الجامعة التكنولوجية

قسم الهندسة الكيمياوية

المرحلة الثالثة

انتقال كتلة

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Example (3):- Water in the bottom of a narrow metal tube, is held at a constant temp. of (293 k). The total pressure of dry air is $1.0132 * 10^5$ Pa. and Temp. = 293 k. Water evaporates and diffuses through the air, the diffusion path (0.15 m) long.
Calculate the rate of evaporation at steady state in Kmol/m².s, given that $D_{AB} = 0.25 * 10^{-4}$ m²/s. and water - vap. press at 20 °C = 17.54 mm Hg.

Sol.:-
$$N_A = \frac{D_{AB} P_T}{RT} \cdot \frac{1}{Z_2 - Z_1} \frac{P_{A_1} - P_{A_2}}{P_{BM}}$$

$P_{A_1} = 17.54$ mm Hg $= 0.0234$ atm

$P_{A_2} =$ Zero (pure air and Large bulk volume)

$P_{B_1} = P_T - P_{A_1} = 1 - 0.0231 \Rightarrow P_{B_1} = 0.977$ atm

$P_{B_2} = P_T - P_{A_2} = 1 - 0 \Rightarrow P_{B_2} = 1.0$ atm

$\because P_{B_1} \simeq P_{B_2}$ $\therefore (P_{BM})$ could be taken as the mean.

$$P_{BM} = \frac{P_{B_1} + P_{B_2}}{2} = \frac{0.977 + 1}{2} = 0.9885 \text{ atm}$$

$$N_A = \frac{0.25 * 10^{-4} * 1.0132 * 10^5}{8.312 * 293} \cdot \frac{1}{0.15} \cdot \frac{\overset{0.0234}{\cancel{2.34 * 10^{-3}}}}{0.9885}$$

$N_A = 0.164 * 10^{-3}$ Kmol/m².s

Example(4) :- Ammonia (A) diffuses through a stagnant Layer of air (B) 1 cm thick at 25°C and 1 atm total press. The partial press. of (A) on the two sides of the air Layer are $P_{A_o} = 0.9$ atm and $P_{A_1} = 0.1$ atm. Air is non diffusing.

Calculate :-

a- The molar flux of NH_3.

b- molar and mass - average velocity for each species.

c- Plot the partial press. distribution of NH_3 and air along the diffusion path.

Given : $D_{AB} = 0.214 \frac{cm^2}{sec}$, $R = 82.1 \frac{cm^3 \cdot atm}{gmol \cdot K}$

Sol. :- for stagnant layer of air (B), ∴ $\boxed{N_B = 0}$

(a) $$N_A = \frac{D_{AB} \cdot P_T}{R \cdot T} \cdot \frac{1}{\Delta Z} \ln \frac{P_T - P_{A_1}}{P_T - P_{A_o}}$$

$$\because \Delta Z = Z_2 - Z_1 = 1 \text{ cm}$$

$$= \frac{(0.214)(1)}{(82.1)(298)(1.0)} \ln \frac{1-0.1}{1-0.9}$$

$$N_A = 1.922 * 10^{-5} \text{ gmol/cm}^2 \cdot s$$

(b) $$N_T = U_m \cdot C_T \Rightarrow U_m = \frac{N_T}{C_T} \Rightarrow \frac{N_A + \cancel{N_B}^{\,0}}{C_T}$$

$$\therefore U_m = \frac{N_A}{C_T} = \frac{N_A}{P_T / RT}$$

$$U_m = \frac{1.922 * 10^{-5}}{\frac{1.0}{(82.1)(298)}} = 0.47 \text{ cm/sec} \quad (\text{molar Avg. velo.})$$

$$N_A = U_A \cdot C_A \Rightarrow U_A = \frac{N_A}{C_A} = \frac{U_m C_T}{C_A} = \frac{U_m}{y_A}$$

(C_A) varies along diffusion path

$\therefore U_A$ also varies.

$$y_A = \frac{P_{Ao}}{P_T} = \frac{0.9}{1.0} = 0.9$$

$$U_{Ao} = \frac{0.47}{0.9} = 0.52 \text{ cm/s.}$$

$U_{Bo} = 0$ stationary (B)

$$\therefore \text{mass avg. velocity } (U) = \frac{U_A \rho_A + U_B \rho_o}{\rho_T}$$

$$U = \frac{U_A \rho_A}{\rho_T}$$

$$\therefore \rho_A = \frac{P_A M_{wt(A)}}{R \cdot T}, \quad \rho_T = \frac{P_T \cdot M_{wt}}{R \cdot T}$$

$$U = \frac{U_A \cdot P_A \cdot M_{wt(A)} / \cancel{RT}}{P_T \cdot M_{wt} / \cancel{RT}} \Rightarrow \frac{U_A (P_A)(M_{wt(A)})}{(P_T)(M_{wt(total)})}$$

$$U = \frac{U_A \cdot y_A \cdot M_{wt(A)}}{M_{wt(total)}}$$

$$M_{wt}(total) = M_{wt}(A) \cdot y_A + M_{wt}(B)\, y_B$$
$$= 17 * 0.9 + 29 * 0.1$$
$$= 18.2$$

$$\therefore u = \frac{(0.52)(0.9)(17)}{18.2} = 0.439 \text{ cm/sec.}$$

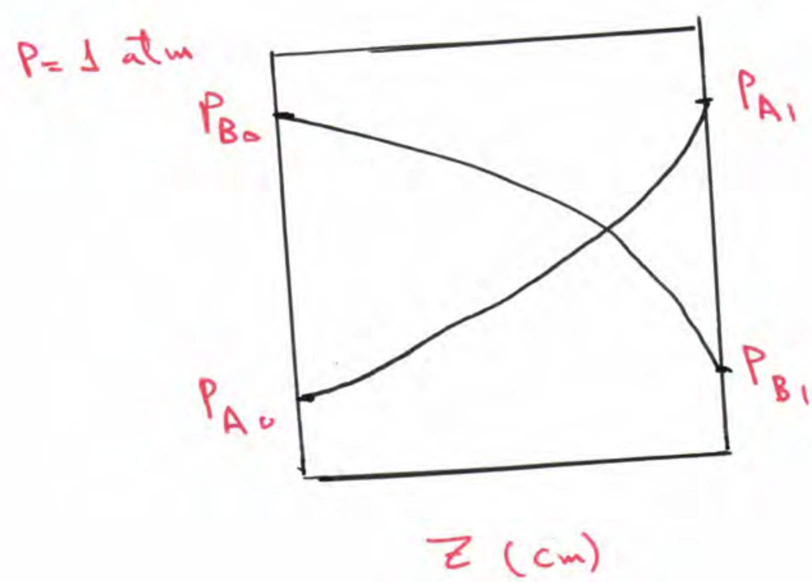
(c) To plot partial press. distrib^n :-

$$N_A = 1.922 * 10^{-5} \frac{gmol}{cm^2 \cdot Sec} = \frac{(0.214)(1.0)}{(82.1)(298} \cdot \frac{1}{Z} \ln \frac{1-P_A}{1-0.9}$$

rearrange above equ^n. to get :-

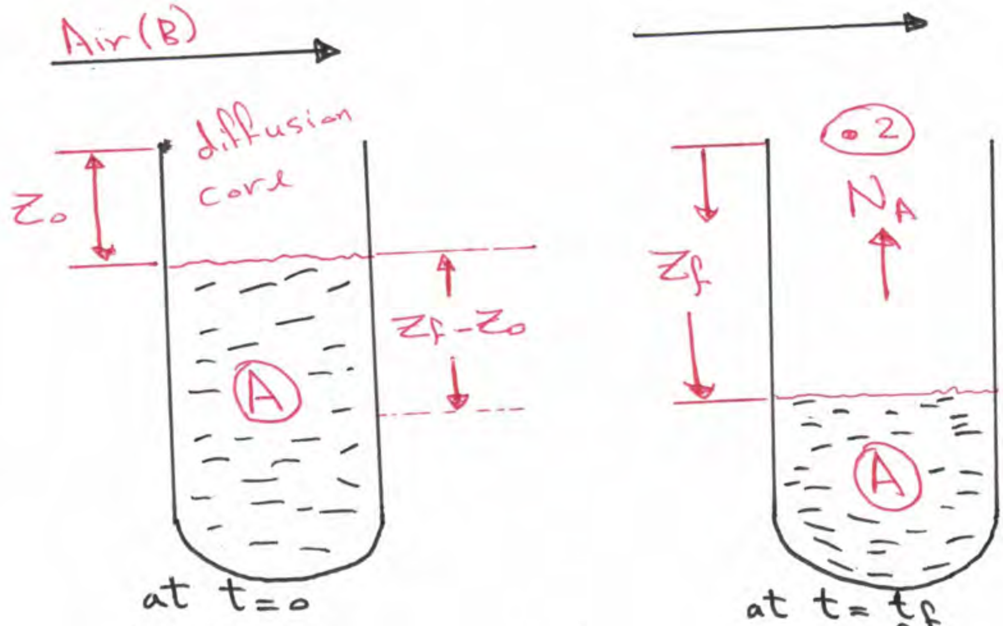
$$\boxed{P_A = 1 - (0.1) \exp(2.197 \cdot Z)}$$ for species (A)

$$\boxed{P_B = (P_T - P_A)}$$ for species (B)



④ Diffusion through a varying path length.

Ⓐ Usually used to determine the <u>time</u> required to drop the level to liquid to a certain height.



A = Acetone

B = Air

(*) At the surface , $P_{A_1} = P_A^\circ$

$X_{A_1} = P_A^\circ / P_T$

(*) At point (2) , $P_{A_2} = 0$

لأن الحسابات عند السطح، حيث أن
(A) هي فقط التي تنفذ، والـ
(B) لا تنفذ، لذلك يبقى تركيز
(A) هو (1) على السطح في
Binary فقط.

$[C_A]$ remain constant always.

For stationary non-diffusing (B) $\therefore N_B = 0$

$$N_A = \frac{-D_{AB}}{RT} \cdot \frac{P_T}{P_{BM}} \cdot \frac{P_{A_2} - P_{A_1}}{Z_2 - Z_1}$$

$\therefore$ We can find (time) from (N_A)

$$N_A = U_A \cdot C_A \Longrightarrow N_A = \frac{dZ}{dt} \cdot \frac{\rho_A}{M_A}$$

Sub. for (N_A):-

$$\frac{dZ}{dt}\,\frac{\rho_A}{M_A} = \frac{-D_{AB}}{RT}\cdot\frac{P_T}{P_{BM}}\cdot\frac{P_{A2}-P_{A1}}{Z} \qquad \text{put } Z_2 - Z_1 = Z$$

separating variables, then solve equation above:-

$$\boxed{t = \frac{\rho_A}{M_A}\left[\frac{RT}{D_{AB}}\cdot\frac{P_{BM}}{P_T\,(P_{A1}-P_{A2})}\right]\int_{Z_o}^{Z_f} Z\,.\,dZ}$$

↳ Used to determine the time required to drop the level to a certain height.

H.W.(4): Find the equation to measure time required to drop level of liquid to certain height, where $(N_A = -N_B)$. (EMD).

④ Diffusion through a varying cross-sectional area

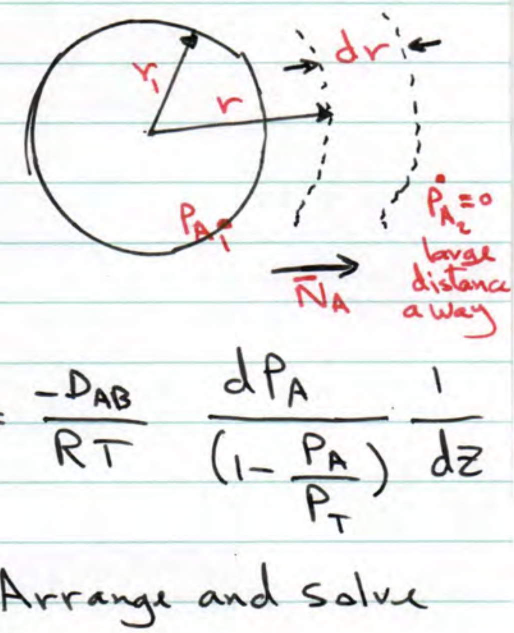
Ⓑ All cases previously mentioned, (N_A) was assumed almost constant, because area of diffusion is constant.
Now for a varying cross-sectional area, then we get:-

$$N_A = \frac{\bar{N}_A}{A}$$ where ($\bar{N}_A$) is constant at steady state
(N_A) is not constant

To study this case, two examples will be taken.
1- Diffusion from a sphere material.
2- Diffusion in Conical vessel.

1- Species (A) diffusing through stagnant (B)

$$N_A = \frac{\bar{N}_A}{A} = \frac{\bar{N}_A}{4\pi r^2}$$



$$\therefore N_A = N_A \frac{P_A}{P_T} - \frac{D_{AB}}{RT} \frac{dP_A}{dz}$$

$$N_A\left(1 - \frac{P_A}{P_T}\right) = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} \Longrightarrow N_A = \frac{-D_{AB}}{RT} \frac{dP_A}{\left(1 - \frac{P_A}{P_T}\right)} \frac{1}{dz}$$

$$\frac{\bar{N}_A}{4\pi r^2} = \frac{-D_{AB}}{RT} \frac{dP_A}{\left(1 - \frac{P_A}{P_T}\right)} \frac{1}{dr} \Longrightarrow \text{Arrange and solve}$$

$$\boxed{\frac{\bar{N}_A}{4\pi}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{D_{AB}}{RT} P_T \ln \frac{P_T - P_{A_2}}{P_T - P_{A_1}}} \quad \cdots \circledast$$

↳ used to find P_{A_2} for certain r_2

1- For $r_2 \ggg r_1 \Rightarrow \frac{1}{r_2} \simeq 0$

and dividing by r_1, then :-

$$\boxed{\frac{\bar{N}_A}{4\pi r_1^2} = N_{A_1} = \frac{D_{AB}}{R \cdot T \cdot r_1} P_T \frac{P_{A_1} - P_{A2}}{P_{BM}}}$$

General form for sphere only

2- For $P_{A_1} \lll P_T$ then $\boxed{P_{BM} = P_1}$ (dilute solution)

$$\frac{\bar{N}_A}{4\pi r_1^{\cancel{2}\,1}} = \frac{D_{AB}}{R \cdot T \cdot \cancel{r_1}} \cancel{P_T} \frac{P_{A_1} - P_{A2}}{\cancel{P_T}}$$

and $D_1 = 2r_1$, then :-

$$\bar{N}_A = 2\pi \cdot D_1 \cdot D_{AB} \left(C_{A_1} - C_{A_2} \right)$$

$$\boxed{\therefore N_A = \frac{2 D_{AB}}{D_1} \left(C_{A_1} - C_{A2} \right)}$$

Can be used for gas and liquids

$$N_{A_1} = \frac{D_{AB}}{R \cdot T \cdot r_1} \left(P_{A_1} - P_{A_2} \right) \quad \text{Gases}$$

$$N_{A_1} = \frac{D_{AB}}{r_1} \left(C_{A_1} - C_{A2} \right) \quad \text{liquids}$$

Ex(1):- A sphere of Naphthalene having radius of 2.0 mm, is suspended in a large volume of still air at 318°K and $1.013 * 10^5$ pas. The surface temp. of the sphere can be assumed to be at 318°K, and vapour pressure is 0.55 mmHg. D_{AB} for naphthalene in air at 318°K is $6.92 * 10^{-6}$ m²/sec. Calculate the rate of evaporation of naphthalene at the surface? given that $R = 8314$ m³.Pa / Kgmol.°K.

Sol. :- $N_{A_1} = \frac{D_{AB}}{R.T.r_1} \cdot P_T \frac{P_{A_1} - P_{A_2}}{P_{BM}}$

$P_{A_1} = \frac{0.55 \text{ mmHg}}{760} * 1.0132 * 10^5 = 73.32$ Pas.

Since $P_{A_1} << P_T$ $\therefore P_{BM} = P_T$

$N_{A_1} = \frac{D_{AB}}{R.T.r_1} P_{A_1}$ where $P_{A_2} = 0$ [large distance]

$$N_{A_1} = \frac{6.92 * 10^{-6} * 73.32}{8314 * 318 * 2 * 10^{-3}} = \frac{507.37 * 10^{-6}}{5287.7}$$

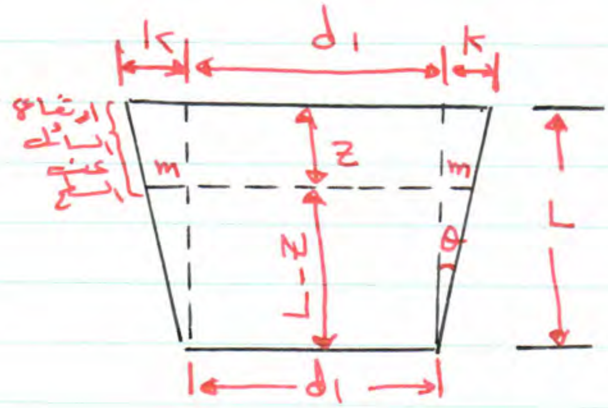
$N_{A_1} = 9.6 * 10^{-8}$ kgmol / m². Sec.

Note:- if it was required to find the partial pressure after a certain distance, then we can use eq. (*) to find (P_{A_2}) at (r_2)

$$\boxed{\frac{\bar{N}_A}{4\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{D_{AB}}{R.T} P_T \ln \frac{P_T - P_{A_2}}{P_T - P_{A_1}}}$$

② Diffusion in Conical vessel

For non-diffusing gas (B)
we have find :-



$$N_A = x_A N_A - C_T D_{AB} \frac{dx_A}{dZ}$$

$$N_A = \frac{-D_{AB}}{R.T} \frac{P_T}{P_T - P_A} \cdot \frac{dP_A}{dZ}$$

$$(N_A) = \frac{-D_{AB}}{R.T} \frac{P_T}{P_T - P_A} \frac{dP_A}{dZ}$$

(N_A) now is not constant, but varies with (Z), then we have to find a relation between them

$$\therefore N_A = \frac{\bar{N}_A}{A} \Rightarrow \frac{\bar{N}_A}{A} = \frac{-D_{AB}}{R.T} \frac{P_T}{P_T - P_A} \frac{dP_A}{dZ}$$

$$\bar{N}_A \int_{Z_1}^{Z_2} \frac{dP_A}{A} = \frac{-D_{AB} \cdot P_T}{R.T} \int_{P_{A_1}}^{P_{A_2}} \frac{dP_A}{P_T - P_A}$$

$\therefore A = \frac{\pi}{4} d^2$ (a relation between (Z) and (d))

$$\tan\theta = \frac{K}{L} = \frac{m}{L-Z} \qquad \therefore \boxed{m = \frac{K(L-Z)}{L}}$$

Total diameter at heigh $(Z) = d$

$d = d_1 + 2m \longrightarrow$ * لأن بنشتغل على مقطع السائل وليس مقطع الوعاء

$$d = d_1 + \frac{2K(L-Z)}{L}$$

$$\therefore \bar{N}_A \int_{Z_1}^{Z_2} \frac{dZ}{\frac{\pi}{4}\left(d_1 + \frac{2K(L-Z)}{L}\right)^2} = \frac{-D_{AB} \cdot P_T}{R \cdot T} \int_{P_{A_1}}^{P_{A_2}} \frac{dP_A}{P_T - P_A}$$

Ex.(2) :- An open conical vessel is filled with water up to 0.5 cm from top. Calculate the time required to drop level by (6 cm), given that $D_{AB} = 0.256\ cm^2/s$, at 1 atm and 25°C. The vapor pressure of water at 25°C is 0.0313 atm. Figure shown below

Sol. :-

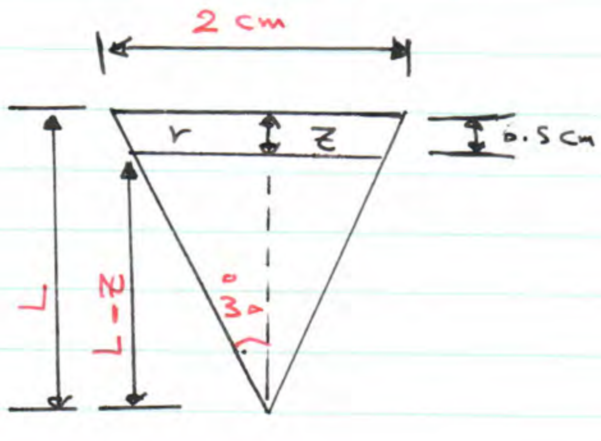
$$\bar{N}_A \int_{Z_1}^{Z_2} \frac{dZ}{A} = \frac{-D_{AB} \cdot P_T}{R \cdot T} \int_{P_{A_1}}^{P_{A_2}} \frac{dP_A}{P_T - P_A}$$

we must find relation between (d) and (Z)

$$\tan\theta = \frac{1}{\sqrt{3}} = \frac{1\ cm}{L} = \frac{r}{L-Z}$$

$$\therefore L = \sqrt{3}$$

$$\frac{1}{\sqrt{3}} = \frac{r}{\sqrt{3} - Z}$$



$$\boxed{\therefore r = \frac{\sqrt{3} - Z}{\sqrt{3}}}$$

$$d = 2r$$

$$d = \frac{2(\sqrt{3} - Z)}{\sqrt{3}}$$

$\therefore A = \frac{\pi}{4} d^2 \implies A = \frac{\pi}{4}\left[\frac{2(\sqrt{3}-Z)}{\sqrt{3}}\right]^2$

$\boxed{\therefore A = \frac{\pi}{3}(\sqrt{3}-Z)^2}$ used to solve for (t)

Solving $(\bar{N}_A)$ equation after substitution (A) and $P_{A_2} = 0$, $(\bar{N}_A)$ can be found:-

$$\bar{N}_A \int_{Z_1}^{Z_2} \frac{dZ}{\frac{\pi}{3}(\sqrt{3}-Z)^2} = \frac{-D_{AB}\,P_T}{R.T} \ln \frac{P_T - P_{A2}}{P_T - P_{A1}}$$

$\therefore \bar{N}_A = 4.41 * 10^{-5}$ Kmol/sec.

Note:- To find the time required to drop the level to a certain new level.

$$N_A = \frac{\bar{N}_A}{A} = v_A . C_A = \frac{dZ}{dt} . \frac{\rho_A}{M_A}$$

$$\bar{N}_A \int_0^t dt = \frac{\rho_A}{M_A} \int_{Z_0}^{Z_f} A . dZ$$

$$t = \frac{1}{\bar{N}_A} . \frac{\rho_A}{M_A} . \int_{Z_0}^{Z_f} \frac{\pi}{3}(\sqrt{3}-Z)^2 \, dZ$$

$$= \frac{\pi}{3\bar{N}_A} . \frac{\rho_A}{M_A} \int_{Z_0}^{Z_f} (3 - 2\sqrt{3} Z + Z^2) dZ$$

$$t = \frac{\pi}{3\bar{N}_A} \frac{\rho_A}{M_A} \left[3Z - \sqrt{3} Z^2 + \frac{Z^3}{3}\right]_{0.5}^{10.5}$$

$\therefore t =$

Diffusivity in gases and vapours

Two kinds of diffusivities can be computed.
① kinematic diffusivity (D_{AB}).
② Dynamic diffusivity (δ_{AB}).

(D_{AB}) is mainly proportional to [Temp. and press.]

① Kinematic diffusivity :- (D_{AB}).

ⓐ Gilliland diffusivity equation.

$$D_{AB} = \frac{b \cdot T^{3/2}}{P_T \left[v_A^{1/3} + v_B^{1/3} \right]^2} \cdot \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}$$ مهمة

D_{AB} = diffusivity m^2/sec.
b = constant = 0.0043
P_T = Total system pressure (P_a) , (N/m^2)
T = Temp. in (°K).
v_A, v_B = molar volume of (A and B) at normal boiling point ($cm^3/gmol$).

M_A, M_B = molecular weight.

ⓑ Andrussow equation.

$$D_{AB} = \frac{b \cdot T^{1.78} \left(1 + \sqrt{M_A + M_B}\right)}{P_T \left(v_A^{1/3} + v_B^{1/3}\right) \cdot \sqrt{M_A + M_B}}$$

$b = 7.98 * 10^{-4}$

(c) Fuller-equation

$$D_{AB} = \frac{B.(T)^{1.75}}{P_T \sqrt{M_{AB}} \left(\Sigma v_A^{1/3} + \Sigma v_B^{1/3}\right)^2}$$

where:-

$$M_{AB} = \frac{1}{(1/M_A + 1/M_B)} \quad , B = 1 * 10^{-7}$$

Σv_A = summation of the atomic and stractural diffusion volumes

ex. $C_6H_6 \Longrightarrow 6 * C + 6 * H$ + aromatic ring.

but for non-cyclic :- No. of atoms * activity + Hetro-cyclic ring.

(2) Dynamic diffusivity (δ_{AB}).

where $\delta_{AB} = D_{AB} \cdot C = D_{AB} / v_m$

$$\delta_{AB} = \frac{b \cdot T^{3/2}}{v_m \cdot P_T \left(v_A^{1/3} + v_B^{1/3}\right)^2} \cdot \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}$$

for ideal gas $v_m * P_T = R.T$, $b = 0.0043$

$$\delta_{AB} = \frac{b \cdot T^{1/2}}{R\left(v_A^{1/3} + v_B^{1/3}\right)^2} \cdot \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}$$

Now to correct (D_{AB}) or (δ_{AB}) at different conditions

$$\frac{D_1}{D_2} = \left(\frac{T_1}{T_2}\right)^{1.5} \cdot \frac{P_2}{P_1}$$

or
$$\frac{D_1}{D_2} = \left(\frac{T_1}{T_2}\right)^{1.75} \frac{P_2}{P_1}$$

and
$$(\delta_{AB})_T = (\delta_{AB})_{Ref.} \cdot \sqrt{\frac{T}{T_{ref}}}$$

Ex(1) :- Normal Butanol (A) is diffusing through air (B) at (1 atm). Estimate the diffusivity (D_{AB}) for the following conditions :-
a- For $0\,^\circ C$, b- $25.9\,^\circ C$, c- $0\,^\circ C$ and 20 atm.

Sol. :- (a) Take $M_A = 74.1$, $M_B = 29$
From Table (8.3) Perry) and using (1) Fuller-equation
$P = 1$ atm, $T = 273$ k
$\upsilon_C = 16.5$, $\upsilon_{H_2} = 1.98$, $\upsilon_{O_2} = 5.48$

$$\Sigma \upsilon_A = 4(16.5) + 10(1.98) + 1(5.48)$$
$$= 91.28$$
$$\Sigma \upsilon_B = 20.1$$

$$D_{AB} = \frac{1.0 * 10^{-7} (273)^{1.75}}{1.0 [(91.28)^{1/3} + (20.1)^{1/3}]^2} \cdot \left[\frac{1}{74.1} + \frac{1}{29}\right]^{0.5}$$

$$D_{AB} = 7.73 * 10^{-6} \ m^2/sec$$

② Using Gilliland corr.

$$D_{AB} = 7.078 * 10^{-6} \ m^2/sec.$$

ⓑ For $T = 25.9 = 298.9 \ K$

$$\underset{\text{at } 25.4}{D_{AB}} = \underset{\text{at zero}^\circ C}{D_{AB}} \cdot \left[\frac{T_{25.9^\circ C}}{T_{0^\circ C}} \right]^{1.75}$$

$$= 7.73 * 10^{-6} \left(\frac{298.9}{273} \right)^{1.75}$$

$$D_{AB} = 9.07 * 10^{-6} \ m^2/sec.$$

ⓒ For $T = 0 \ ^\circ C$ and $P_2 = (2 \ atm)$.

$$D_{AB} = D_{AB} \left(\frac{P_1}{P_2} \right) = 7.73 * 10^{-6} \left(\frac{1}{2} \right)$$

$$D_{AB} = 3.87 * 10^{-6} \ m^2/sec.$$

Maxwell's Law of diffusion For Binary System

Maxwell's postulated that the pressure gradient (dP_A) in the direction of diffusion for a constituent of two components gaseous mixture was proportional to:

A) The relative velocity of the molecules in the direction of diffusion.

B) The product of the molar concentration of the component.

Thus;

$-dp_A \alpha\ C_A\ C_B\ (\nu_A - \nu_B)dz$

$-dp_A = \alpha_{AB}\ C_A\ C_B\ (\nu_A - \nu_B)dz$

Where ν_A&ν_B= mean molecule of (A)&(B) respectively.

C_A & C_B = molar concentration of the component (A)&(B) respectively.

At equilibrium, the partial pressure gradient of the diffusing gas = dp_A/dz.

$$\frac{-dp_A}{dz} = \alpha_{AB}\left(\frac{\rho_A}{M_A}\right)\left(\frac{\rho_B}{M_B}\right)(\nu_A - \nu_B)$$

$$N_A = \nu_A.C_A \ \rightarrow\ N_A = \nu_A.\left(\frac{\rho_A}{M_A}\right),\quad N_B = \nu_B.\left(\frac{\rho_B}{M_B}\right)$$

$$\nu_A = N_A.\frac{M_A}{\rho_A},\ \nu_B = N_B.\frac{M_B}{\rho_B}$$

$$\frac{-dp_A}{dz} = \alpha_{AB}\left(\frac{\rho_A}{M_A}\right)\left(\frac{\rho_B}{M_B}\right)\left(N_A.\frac{M_A}{\rho_A} - N_B.\frac{M_B}{\rho_B}\right)$$

$$\frac{-dp_A}{dz} = \alpha_{AB}\left(N_A.\frac{\rho_B}{M_B} - N_B\frac{\rho_A}{M_A}\right)$$

For ideal gas law; P.V = n. R.T, $C_A = \frac{P_A}{R.T} = \frac{\rho_A}{M_A}$

$$\frac{-dp_A}{dz} = \alpha_{AB}\left(N_A.\frac{P_B}{R.T} - N_B.\frac{P_A}{R.T}\right)$$

$$\frac{-dp_A}{dz} = \frac{\alpha_{AB}}{R.T}(N_A.P_B - N_B.P_A)$$

$$\frac{-dp_A}{dz} = \frac{\alpha_{AB}}{R.T}(N_A.(P_T - P_A) - N_B.P_A)$$

Diffusivity coefficient (D_{AB}) $= \frac{R^2.T^2}{\alpha_{AB}.P_T}$

$$\frac{-dp_A}{dz} = \frac{R.T}{D_{AB}\,P_T}(N_A.P_T - N_A.P_A - N_B.P_A)$$

The above equation is Maxwell-equation.

Where: R=gas constant, T=temperature in ^oK,^oR.
P_A,P_B=partial pressure, P_T=total pressure.

Now, applying the two cases that have been considered before, *i.e.* equimolardiffusion, and diffusion through stagnant layer, then we can reach to the final equation.

a) For equimolar-counter-diffusion (EMD).

$N_A = - N_B$

$$\frac{-dp_A}{dz} = \frac{R.T}{D_{AB}\,P_T}(N_A.P_T)$$

$$NA = \left(\frac{-DAB}{R.T}\right)\left(\frac{P_{A2} - P_{A1}}{Z_2 - Z_1}\right) \quad \text{[Maxwell eq. for EMD]}$$

b) For $N_B = 0$

$$-dp_A = N_A\left(\frac{R.T}{D_{AB}\,P_T}\right)(P_T - P_A)dz$$

$$NA = \left(\frac{-D_{AB}}{R.T}\right)\left(\frac{P_T}{(Z_2 - Z_1)}\right)\ln\left(\frac{P_T - P_{A2}}{P_T - P_{A1}}\right)$$

Note: $P_T = P_{A1} + P_{B1}$ and $P_T = P_{A2} + P_{B2}$,
$P_{A1} + P_{B1} = P_{A2} + P_{B2} \rightarrow P_{A1} - P_{A2} = P_{B2} - P_{B1}$
Therefore:

$$NA = \left(-\frac{D_{AB}\,.\,P_T}{R.\,T}\right)\left(\frac{P_T}{P_{BM}}\right)\left(\frac{P_{A2} - P_{A1}}{Z_2 - Z_1}\right) \quad \text{[Maxwell eq. for } N_B\text{=0]}$$

Where: $\left(\frac{P_T}{P_{BM}}\right)$ = Drift factor.
Drift factor $\left(\frac{P_T}{P_{BM}}\right)$ = Represent the enhancement effect on mass transfer due to thenon diffusing component (B) and to total flow in the direction of diffusion.

<u>Maxwell's Law for Multi-Component Mass Transfer</u>

Consider the transfer of component (A) through a stationary gas consisting of component B, C, D …

Suppose that the total partial pressure gradient can be regardedas being made up of series of terms, each represent the contribution of the individual component gases,

$-dp_A = \alpha_{AB}\, C_A\, C_B\, (\nu_A - \nu_B)dz$ (for binary system)

But now we have:

$$\frac{-dp_A}{dz} = \boldsymbol{\alpha_{AB}\, C_A\, C_B\, (\nu_A - \nu_B) + \alpha_{AC}\, C_A\, C_C\, (\nu_A - \nu_C) + \alpha_{AD}\, C_A\, C_D\, (\nu_A - \nu_D) + \ldots\ldots\ldots\ldots}$$

For stationary gas (B, C, D) velocities of (B, C, D) = 0

$$N_A = \nu_A . C_A \;\rightarrow \nu_A = \frac{N_A}{C_A}$$

$$\frac{-dp_A}{dz} = [\alpha_{AB}\; C_B + \alpha_{AC}\, C_C + \alpha_{AD}\, C_D\,].N_A$$

Since $P_B = C_B.\, R.\, T$, $P_C = C_C.\, R.\, T$, $P_D = C_D.\, R.\, T$

$$\frac{-dp_A}{dz} = \left(\frac{N_A}{R.T}\right) [\alpha_{AB}\; P_B + \alpha_{AC}\, P_C + \alpha_{AD}\, P_D]$$

When $\alpha_{AB} = \frac{R^2.T^2}{D_{AB}.P_T}$, therefore:

$$\frac{-dp_A}{dz} = N_A \left(\frac{R.T}{P_T}\right) \left[\frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \frac{P_D}{D_{AD}}\right]$$

$$N_A = \left(\frac{-P_T}{R.T}\right) \left[\frac{1}{\frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \frac{P_D}{D_{AD}}}\right] \frac{dp_A}{dz} \ldots\ldots.(*)$$

$$N_A = \left(\frac{-P_T}{R.T}\right) D^{-} \frac{dp_A}{dz}$$

Put $\frac{1}{D^-} = \frac{P_B}{D_{AB}} + \frac{P_C}{D_{AC}} + \frac{P_D}{D_{AD}}$

Divided equation (*) by $\frac{(P_T - P_A)}{(P_T - P_A)}$:

$$N_A = \frac{-1}{R.T} \frac{P_T}{(P_T - P_A)} \left[\frac{1}{\frac{y_{B^-}}{D_{AB}} + \frac{y_{C^-}}{D_{AC}} + \frac{y_{D^-}}{D_{AD}}} \right] \frac{dp_A}{dz}$$

Where $\frac{1}{D^-} = \frac{y_{B^-}}{D_{AB}} + \frac{y_{C^-}}{D_{AC}} + \frac{y_{D^-}}{D_{AD}}$

D^- = effective Diffusivity = function of mole fraction

$$N_A = \frac{-D^-}{R.T} \frac{P_T}{(P_T - P_A)} \frac{dp_A}{dz}$$

$$-\int_{P_A}^{P_A} \frac{dp_A}{(P_T - P_A)} = \left(\frac{R.T}{D^-.P_T}\right) N_A \int_{Z1}^{Z2} dZ$$

After the integration:

$$N_A = \left(\frac{-D^-}{R.T}\right) \left(\frac{P_T}{P_{im}}\right) \left(\frac{P_{A2} - P_{A1}}{Z_2 - Z_1}\right) \text{[for multi-component gas system]}$$

Where:

$P_T = P_A + P_B + P_C + P_D$ [diffusing + nondiffusing]

$P_t = (P_T - P_A) = P_B + P_C + P_D$ [nondiffusing only]

$y^-_B = P_B/P_t$, $y^-_C = P_C/P_t$, $y^-_D = P_D/P_t$

and $P_{im} = \frac{(P_T - P_{A2}) - (P_T - P_{A1})}{\ln \frac{(P_T - P_{A2})}{(P_T - P_{A1})}}$