الجامعة التكنولوجية

قسم الهندسة الكيمياوية

المرحلة الثالثة

انتقال كتلة

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Example 1:

Oxygen (O_2) (A) is diffusing through non-diffusing gas mixture of methane (CH_4)(B) andhydrogen (H_2) (C) in the volume ratio of 2:1.The total pressure is$1*10^5$ N/m^2 and thetemperature is 0 ^oC.The partial pressure of oxygen at two planes (2 mm) is 13 kN/$m^2$and 6.5 kN/m^2. The diffusivity of Oxygen in Hydrogen (D_{O2}/H_2) = $6.99*10^{-5}$ m^2/sand the diffusivity of Oxygen in Methane (D_{O2}/CH_4) = $1.88*10^{-5}$$m^2$/s.
Calculate the rate of diffusion of oxygen in kmol/m^2s through each square meter of the two planes.

Solution:

$$N_A = \left(\frac{-D^-}{R.T}\right)\left(\frac{P_T}{P_{im}}\right)\left(\frac{P_{A2}-P_{A1}}{Z_2-Z_1}\right)$$

$$D^- = \frac{1}{\frac{y_{B^-}}{D_{AB}} + \frac{y_{C^-}}{D_{AC}}}$$

$y_{B^-}=\frac{2}{3}$ =0.667 ,$y_{C^-}=\frac{1}{3}$ =0.333

D^- = $2.46*10^{-5}$ m^2/s

$$P_{im} = \frac{(P_T-P_{A2})-(P_T-P_{A1})}{\ln\frac{(P_T-P_{A2})}{(P_T-P_{A1})}} = \frac{(1*10^5-6500)-(1*10^5-13000)}{\ln\frac{(1*10^5-6500)}{(1*10^5-13000)}} = 9*10^4 \text{ N/m}^2$$

Sub. For $Z_2 - Z_1 = 2*10^{-3}$ m

N_A = $3.91*10^{-5}$ kmol/m^2.sec.

Molecular Diffusion in Liquid Phase

Molecular diffusion in liquid phase takes place in many separation operations, such as:

1- Liquid – liquid extraction.
2- Gas absorption.
3- Distillation.
4- Oxygenation of rivers by air.
5- Diffusion of salts in blood.

Some important notes in liquid diffusion, these are:

1- Slower than in gas phase because of the density and attractive forces between molecules.
2- Diffusivities are dependent on the concentration of the diffusing component.

General form for diffusing equation is :

$$N_A = -D_L\left(\frac{dC_A}{dZ}\right) + \frac{C_A}{C_T}(N_A + N_B)$$

1-forequimolar diffusion (EMD), where $N_A = -N_B$, then:

$$N_A = -D_L\left(\frac{C_{A2} - C_{A1}}{Z_2 - Z_1}\right), \text{ Or}$$

$$NA = -D_L . C_{av}\frac{(X_{A2} - X_{A1})}{(Z_2 - Z_1)}$$

Where:-

D_L: diffusivity of solute (A) in (B) (m^2/s).

X_A: mole fraction of (A) at any point.

$$C_{av} = \frac{1}{2}\left[\frac{\rho_1}{Mwt_1} + \frac{\rho_2}{Mwt_2}\right]$$

Where:-

C_{av}: average concentration of (A+B) in ($kmol/m^3$).

Mwt_1 & Mwt_2: average molecular weight of the solution at points 1 & 2 respectively(kg/kmol).

ρ_1& ρ_2: average density of the solution at points 1 & 2 in (kg/m^3).

2- WhemN$_B$= 0 (stationary).

$$N_A = - D_L . \left(\frac{C_{av}}{X_{BLM}}\right) \frac{(X_{A2} - X_{A1})}{(Z_2 - Z_1)}$$

Where $X_{BLM} = \frac{(X_{B2} - X_{B1})}{\ln \frac{X_{B2}}{X_{B1}}}$

An important note, that is $X_{A1} + X_{B1} = X_{A2} + X_{B2} = 1$ and for dilute solution:

$X_{BLM} \sim 1.0$, then:

$N_A = - D_L\left(\frac{C_{A2} - C_{A1}}{Z_2 - Z_1}\right)$(Dilute solution only)

Diffusivities in Liquids:

Diffusion coefficient in liquids at 293 K is given in table (10.7) in volume (1) of chemical engineering by(Coulson, J.M.; Richardson, J.F.,) fifth edition (page 506).

1- Wilke& Chang equation:

$$D_{AB} = \frac{7.4*10^{-8} (\varphi_B . Mwt_B)^{0.5} . T}{\mu_B . \upsilon_A^{0.6}}$$

Where:

D_{AB} = diffusivity of solute (A) in very dilute solution in solvent (B), (cm^2/s).

Mwt_B= molecular weight of solvent (B).

T = temperature (K).

μ_B= viscosity of solvent (B), (C_P or gm/ cm.s).

υ_A= Solute molar volume at its normal boiling point (cm^3/mol).

Diffusion of (A)Through Multi-Component Stagnant Layer Mixture.

$$N_A = D^- \left(\frac{-C_T}{C_{RM}}\right)\left(\frac{C_{A2} - C_{A1}}{Z_2 - Z_1}\right), \quad \text{Or, } N_A = D^- \left(\frac{-C_{av}}{X_{RM}}\right)\left(\frac{X_{A2} - X_{A1}}{Z_2 - Z_1}\right)$$

Where: $D^- = \frac{1}{\frac{X_B}{D_{AB}} + \frac{X_C}{D_{AC}} + \frac{X_D}{D_{AD}}}$

X_{RM}(same as P_{im})= remaining mole fraction log mean. (all component except (A)).

Example:

Calculate the rate of diffusion of CH_3COOH (A) across a film of non-diffusing water (B) solution (1 mm) thick at 17°C when the concentration on opposite sides of the film are 9 and 3 wt% respectively. The $D_{AB} = 0.95*10^{-9}\ m^2/s$.
Give that:-
$M_A = 60,\ M_B = 18.$
$\rho = 1012\ kg/m^3$ (9% solution).
$\rho = 1003\ kg/m^3$ (3% solution).

Solution:

$$N_A = -D_L.\left(\frac{C_{av}}{X_{BLM}}\right)\frac{(X_{A2}-X_{A1})}{(Z_2-Z_1)}$$

$$X_{A1} = \frac{\frac{0.09}{60}}{\frac{0.09}{60}+\frac{0.91}{18}} = 0.0288 \quad \text{at position (2).}$$

$$X_{A2} = \frac{\frac{0.03}{60}}{\frac{0.03}{60}+\frac{0.97}{18}} = 0.0092 \quad \text{at position (1).}$$

$$\frac{1}{M_1} = \frac{0.09}{60} + \frac{0.91}{18} \longrightarrow M_1 = 19.8\ kg/kmol$$

$$\frac{1}{M_2} = \frac{0.03}{60} + \frac{0.97}{18} \longrightarrow M_2 = 18.4\ kg/kmol$$

$$C_{av} = \frac{1}{2}\left[\frac{\rho_1}{Mwt_1} + \frac{\rho_2}{Mwt_2}\right] \longrightarrow = \frac{1}{2}\left[\frac{1012}{19.8} + \frac{1003}{18.4}\right]$$

$C_{av} = 52.8\ kmol/m^3$.

$X_{B1} = 1 - X_{A1} \longrightarrow = 1 - 0.0288 \longrightarrow X_{B1} = 0.9712$

$X_{B2} = 1 - X_{A2} \longrightarrow = 1 - 0.009 \longrightarrow X_{B1} = 0.9908$

$$X_{BLM} = \frac{(X_{B2}-X_{B1})}{\ln\frac{X_{B2}}{X_{B1}}} \longrightarrow = \frac{(0.9908-0.971)}{\ln\frac{0.9908}{0.971}}, \qquad X_{BLM} = 0.98$$

$$N_A = -0.95*10^{-9} * \frac{52.8}{0.98} * \frac{0.0092-0.0283}{0.001}$$

$N_A = 0.977*10^{-6} kmol/m^2.sec$

Molecular Diffusion in Solid Phase

Diffusion in solids takes place by different mechanisms depending on the diffusing atom, molecule, or ion; the nature of the solid structure, whether it be porous or nonporous, crystalline, or amorphous; and the type of solid material, whether it be metallic, ceramic, polymeric, biological, or cellular. Diffusion in solid phase can be c classified as:

1- Diffusion which follows Fick's law and does not depend on the structure of solid.
2- Diffusion in which the structure of solid are important.

1- Diffusion that follows Fick's law:

That is

$$N_A = -D_{AB}\left(\frac{dC_A}{dZ}\right)$$

Where:

D_{AB} = Diffusivity of fluid A in solid B in m^2/s. It is independent of pressure, but a function of Temperature.

A) For diffusion through a solid slab at steady – state:

$$N_A = -D_{AB}\left(\frac{C_{A_2} - C_{A_1}}{Z_2 - Z_1}\right)$$

B) For diffusion through a solid hollow cylinder of r_1 (inner radius) and r_2 (outer radius) with length L:

$$N_A = \frac{\overline{N}_A}{A} = \frac{\overline{N}_A}{2\pi rL} = -D_{AB}\left(\frac{dC_A}{dZ}\right)$$

$$\overline{N}_A = -D_{AB}\left(C_{A_2} - C_{A_1}\right)\frac{2\pi L}{\ln\frac{r_2}{r_1}}$$

C) For diffusion through a solid hollow spherical shape of r_1 (inner radius) and r_2 (outer radius):

$$N_A = \frac{\overline{N}_A}{A} = \frac{\overline{N}_A}{4\pi r^2} = -D_{AB}\left(\frac{dC_A}{dZ}\right)$$

$$\frac{\overline{N}_A}{4\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = D_{AB}\left(C_{A_2} - C_{A_1}\right)$$

Where:

C_{A_2} & C_{A_1} = concentrations at opposite side of the sphere.

In the case that gas diffuses through a solid, then the solubility of the gas in the solid is directly proportional to the partial pressure of solute.
The solubility of a solute gas (A) in a solid is expressed as (S) in cm^3 solute (at STP, e.g. 0°C, and 1 atm) per cm^3 solid per (atm, partial pressure of solute A), then:

$$S = \frac{cm^3\,(STP)\text{ of A}}{cm^3\text{ solid . atm}}$$

To convert the solubility (S) of A to concentration (C_A):

$$C_A = S * \frac{1}{22.414} * P_A$$

Where:

$$C_A = \frac{mol\text{ A}}{\text{cm}^3}, S = \frac{cm^3\,(STP)\text{ of A}}{\text{atm . cm}^3\,(solid)}, \text{ P}_\text{A} = \text{atm, and } 22.414 = \frac{cm^3\text{ A}}{\text{mol}}$$

In many cases the experimental data for diffusion of gases in solid are given as permeability (P_M) in cm^3 of solute gas (A) at STP {0°C, and 1 atm} per diffusing per second per cm^2 cross-sectional area through of solid of 1 cm thick under a pressure difference of 1 atm.

$$\text{N}_\text{A} = -D_{AB}\left(\frac{C_{A_2} - C_{A_1}}{Z_2 - Z_1}\right)$$

Applying (Eq. 84), then

$$C_{A_1} = \frac{S.P_{A_1}}{22.414} \;\&\; C_{A_2} = \frac{S.P_{A_2}}{22.414}, \text{ then:}$$

$$N_A = -\frac{D_{AB}.S}{22.414}\left(\frac{P_{A_2} - P_{A_1}}{Z_2 - Z_1}\right) = \frac{\text{P}_\text{m}(P_{A_1} - P_{A_2})}{22.414(Z_2 - Z_1)}$$

Where:

$$P_m = D_{AB}.S = \frac{cm^3\,(STP)\text{ of A}}{\text{cm}^2.S.\frac{atm}{1cm}}$$

If series of solids are presented in series 1, 2, 3, .., etc of thickness of each L_1, L_2, L_3,…, etc respectively, then:

$$N_A = \frac{(P_{A_1} - P_{A_2})}{22.414}\left(\frac{1}{\frac{\text{L}_1}{\text{P}_{\text{m}_1}} + \frac{\text{L}_2}{\text{P}_{\text{m}_2}} + \frac{\text{L}_3}{\text{P}_{\text{m}_3}} + \ldots\ldots}\right)$$

Where:

$(P_{A_1} - P_{A_2})$ is the overall partial pressure difference.

Diffusion in which the structure of solid are important:

Porous Solids

When solids are porous, predictions of the diffusivity of gaseous and liquid solute species in the pores can be made. This type of diffusion is also of great importance in the analysis and design of reactors using porous solid catalysts. It is sufficient to mention here that any of the following four mass transfer mechanisms or combinations thereof may take place:

1. Ordinary molecular diffusion through pores, which present tortuous paths and hinder the movement of large molecules when their diameter is more than 10% of the pore diameter.
2. Knudsen diffusion, which involves collisions of diffusing gaseous molecules with the pore walls when the pore diameter and pressure are such that the molecular mean free path is large compared to the pore diameter.
3. Surface diffusion involving the jumping of molecules, adsorbed on the pore walls, from one adsorption site to another based on a surface concentration-driving force.
4. Bulk flow through or into the pores.

When treating diffusion of solutes in porous materials where diffusion is considered to occur only in the fluid in the pores, it is common to refer to an effective diffusivity, D_{eff}, which is based on (1) the total cross-sectional area of the porous solid rather than the cross-sectional area of the pore and (2) on a straight path, rather than the pore path, which may be tortuous. If pore diffusion occurs only by ordinary molecular diffusion, the effective diffusivity can be expressed in terms of the ordinary diffusion coefficient, D, by

$$D_{eff} = \frac{D_{AB}\,\varepsilon}{\tau}$$

And the rate of diffusion is calculated by

$$N_A = -D_{eff}\left(\frac{C_{A_2} - C_{A_1}}{Z_2 - Z_1}\right)$$

Where ε the fractional porosity (typically 0.5) of the solid and τ is the pore-path tortuosity (typically 2 to 3), which is the ratio of the pore length to the length if the pore were straight in the direction of diffusion.

Example

A sintered solid of silica (2mm) thick is porous with void fraction of 0.3 and tortuosity of 4. the pores are filled with water at 298 K. at one face the concentration of KCl is held at 0.1 mol / liter and fresh water flow rapidly by other face. Neglecting any other resistance but that in porous solid, calculate the diffusion of KCl at steady state. Given the diffusivity D_{AB} is equal to $1.98*10^{-9}$ m^2/s.

Solution

Since neglecting any other resistance but that in porous solid, then ordinary diffusion is occurred

$$N_A = -D_{eff}\left(\frac{C_{A_2} - C_{A_1}}{Z_2 - Z_1}\right)$$

And D_{eff} is calculated by equation 89

$$D_{eff} = \frac{D_{AB}\,\varepsilon}{\tau} = \frac{1.98*10^{-9}*0.3}{4} = 1.4025*10^{-10}\,\frac{m^2}{s}$$

$$N_A = -1.4025*10^{-10}\left(\frac{0-0.1}{0.002}\right) = 7.01*10^{-9}\,\frac{kmol\ KCl}{m^2*s}$$

Convective Mass Transfer for Binary gas Mixture

In previous sections we have considered molecular diffusion in stagnant fluid (laminar flow) where the rate of diffusion is slow. To increase the rate of mass transfer, the fluid velocity is increased until turbulent mass transfer occurs. As given before in equation (1):

Ni = Xi.N + molecular diffusion flux of i + eddy diffusion flux of i .... (1)

The eddy diffusion flux term is given by:

$$J^{*}_{A_Z} = -E_d \frac{dC_A}{dZ} \quad \cdots\cdots (2)$$

Then

$$N_A = -C_T^{*} (D_{AB} + E_D) * (\frac{dX_A}{dZ}) + X_A (N_A + N_B) \cdots\cdot (3)$$

This is the general equation used to calculate the mass transfer.

1- For equi - molar mass transfer

A) For gases:

$$N_A = \frac{-(D_{AB} + E_D)}{R*T} * \frac{dP_A}{dZ} + \frac{P_A}{P_T} * (N_A + N_B) \; \cdots\cdots (4)$$

For equi – molar transfer: $(N_A = -N_B)$

Then :

$$N_A = \frac{-(D_{AB} + E_D)}{R*T} * \frac{dP_A}{dZ} \quad \cdots\cdot (5)$$

By integrating equation (5) from P_{A_1} at Z_1 to P_{A_2} and Z_2 then:

$$N_A = \frac{(D_{AB} + E_D)}{R*T} * \frac{(P_{A_1} - P_{A_2})}{(Z_2 - Z_2)} \quad \cdots\cdot (6)$$

Because the film thickness can not be measured or is not known, then the value (Z_2-Z_1) is not known (the distance of the path), also the amount of E_D can not be measured, then, the term (individual mass transfer coefficient) is used as shown below:

$$N_A = K'_G (P_{A_1} - P_{A_2}) \quad \cdots\cdot (7)$$

Where:

$$K'_G = \frac{(D_{AB} + E_D)}{R*T(Z_2 - Z_1)} \quad \cdots\cdot (8)$$

Also for gases another form of equation (8) can be written as:

$$N_A = K'_y (y_{A_1} - y_{A_2}) \; \cdots\cdot (9)$$

A) For Liquids:

Similar to what done for gases, the rate of mass transfer is:

$$N_A = K'_L (C_{A_1} - C_{A_2}) = K'_x (X_{A_1} - X_{A_2}) \quad \cdots\cdot (10)$$

The general form for the mass transfer is:

$$N_A = K'_C\,(C_{A_1} - C_{A_2}) \quad \cdots\cdots(11)$$

All these individual mass transfer coefficients are related to each other.

For gases:

$$N_A = K'_C\,(C_{A_1} - C_{A_2}) = K'_G\,(P_{A_1} - P_{A_2}) = K'_y\,(y_{A_1} - y_{A_2}) \cdots(12)$$

So

$$K'_C = K'_G \cdot R.T = \frac{K'_y}{C_T}$$

Home work: find the relations for the liquid phase.

2- For uni - molecular mass transfer

A) For gases:

$$N_A = \frac{-(D_{AB} + E_D)}{R*T} * \frac{dP_A}{dZ} + \frac{P_A}{P_T} * (N_A + N_B) \quad \cdots\cdots (4)$$

For uni – molar transfer: ($N_B = 0$)

Then equation (4) will be:

$$N_A = \frac{-(D_{AB} + E_D)}{R*T} * \frac{dP_A}{dZ} + \frac{P_A}{P_T} * (N_A) \quad ---- (13)$$

By integrating equation (13) from P_{A_1} at Z_1 to P_{A_2} and Z_2 then

$$N_A = \frac{(D_{AB} + E_D) * P_T}{R * T * P_{BLM}} * \frac{(P_{A_1} - P_{A_2})}{(Z_2 - Z_2)} \quad ---- (14)$$

Similarly as done in section 1:

$$N_A = K_G\,(P_{A_1} - P_{A_2}) = K_y\,(y_{A_1} - y_{A_2}) = K_c\,(C_{A_1} - C_{A_2}) \quad ------(15)$$

B) For Liquids:

Similar to what done for gases, the rate of mass transfer is:

$$N_A = K_L\,(C_{A_1} - C_{A_2}) = K_x(X_{A_1} - X_{A_2}) \quad ----(16)$$

The general form for the mass transfer is:

$$N_A = K_C\,(C_{A_1} - C_{A_2}) \quad -------(17)$$

Home work: find the relations for the gas phase and liquid phase. Also write the units of each coefficient.

Methods to Determine the Mass transfer Coefficient

The mass transfer coefficient can be evaluated using empirical correlations involving dimensionless numbers. These empirical equations were obtained from experimental data using various types of fluids, different velocities, and different geometrics. The most important groups (dimensionless numbers) are:

1- Reynolds Number (Re No.):

Re. No indicate the degree of turbulence.

$$R_e No = \frac{\rho u d}{\mu} = \frac{\text{interfial force}}{\text{Viscous force}}$$

Where:

ρ = density of flowing mixture fluid (solute A and solvent B).

μ = viscosity of flowing mixture fluid (solute A and solvent B).

d = diameter of the pipe in which the fluid is flow.

D_p May be used in Reynolds number equation instead of d when flow across a sphere, also L may be used if the flow was above a flat plate.

U = mean average velocity in the pipe.

If the flow was across a packed bed, U will be substituted by:

$$U = \frac{\overline{U}}{\varepsilon}$$

Where

$\overline{U}$ is the superficial velocity of empty cross- section of packed bed column.

ε = void fraction.

2- Schmidt Number (Sc No.):

$$Sc\ No = \frac{\mu}{\rho D_{AB}} = \frac{\nu}{D_{AB}} = \frac{\text{momentum diffusivity}}{\text{molecular diffusivity}}$$

3- Sherwoood Number (Sh No.):

$$Sh\ No = \frac{K_C^{'} L}{D_{AB}}$$

4- Stanton Number (St No.):

$$Sh\ No = \frac{Sh}{\text{Re} * \text{Sc}} = \frac{K_C^{'}}{u} = \frac{K_C^{'} \rho}{G_m} \quad \text{, where } \overset{\text{(flow rate)}}{G_m} = \frac{\rho . u}{M_{avg.}} = u . C_{avg.} \quad \ldots (1)$$

5- *J*-factor

In general *j*-factors are uniquely determined by the geometric configuration and the Reynolds number.

Now after introducing the dimensionless groups, now we will present the correlations used to calculate the mass transfer coefficient.

1- Using the analogy or similarity of momentum, heat, and mass transfer using Chilton – Colburn Analogy:

$$j_M = j_H = j_D = \frac{f}{2} = \frac{K_C^{'} * \rho}{G_m}(Sc)^{2/3} = \frac{k_c^{'}}{u}(S_c)^{2/3} = \frac{k_G^{'} P_T}{G_m}(S_c)^{2/3} \quad \ldots\ldots (2)$$

2- For flow inside pipe

For both gases and liquids, where Re > 2100

$$Sh\ No = \frac{K_C^{'}\, d}{D_{AB}} = 0.023(\text{Re})^{0.833}(Sc)^{0.333} \quad \ldots\ldots (3)\ (\text{for turbulent})$$

Note:

Sc = 0.5 – 3 for gases

Sc > 100 for liquids

Or

$$j_M = j_H = j_D = 0.023(\text{Re})^{-0.2} \quad \ldots\ldots (4)$$

For $10000 < \text{Re} < 1000000$

$$Sh = k_L \frac{d}{D_{AB}} = 1.62\left[(Re)(Sc)\frac{d}{L}\right]^{1/3} \quad \ldots\ldots (5)\ (\text{for Laminar})$$

3- For flow parallel to flat plate of length L

A- for gases

$$j_M = j_H = j_D = 0.664(\text{Re})^{-0.5} \quad \ldots\ldots (6)$$

For $\text{Re} < 15000$ where $\text{Re} = \frac{L\,U\,\rho}{\mu}$

And

$$j_M = j_H = j_D = 0.036(\text{Re})^{-0.2} \quad \ldots\ldots (7)$$

For $15000 < \text{Re} < 300000$ where $\text{Re} = \frac{L\,U\,\rho}{\mu}$

B- for liquids

$$j_M = j_H = j_D = 0.99(\text{Re})^{-0.5} \quad \ldots\ldots (8)$$

For $600 < \text{Re} < 50000$ where $\text{Re} = \frac{L\,U\,\rho}{\mu}$

For both gases and liquids

$$j_M = j_H = j_D = 0.037(\text{Re})^{-0.2} \quad \ldots\ldots (9)$$

For $500000 < \text{Re} < 5*10^8$ where $\text{Re} = \frac{L\,U\,\rho}{\mu}$

4- For flow normal to a long circular cylinder of diameter D, where the drag coefficient includes both form drag and skin friction, but only the skin friction contribution applies to the analogy:

$$(j_M)_{skin\,\text{friction}} = j_H = j_D = 0.193(\text{Re})^{-0.382} \qquad \text{- - - - . . . (10)}$$

For $4000 < \text{Re} < 40000$ where $\text{Re} = \dfrac{D_{\text{cylind}}\, U\, \rho}{\mu}$

$$(j_M)_{skin\,\text{friction}} = j_H = j_D = 0.0266(\text{Re})^{-0.195} \qquad \text{- - - - . . (11)}$$

For $40000 < \text{Re} < 250000$ where $\text{Re} = \dfrac{D_{\text{cylind}}\, U\, \rho}{\mu}$

5- For flow past a single sphere of diameter D_p

$$(j_M)_{skin\,\text{friction}} = j_H = j_D = 0.37 * (\text{Re})^{-0.4} \qquad \text{- - - - - . (12)}$$

For $20 < \text{Re} < 100000$ where $\text{Re} = \dfrac{D_p u \rho}{\mu}$

The above equation is used for both gases and liquids. Another method is used to calculate the mass transfer coefficient.

For very low Re No. (Re < 1) the Sh No. approach a value of 2, then:

$$Sh\ No = \frac{K_C' \, d}{D_{AB}} = 2$$

$$\therefore K_C' = Sh * \frac{D_{AB}}{D_P} = \frac{2 * D_{AB}}{D_p} \qquad \text{- - - - - - . (13)}$$

For gases where Re = 1 – 48000 and Sc = 0.6 – 2.7, a modified equation can be used:

$$Sh\ No = \frac{K_C' \, d}{D_{AB}} = 2 + \left(0.552\,\text{Re}^{0.53} * Sc^{0.333}\right) \qquad \text{- - - - - - . (14)}$$

For liquids:

$$Sh\ No = \frac{K_C' \, d}{D_{AB}} = 2 + \left(0.95\,\text{Re}^{0.5} * Sc^{0.333}\right) \qquad \text{- - - - - . (15)}$$

For $\text{Re} = 2 - 2000$

And also for liquids

$$Sh\ No = \frac{K_C' \, d}{D_{AB}} = 2 + \left(0.347\,\text{Re}^{0.62} * Sc^{0.333}\right) \qquad \text{- - - - . . (16)}$$

For $\text{Re} = 2000 - 17000$

6- For flow through beds packed with spherical particles of uniform size D_p

$$j_H = j_D = 1.17(\text{Re})^{-0.415} \qquad \text{------} (17)$$

For $10 < \text{Re} < 2500$ where $\text{Re} = \dfrac{D_p U \rho}{\mu}$

For other shapes of packing a correction factor can be used such as:
j_D (cylinder) = 0.79 * j_D (sphere)
j_D (cube) = 0.71 * j_D (sphere)

7- For fluidized beds of packed with sphere, for both gases and liquids:

$$j_D = 0.01 + \frac{0.86}{\text{Re}^{0.58} - 0.483} \qquad \text{------} (18)$$

Methods for Mass transfer at Fluid – Fluid Interface (Phase Boundary)

In the previous sections, diffusion and mass transfer within solids and fluids were considered, where the interface was a smooth solid surface. Of greater interest in separation processes is mass transfer across an interface between a gas and a liquid or between two liquid phases. Such interfaces exist in absorption, distillation, extraction, and stripping. At fluid-fluid interfaces, turbulence may persist to the interface. Mass transfer rate between two fluid phases will depend on:-

1-Physical properties of the two phases.
2-Concentration difference ΔC.
3-Interfacial area.
4-Degree of turbulence.

The following theoretical models have been developed to describe mass transfer from a fluid to such an interface.

FilmTheory

A simple theoretical model for turbulent mass transfer to or from a fluid-phase boundary was suggested in 1904 by Nernst, who postulated that the entire resistance to mass transfer in a given turbulent phase is in a thin, stagnant region of that phase at the interface, called a film. This film is similar to the laminar sub – layer that forms when a fluid flows in the turbulent regime parallel to a flat plate. This is shown schematically in Figure 4. For the case of a gas-liquid interface, where the gas is pure component A, which diffuses into nonvolatile liquid B. Thus, a process of absorption of A into liquid B takes place, without desorption of B into gaseous A. Because the gas is pure A at total pressure $P = P_A$, there is no resistance to mass transfer in the gas phase. At the gas-liquid interface, equilibrium is assumed so the concentration of A, C_{Ai} is related to the partial pressure of A, P_A, by some form of Henry's law, for

example, $C_{Ai} = H_A P_A$. In the thin stagnant liquid film of thickness δ, molecular diffusion only occurs with a driving force of $(C_{A_i} - C_{A_B})$. Since the film is assumed to be very thin, the entire diffusing A passes through the film and into the bulk liquid. If, in addition, bulk flow of A is neglected, the concentration gradient is linear as in Figure . Accordingly, Fick's first law, for the diffusion flux integrates to:

$$J_A = \frac{D_{AB}}{\delta}\left(C_{A_i} - C_{A_b}\right) = \frac{C_T^* D_{AB}}{\delta}\left(X_{A_i} - X_{A_b}\right) \quad - - - - - (1)$$

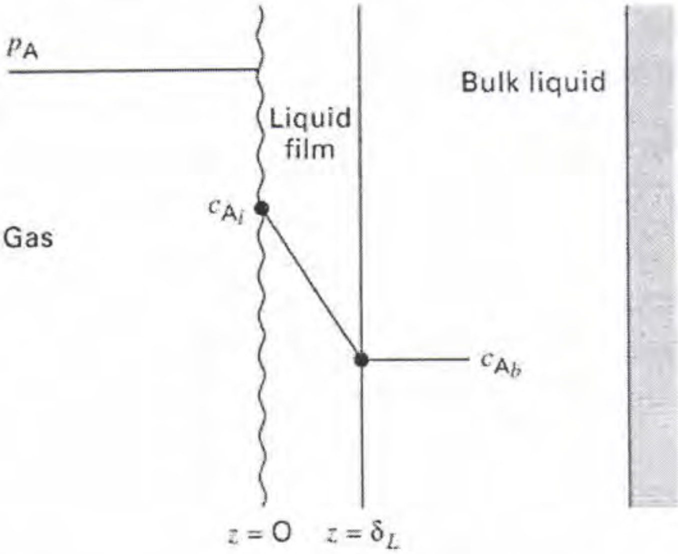


Figure 1 Film theory for mass transfer from a fluid – fluid interface into a liquid.

If the liquid phase is dilute in A, the bulk-flow effect can be neglected and (Eq. 1) applies to the total flux:

$$N_A = \frac{D_{AB}}{\delta}\left(C_{A_i} - C_{A_b}\right) = \frac{C_T^* D_{AB}}{\delta}\left(X_{A_i} - X_{A_b}\right) \quad \cdots \cdots\cdots (2)$$

If the bulk-flow effect is not negligible, then,

$$N_A = \frac{C.D_{AB}}{Z_2 - Z_1}\ln\left(\frac{1 - X_{Ab}}{1 - X_{A_i}}\right) = \frac{C.D_{AB}}{\delta(1 - X_A)_{LM}}\left(X_{A_i} - X_{A_b}\right) \quad - - - (3)$$

Where:

$$(1 - x_A)_{LM} = \frac{x_{A_i} - x_{A_b}}{\ln[(1 - x_{A_b})/(1 - x_{A_i})]} = (x_B)_{LM}$$

In practice, the ratios D_{AB}/δ in (Eq. 2) and $D_{AB}/\delta^*(1 - X_A)_{LM}$ in (Eq. 3) are replaced by mass transfer coefficients and K_c' and K_c, respectively, because the film thickness, δ, which depends on the flow conditions, is not known.

The film theory, which is easy to understand and apply, is often criticized because it appears to predict that the rate of mass transfer is directly proportional to the molecular diffusivity. This dependency is at odds with experimental data, which indicate a dependency of D^n, where n ranges from about 0.5 to 0.75. However, if D_{AB}/δ is replaced with K_c', which is then estimated from the Chilton-Colburn analogy, (Eq. $\overset{\text{Page 51}}{2}$), we obtain K_c' proportional to $D_{AB}^{2/3}$, which is in better agreement with experimental data. In effect, δ depends on D_{AB} (Sc No.). Regardless of whether the criticism of the film theory is valid, the theory has been and continues to be widely used in the design of mass transfer separation equipment.

Example

A wetted wall column of inside diameter (2 in) contains air and CO_2 flowing at 3 ft/s. at a certain point in the column, the CO_2 concentration in the air is 0.1 mol fraction, at the same point in the column, the concentration of CO_2 in the water at the water – air interface is 0.005 mole fraction. The column operates at 10 atm and 25 °C. Calculate the mass transfer coefficient and the mass flux at the point of consideration, given the following data:

The diffusivity of CO_2 in air at 25 °C and 1 atm = $0.164\ cm^2/s$

The density of air at STP = $0.0808\ lb/ft^3$

The viscosity of air at 25 °C and 10 atm = 0.018 cP

Hennery constant = 1640 atm / mol fraction

Solution

Since we are dealing with a case of fluid flow inside a pipe, then to calculate the mass transfer coefficient use the following equation:

$$Sh\ No = \frac{K_C^{'}\ d}{D_{AB}} = 0.023(\mathrm{Re})^{0.833}(Sc)^{0.333}$$

But we are dealing with mass transfer through stagnant layer (transfer of CO_2 from air to water only), then

$$K_C = K_C^{'}\frac{P_T}{P_{B_{LM}}}$$

Or

$$Sh\ No = \frac{K_C * \mathrm{P}_{\mathrm{B}_{\mathrm{LM}}} * d}{\mathrm{P}_\mathrm{T} * D_{AB}} = 0.023(\mathrm{Re})^{0.833}(Sc)^{0.333}$$

And

$$K_C = \frac{P_T * D_{AB}}{P_{B_{LM}} * d} * 0.023(\mathrm{Re})^{0.833}(Sc)^{0.333}$$

To calculate the dimensionless groups we must first correct the physical properties from their conditions to the operation condition (10 atm and 25 °C), also we must convert all the units of all the quantities to the SI system.

$$u = 3\frac{ft}{s} * \frac{1m}{3.28ft} = 0.915\frac{m}{s}$$

$$d = 2in = 2in * \frac{2.54cm}{in} * \frac{1m}{100cm} = 0.0508m$$

$$\mu = 0.018cP = 0.018 * 10^{-3}\frac{kg}{m * s}$$

$$\rho = \frac{p * mwt}{R * T}$$

Then

$$\frac{(\rho_2)_{298K,10atm}}{(\rho_1)_{STP}} = \frac{\left(\frac{P_2}{T_2}\right)}{\left(\frac{P_1}{T_1}\right)} = \frac{\frac{10}{298}}{\frac{1}{273}}$$

$$\rho_1 = 0.0808 \frac{lb}{ft^3} = 0.0808 \frac{lb}{ft^3} * \frac{1kg}{2.2lb} * \left(\frac{3.28 ft}{1m}\right)^3 = 1.296 \frac{kg}{m^3}$$

$$\therefore \rho_2 = 1.296 \frac{kg}{m^3} * \frac{10 * 273}{298} = 11.973 \frac{kg}{m^3}$$

$$\frac{(D_{AB})_2}{(D_{AB})_1} = \left(\frac{P_1}{P_2}\right) * \left(\frac{T_2}{T_1}\right)^{1.5}$$

$$(D_{AB})_1 = 0.164 \frac{cm^2}{s} = 0.164 \frac{cm^2}{s} * \left(\frac{1m}{100cm}\right)^2 = 1.64 * 10^{-5} \frac{m^2}{s}$$

$$(D_{AB}) = 1.64 * 10^{-5} * \left(\frac{1}{10}\right) * \left(\frac{298}{298}\right)^{1.5} = 1.64 * 10^{-6} \frac{m^2}{s}$$

$$P_{B_{Lm}} = \frac{P_{B_2} - P_{B_1}}{\ln\left(\frac{P_{B_2}}{P_{B_1}}\right)}$$

$$P_{A_1} = y_1 * P_T = 0.1 * 10 = 1 atm$$

$$P_{B1} = 10 - 1 = 9 atm$$

P_{A2} can be estimated from Henery law

$$\mathrm{P_A} = x_A * H$$

$$P_{A_2} = 0.005 * 1640 = 8.2 atm$$

$$P_{B_2} = 10 - 8.2 = 1.8 atm$$

$$\therefore P_{B_{LM}} = \frac{1.8 - 9}{\ln(\frac{1.8}{9})} = 4.473612$$

Now we can calculate the mass transfer coefficient

$$K_C = \frac{P_T * D_{AB}}{P_{B_{LM}} * d} * 0.023(\mathrm{Re})^{0.833}(Sc)^{0.333}$$

$$K_C = \frac{P_T * D_{AB}}{P_{B_{LM}} * d} * 0.023 * \left(\frac{\rho * u * d}{\mu}\right)^{0.833}\left(\frac{\mu}{\rho * D_{AB}}\right)^{0.333}$$

$$K_C = \frac{10 * 1.64 * 10^{-6} * 0.023}{4.473612 * 0.0508} * \left(\frac{11.973 * 0.915 * 0.0508}{0.018 * 10^{-3}}\right)^{0.833} * \left(\frac{0.018 * 10^{-3}}{11.973 * 1.64 * 10^{-6}}\right)^{0.333}$$

$$K_C = 0.008868\frac{m}{s}$$

$$N_A = K_g(P_{A_1} - P_{A_2})$$

$$K_g = \frac{K_C}{R * T} = 0.008868\frac{m}{s} * \frac{1}{0.082\frac{atm * l}{mol * K} * \frac{m^3}{1000l} * \frac{1000mol}{kmol} * 298K}$$

$$K_g = 3.62642 * 10^{-4}\frac{kmol}{m^2 * s * atm}$$

$$N_A = 3.62642 * 10^{-4} * (8.2 - 1) = 2.611 * 10^{-3}\frac{kmol}{m^2 * s}$$

	Rate equation		Units of coefficient
	(EMD)	Non-diffusing (B)	
For Gas	$N_A = K^{-}_{G}\ .\ \Delta P_A$	$N_A = K_G\ .\ \Delta P_A$	$\frac{\text{Moles transferred}}{(\text{time})\ (\text{area})(\text{press.})}$
	$N_A = K^{-}_{y}\ .\Delta y_A$	$N_A = K_y\ .\Delta y_A$	$\frac{\text{Moles transferred}}{(\text{time})\ (\text{area})\left(\frac{\text{mole}}{\text{fraction}}\right)}$
	$N_A = K^{-}_{C}\ .\ \Delta C_A$	$N_A = K_C\ .\ \Delta C_A$	$\frac{\text{Moles transferred}}{(\text{time})\ (\text{area})\left(\frac{\text{mole}}{\text{vol.}}\right)}$
conversions	$F = K_G.\ P_{BM} = K_y \frac{P_{BM}}{P_T} = K_C.\frac{P_{BM}}{R.\ T} = K^{-}_{G}.\ P_T = K^{-}_{y} = K^{-}_{C}\frac{P_T}{R.\ T} = K^{-}_{C}\ .G_T$		
For Liquid	$N_A = K^{-}_{L}\ .\ \Delta C_A$	$N_A = K_L\ .\ \Delta C_A$	$\frac{\text{Moles transferred}}{(\text{time})\ (\text{area})\left(\frac{\text{mole}}{\text{vol.}}\right)}$
	$N_A = K^{-}_{X}\ .\ \Delta X_A$	$N_A = K_X\ .\ \Delta X_A$	$\frac{\text{Moles transferred}}{(\text{time})\ (\text{area})\left(\frac{\text{mole}}{\text{fraction}}\right)}$
conversions	$F = K_X.\ X_{BM} = K_L.\ X_{BM}\ .C_T = K^{-}_{L}.\ C_T = K^{-}_{L}\frac{\rho}{M} = K^{-}_{X}$		

Relations among mass – transfer – coefficients.

Conversions	
$(N_B = 0)$	$(N_A = - N_B)$
$K_C = K_G.\ R.\ T$	$K^{-}_{C} = K^{-}_{y}.\ \frac{R.T}{P_T}$
$K_y = K_G.\ P_T$	$K_C = K^{-}_{G}.\ R.\ T$
$K_x = K_L.\ \left(\frac{\rho}{M_{WT}}\right)_{avg.}$	$K^{-}_{X} = K^{-}_{L}.\ C_{avg..}$ $= K^{-}_{L}.\left(\frac{\rho}{M_{WT}}\right)_{avg.}$

Film – Penetration theory

Toor and Marchello, in 1958, combined features of the film, penetration, and surface renewal theories to develop a film-penetration model, which predicts a dependency of the mass transfer coefficient Kc on the diffusivity, that varies from $\sqrt{D_{AB}}$ to D_{AB}. Their theory assumes that the entire resistance to mass transfer resides in a film of fixed thickness δ. Eddies move to and from the bulk fluid and this film. Age distributions for time spent in the film are of the Higbie or Danckwerts type.

Fick's second law, (Eq. 100), still applies, but the boundary conditions are now

At $t=0$ $\quad 0 \le Z \le \infty \quad C_A = C_{Ab}$

$t>0 \quad Z=0 \quad C_A = C_{Ai}$ $\quad$ (C_{Ai} : initial concentration)

$t>0 \quad Z=\delta \quad C_A = C_{Ab}$

Infinite-series solutions are obtained by the method of Laplace transforms. The rate of mass transfer is then obtained in the usual manner by applying Fick's first law at the fluid-fluid interface.

$$N_A = -D_{AB}\frac{\partial C_A}{\partial Z}\bigg|_{Z=0} = K_c\left(C_{A_i} - C_{A_0}\right)$$

Two – Film Theory

Separation processes that involve contacting two fluid phases generally require consideration of mass transfer resistances in both phases. In 1923, Whitman suggested an extension of the film theory to two fluid films in series. Each film presents a resistance to mass transfer, but concentrations in the two fluids at the interface are in equilibrium. That is, there is no additional interfacial resistance to mass transfer. This concept has found extensive application in modeling of steady-state gas-liquid and liquid-liquid separation processes, when the fluid phases are in laminar or turbulent flow. The assumption of equilibrium at the interface is satisfactory unless mass transfer rates are very high or surfactants accumulate at the interface.

Gas- Liquid Case

Consider the steady-state mass transfer of A from a gas phase, across an interface, into liquid phase. It could be postulated, as shown in Figure 2a, that a thin gas film exists on one side of the interface and a thin liquid film exists on the other side with the controlling factors being molecular diffusion through each of the films. However, this postulation is not necessary, because instead of writing the mass transfer rate as:

$$N_A = \frac{(D_{AB})_G}{\delta_G}\left(C_{A_b} - C_{A_i}\right)_G = \frac{(D_{AB})_L}{\delta_L}\left(C_{A_i} - C_{A_b}\right)_L \qquad \cdots \text{——} (4)$$

We can express the rate of mass transfer in terms of mass transfer coefficients that can be determined from any suitable theory, with the concentration gradients visualized more realistically as in Figure 2b. In addition, we can use any number of different mass transfer coefficients, depending on the selection of the driving force for mass transfer.

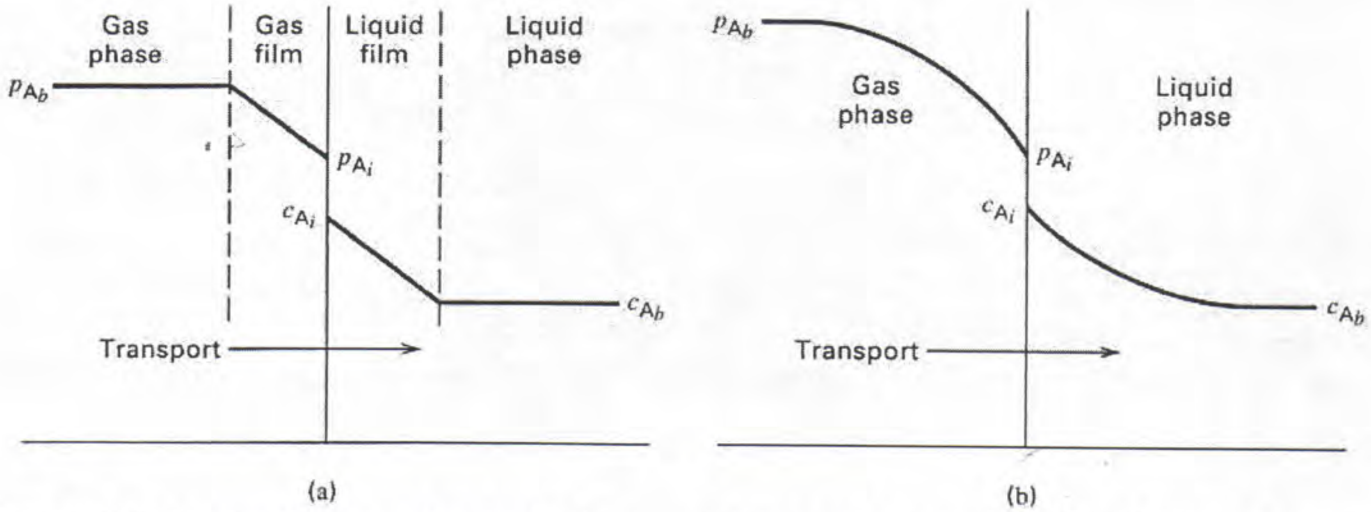


Figure 2 Concentration gradients for two – resistance theory: (a) film theory; (b) more realistic gradients.

For gas phase, under dilute or equimolar counter diffusion (EMD) conditions, we write the mass transfer rate in terms of partial pressure:

$$N_A = K_g^{'}\left(P_{A_b} - P_{A_i}\right) \qquad \text{------(5)}$$

Or for stagnant layer mass transfer (non – volatile liquid), the rate of mass transfer can be written as:

$$N_A = K_g\left(P_{A_b} - P_{A_i}\right) \qquad \text{----(6)}$$

Where (') refers to the equimolar counter diffusion case.

The above equations can be written in terms of mole fractions as:

$$N_A = \frac{K_g^{'}}{P_T}\left(y_{A_b} - y_{A_i}\right) = K_y^{'}\left(y_{A_b} - y_{A_i}\right) \qquad \text{---(7)}$$

And

$$N_A = \frac{K_g}{P_T}\left(y_{A_b} - y_{A_i}\right) = K_y\left(y_{A_b} - y_{A_i}\right) \qquad \text{-----(8)}$$

For the liquid phase, we might use molar concentrations:

$$N_A = K_l^{'}\left(C_{A_i} - C_{A_b}\right) \qquad \text{------(9)} \text{ for equi – molar mass transfer}$$

$$N_A = K_l\left(C_{A_i} - C_{A_b}\right) \qquad \text{------(10)} \text{ for uni - molar mass transfer}$$

Also can be written in terms of mole fractions:

$$N_A = \frac{K_l'}{C_T}\left(x_{A_i} - x_{A_b}\right) = K_x'\left(x_{A_i} - x_{A_b}\right) \quad ----(11) \text{ for equi – molar mass transfer}$$

And

$$N_A = \frac{K_l}{C_T}\left(x_{A_i} - x_{A_b}\right) = K_x\left(x_{A_i} - x_{A_b}\right) \quad ......(12) \text{ for uni - molar mass transfer}$$

At the phases interface, C_{A_i} and P_{A_i} are in equilibrium. Applying a version of Henry's law:

$$C_{A_i} = H_A * P_{A_i} \quad ------(13)$$

Equations (5), (9) and (13) are a commonly used combination for vapor-liquid mass transfer. Computations of mass transfer rates are generally made from knowledge of bulk concentrations, which in this case are C_{A_b} and P_{A_b}.

The equilibrium relationship for dilute solution (Henry's law) is:

$$P_A^* = H * x_A \quad -----(14)$$

And for gases (Dalton's Law):

$$P_A = P_T * y_A \quad ------(15)$$

Then

$$y_A^* = \frac{H}{P_T} * x_A = m * x_A \quad -----(16)$$

Or

$$y_A = m * x_A^* \quad ------(17)$$

And for the interface

$$y_i = m * x_i \quad -------(18)$$

Equilibrium data can be presented as a curve (for concentrated solution) and straight line (for dilute solution).

The task now is how to calculate the interfacial concentration (interfacial mole fraction) because we need them in the calculation of mass transfer rate.

Depending on the two film theory, and as shown in above sections we have two cases, the EMD (for example distillation), and the UMD (absorption through non – volatile liquid).

Case 1:

For equi-molecular counter diffusion

Let $y_A = y_{Ab}$

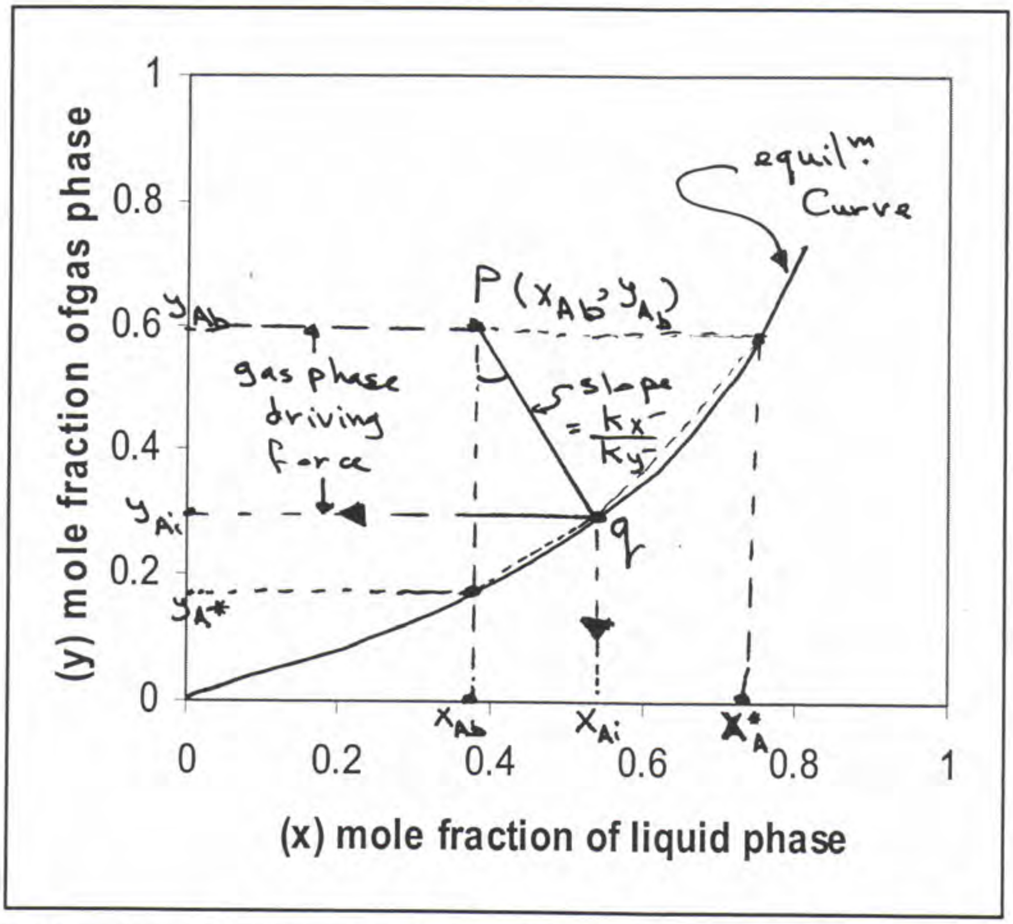
and $x_A = x_{Ab}$

$$N_A = K_y'\left(y_A - y_{A_i}\right) = K_x'\left(x_{A_i} - x_A\right) \quad ------(19)$$

Then

$$-\frac{K'_x}{K'_y} = \frac{(y_A - y_{A_i})}{(x_A - x_{A_i})} \quad \text{......(20)}$$

Assume a column where a gas and a liquid are contacted. At any point (P) in the column, the gas phase has a mole fraction of A (certain composition of A) that is y_A. And the liquid has a mole fraction of A (certain composition of A) that is x_A. Then at that point an equilibrium between the gas phase and the liquid phase exist at y_{A_i} and x_{A_i} at point (m)



In the above figure plot the equilibrium data and also and point (p). Draw a straight line from point (P) to intersect the equilibrium curve at point (1), let it be q.
The slop of this straight line (pq) is:

$$Slop = \frac{y_A - y_{A_i}}{x_A - x_{A_i}} = -\frac{K'_x}{K'_y}$$

Then, to estimate the interface composition, we must know the mass transfer coefficient for both gas phase and liquid phase also the equilibrium data must be known.
The equilibrium data are presented by a curve for concentrated solution, and by a straight line for dilute solution.

Case 2:

For uni-molecular diffusion (mass transfer of A through stagnant layer of B)

Let $y_A = y_{Ab}$

and $x_A = x_{Ab}$

$$N_A = K_y\left(y_A - y_{A_i}\right) = K_x\left(x_{A_i} - x_A\right) \qquad \text{------ --(21)}$$

Or

$$N_A = \frac{K_y'}{y_{A_{iLM}}}\left(y_A - y_{A_i}\right) = \frac{K_x'}{x_{A_{iLM}}}\left(x_{A_i} - x_A\right) \qquad \text{- - - ----(22)}$$

Then

$$-\frac{K_x}{K_y} = \frac{\left(y_A - y_{A_i}\right)}{\left(x_A - x_{A_i}\right)} \qquad \text{- ------ --(23)}$$

But the slop of the drawn between points (P) & (m) is

$$Slop = \frac{y_A - y_{A_i}}{x_A - x_{A_i}} = -\frac{K_x'}{K_y'}$$

Therefore substitute K_y and K_x by there equivalents as

$$K_y = \frac{K_y'}{y_{A_{iLM}}}$$

And

$$K_x = \frac{K_x'}{x_{A_{iLM}}}$$

These relations must be found from the previous section

Therefore the slop for this case is:

$$Slop = -\frac{K_x' / x_{A_{iLM}}}{K_y' / y_{A_{iLM}}} = \frac{y_A - y_{A_i}}{x_A - x_{A_i}}$$

Where

$$x_{A_{iLM}} = \frac{(1-x_A)-(1-x_{A_i})}{\ln\frac{(1-x_A)}{(1-x_{A_i})}}$$

And

$$y_{A_{iLM}} = \frac{(1-y_{A_i})-(1-y_A)}{\ln\frac{(1-y_{A_i})}{(1-y_A)}}$$

From the slop equation we can conclude that there is a difficulty in calculating the interface composition (y_{A_i} and x_{A_i}), because they are already exist in the left hand side of the slop equation. This problem will be solved by trail and error by following these steps:

1- Assume a value for ($y_{A_{iLM}}$ and $x_{A_{iLM}}$) and let it be equal to (1).
2- Calculate the slop.
3- Plot the line (pq).
4- From the intersection point read x_{Ai} and y_{Ai} .
5- Calculate ($y_{A_{iLM}}$ and $x_{A_{iLM}}$), then recalculate the slop of the line (pq) and let it be (slop)$_2$ if the value of the two slops are equal then the assumed values of ($y_{A_{iLM}}$ and $x_{A_{iLM}}$) is correct. If not, then use the value of (slop)$_2$ to estimate a new values of x_{Ai} and y_{Ai} by repeating steps (3-5) until you will reach not more the 10% change in the value of the slop.

Note:

For dilute solutions ($y_{A_{iLM}}$ and $x_{A_{iLM}}$) are equal to (1)

Overall driving force and overall mass transfer coefficient

Because of difficulties in measuring the interface composition and the individual mass transfer coefficients in some cases, another driving force and coefficients are used, that is the overall driving force and the overall mass transfer coefficient.

Again, assume a column where a gas and a liquid are contacted. At any point (P) in the column, the gas phase has a mole fraction of A (certain composition of A) that is y_A, and the liquid has a mole fraction of A (certain composition of A) that is x_A. Then at that point which we will call point (1), equilibrium between the gas phase and the liquid phase exist at y_{A_i} and x_{A_i}. From point P plot a vertical line to intersect the equilibrium curve at point (2), where $y = y_A^*$ and $x = x_A$. And plot a horizontal line from point (p) to intercept the equilibrium curve at point (3) at this point $y = y_A$ and $x = x_A^*$.

$(y_A - y_A^*)$ is called the overall driving force for the gas phase, and

$(x_A^* - x_A)$ is called the overall diving force for the liquid phase.

The slope of the equilibrium curve at point (1) is m_1:

$$m_1 = \frac{y_A - y_{A_i}}{x_A - x_{A_i}}$$

The slope of the equilibrium curve at point (2) is m_2:

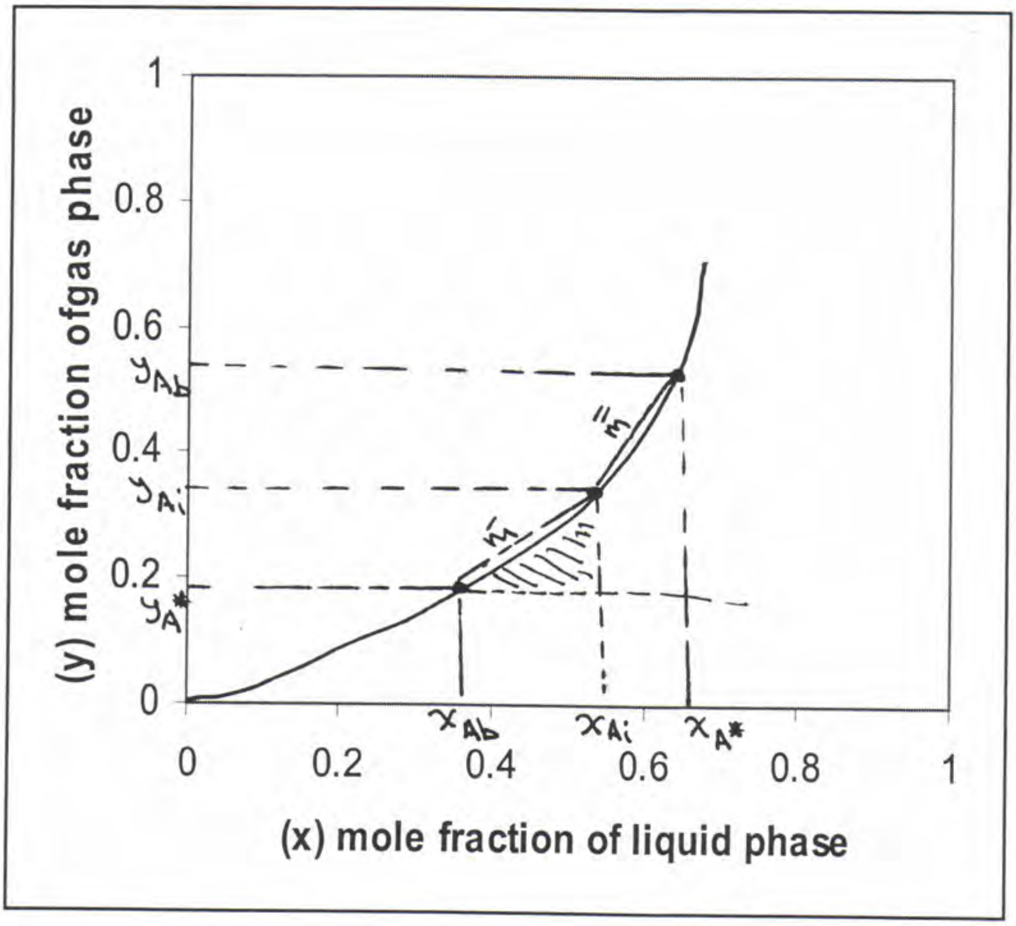
$$m_2 = \frac{y_{A_i} - y_A^*}{x_A - x_{A_i}}$$

The slope of the equilibrium curve at point (3) is m_3:

$$m_3 = \frac{y_A - y_{A_i}}{x_A^* - x_{A_i}}$$

The slops of the equilibrium curve at points 1, 2, and 3 are equivalent if the solution is a dilute solution, that is

$m_1 = m_2 = m_3$



Now, the rate of mass transfer which will be calculated based on overall driving force can be written as follow:

A) For EMD

$$N_A = K'_{oy}(y_A - y_A^*) \quad \text{------(24)}$$

B) For UMD

$$N_A = K_{oy}(y_A - y_A^*) \quad \text{------(25)}$$

Where

K'_{oy} and K_{oy} are the overall mass transfer coefficients with units (kmol/m^2. s.mol fract), which are based on the overall driving force in the gas phase.

y_A^* : is the value of the mole fraction of A in the gas phase that would be in equilibrium with x_A.

Or, and for the two cases

$$N_A = K'_{ox}(x_A^* - x_A) \quad \text{------(26)}$$

$$N_A = K_{ox}(x_A^* - x_A) \quad \text{------(27)}$$

Where

K'_{ox} and K_{ox} are the overall mass transfer coefficients with units (kmol/m^2. s.mol fract), which are based on the overall driving force in the liquid phase.

x_A^*: is the value of the mole fraction in the liquid phase that would be in equilibrium with y_A.

The relationship between the overall mass transfer coefficient and the individual mass transfer coefficient

A)Case 1 EMD

We can write the overall driving force as follow by adding and subtracting y_{A_i} , that is:

$$y_A - y_A^* = (y_A - y_{A_i}) + (y_{A_i} - y_A^*)$$

But

$$m_2 = \frac{y_{A_i} - y_A^*}{x_A - x_{A_i}}$$

Then

$$y_A - y_A^* = (y_A - y_{A_i}) + m_2(x_A - x_{A_i})$$

Substitute each driving force by its equivalent in the EMD case, that is:

$$\frac{N_A}{K'_{oy}} = \frac{N_A}{K'_y} + \frac{m_2 * N_A}{K'_x}$$

Finally

$$\frac{1}{K'_{oy}} = \frac{1}{K'_y} + \frac{m_2}{K'_x} \tag{28}$$

Equation 174 can be explained as follow:

The total resistance for mass transfer is equal to the summation of individual resistance for each phase 9the gas phase & the liquid phase).

The same procedure could be made to find the relationship between the over all mass transfer coefficient for the liquid phase and the individual mass transfer coefficients, the relation is:

$$\frac{1}{K'_{ox}} = \frac{1}{m_3 K'_y} + \frac{1}{K'_x} \tag{29}$$

For the dilute solutions:

$m_1 = m_2 = m_3 = m$

Then:

$$\frac{1}{K'_{oy}} = \frac{1}{K'_y} + \frac{m}{K'_x} \tag{30}$$

$$\frac{1}{K'_{ox}} = \frac{1}{m K'_y} + \frac{1}{K'_x} \tag{31}$$

B) Case 1 UMD

In this case only solute A is transferred through the interface and no B is transferred. As shown in above:

$$K_y = \frac{K'_y}{y_{A_{iLM}}}$$

And

$$K_x = \frac{K'_x}{x_{A_{iLM}}}$$

Then

$$N_A = \frac{K'_y}{y_{A_{iLM}}}\left(y_A - y_{A_i}\right) = \frac{K'_x}{x_{A_{iLM}}}\left(x_{A_i} - x_A\right) \tag{32}$$

For the over all driving force, and by using the over all mass transfer coefficient, the mass transfer rate is calculated by:

$$N_A = \frac{K'_{oy}}{y^*_{A_{LM}}}\left(y_A - y^*_A\right) = \frac{K'_{ox}}{x^*_{A_{LM}}}\left(x^*_A - x_A\right) \qquad \text{-------- (33)}$$

Where:

$$x^*_{A_{LM}} = \frac{(1-x_A)-(1-x^*_A)}{\ln\frac{(1-x_A)}{(1-x^*_A)}}$$

And

$$y^*_{A_{LM}} = \frac{(1-y^*_A)-(1-y_A)}{\ln\frac{(1-y^*_A)}{(1-y_A)}}$$

Using the same procedure done in the EMD, we can find the relationship between the overall mass transfer coefficient and the individual mass transfer coefficients, as shown:

For the gas phase:

$$\frac{y^*_{A_{LM}}}{K'_{oy}} = \frac{y_{A_{iLM}}}{K'_y} + \frac{m_2 x_{A_{iLM}}}{K'_x} \qquad \text{-------- (34)}$$

For the liquid phase:

$$\frac{x^*_{A_{Lm}}}{K'_{ox}} = \frac{y_{A_{iLM}}}{m_3 K'_y} + \frac{x_{A_{iLM}}}{K'_x} \qquad \text{-------- (35)}$$

Again for dilute solution when:

$m_1 = m_2 = m_3 = m$

Then

$x^*_{A_{LM}} = y^*_{A_{LM}} = x_{A_{iLM}} = y_{A_{iLM}} = 1$

Question:

Is there an overall mass transfer coefficient base on partial pressure for the gas phase, and other one based on concentration for the liquid phase?

Answer:

Yes, these are presented as K_{og}, and K_{ol}.

Question:

Find the relationships between these overall mass transfer coefficients and the individual mass transfer coefficient?