The gas flowrate 13.65 kmol inner air/hr. The pure water inlet flow is 45.36 kmol/hr. film coefficient for the given flow in the tower are:

$$k_y \cdot a = 3.78 \times 10^{-2} \text{ kmol/m.s. mol fraction and}$$

$$k_x \cdot a = 6.16 \times 10^{-2} \text{ s s s s}$$

The equilibrium line equation  $y = 1.186 X$

1 - Calculate the tower height using  $k_y \cdot a$  &  $k_x \cdot a$

2 -  $s s , s s K_{OG} \cdot a$

Sol. Overall M.B. to find ( $X_B$ ) :-

$$L \cdot x_T + G \cdot y_B = L \cdot x_B + G \cdot y_T$$

$$45.65 \times 0 + 13.65 \times 0.026 = 45.65 \times X_B + 13.65 \times 0.05$$

$$0.355 = 45.65 X_B + 0.0683$$

$$X_B = 0.0063$$

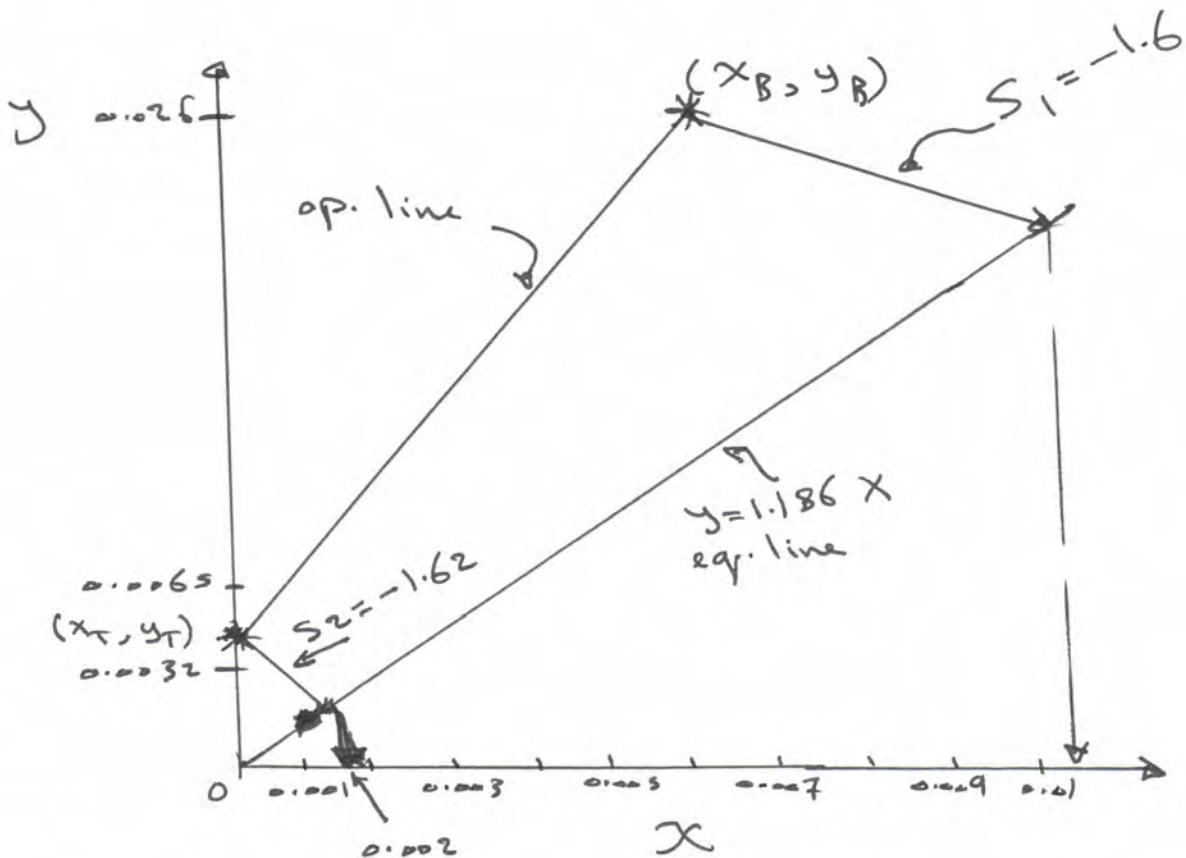
$$\textcircled{1} Z = (HTU)_G * (NTU)_G$$

$$(HTU)_G = \frac{G_m}{k_y \cdot a \cdot s} = \frac{0.0038}{3.78 \times 10^{-2} \times 0.186} = 0.54 \text{ m}$$

$$(NTU)_G = \frac{y_B - y_T}{(y - y_i) L_m}$$

$$(y - y_i)_{L_m} = \frac{\Delta y_B - \Delta y_T}{\ln \frac{\Delta y_B}{\Delta y_T}} = \frac{(y_B - y_{iB}) - (y_T - y_{iT})}{\ln \frac{y_B - y_{iB}}{y_T - y_{iT}}}$$

To find ( $y_{iB}$  &  $y_{iT}$ )



To find  $s_1$  from  $(x_B, y_B)$  &  $s_2$  from  $(x_T, y_T)$

$$s_1 = \frac{-k_x \cdot a / (1 - x_B)}{k_y \cdot a / (1 - y_B)} = -1.6$$

$$s_2 = \frac{-k_x \cdot a / (1 - x_T)}{k_y \cdot a / (1 - y_T)} = -1.62$$

- Intercept of ( $S_1$ ) with E.L. gives :-

$$x_{iB} = 0.013, y_{iB} = 0.0154$$

- Intercept of ( $S_2$ ) with E.L. gives :-

$$x_{iT} = 0.0018, y_{iT} = 0.002$$

$$\therefore (y - y_i)_{Lm} = 0.006$$

$$NTU)_G = \frac{0.026 - 0.005}{0.006} = 3.5$$

$$\therefore Z = 0.54 * 3.5 = \underline{\underline{1.91 \text{ m}}}$$

$$(2) Z = HTU)_{OG} \cdot NTU)_{OG}$$

$$HTU)_{OG} = \frac{G_m}{k_{OG} \cdot a.s. P_T}$$

$$NTU)_{OG} = \frac{y_B - y_T}{(y - y^*)_{Lm}}$$

$$(y - y^*)_{Lm} = \frac{(y_B - y_B^*) - \ell(y_T - y_T^*)}{\ell_n \frac{(y_B - y_B^*)}{(y_T - y_T^*)}}$$

$$y_B^* = 1.186 x_B \quad , \quad y_T^* = 1.186 \overrightarrow{x_T}$$

T-find ( $K_{DG}$ ):-

$$\frac{1}{K_{DG} \cdot a} = \frac{1}{k_y \cdot a} + \frac{m}{K_x \cdot a} \quad (\text{T-find } \frac{k_y \cdot a}{K_x \cdot a})$$

$$k_y \cdot a = \frac{k_y \cdot a}{y_{im}} \quad \text{and} \quad K_x \cdot a = \frac{k_x \cdot a}{x_{im}}$$

$$y_{im} = \frac{(1-y_{iB})(1-y_B)}{\ln \frac{(1-y_{iB})}{(1-y_B)}} \quad , \quad x_{im} = \frac{(1-x_B)-(1-x_{iB})}{\ln \frac{1-x_B}{1-x_{iB}}}$$

$$K_{DG} \cdot a = 2.188 * 10^{-2} \text{ kmol / s. m}^3 \cdot \text{mol. fract.}$$

$$Z = 1.99 \text{ m}$$