الجامعة التكنولوجية

قسم الهندسة الكيمياوية

المرحلة الثالثة

انتقال كتلة

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(3) Continuous - multistage - fractionation of binary mixture

Seperation of a volatile liquid mixture to relatively pure products is very often done in a continuous fractionating column. A continuous column is much more effective than multistage flash or batch dist^n. A tray column and flow rates and the conc^n. of the vapour and the liquid phases at different trays are shown in Fig. :-

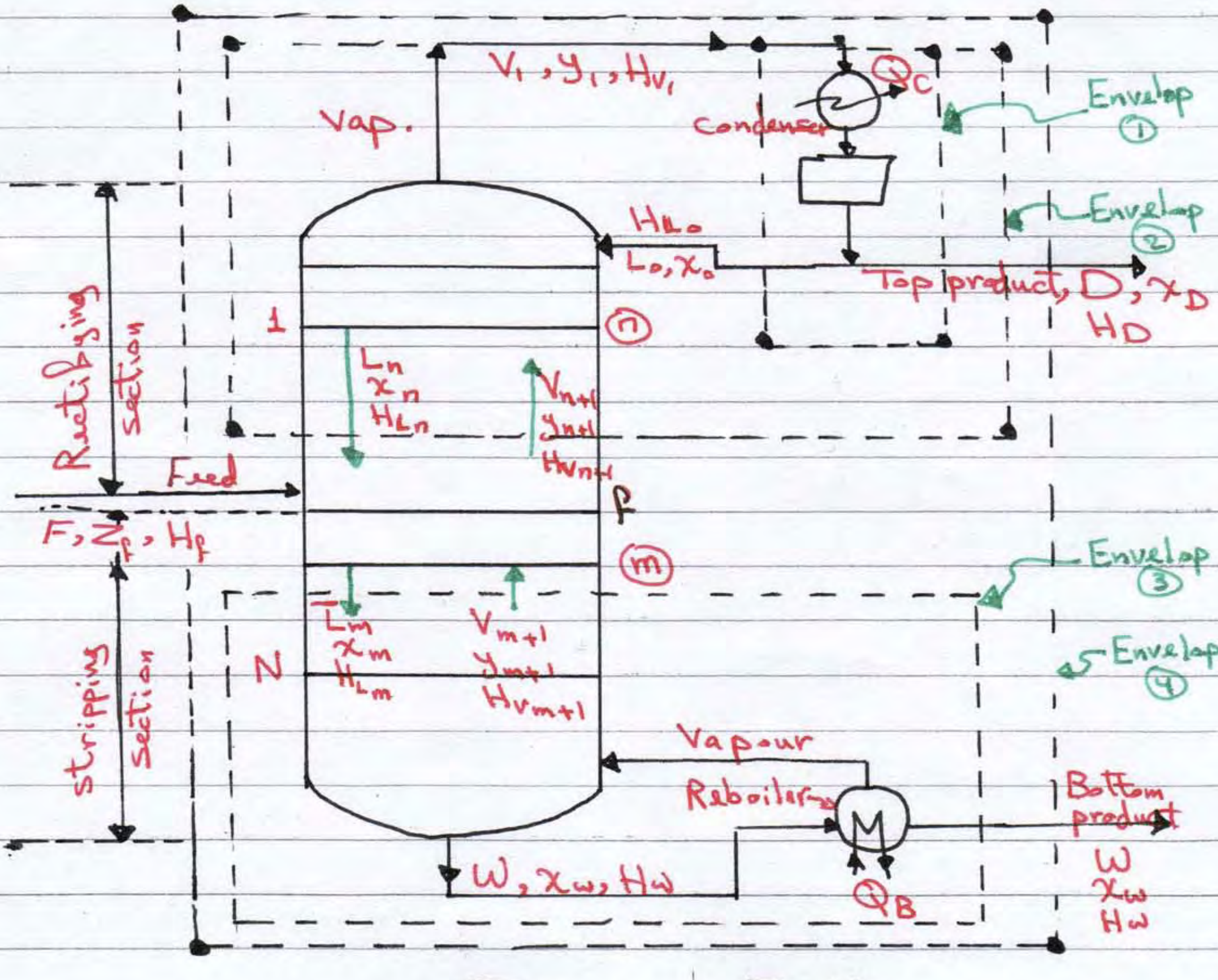


Fig.(1)

* For a binary liquid mixture, the feed (liquid, vapour or two-phase mixture) containing component A and B. (A) is the M.V.C enters the column at a suitable location. The liquid stream flows down the column from one tray to the next lower tray, the vapour stream flows up bubbling through the liquid on the trays.
* the vapour from the top tray is condensed and the condensate is collected in a reflux drum. A part of this liquid is drawn as the "top-product" and the other part is fed back to the top tray (no. 1) as reflux. The top product contains the M.V.C (A) and a little of the L.V.C. (B).
* The liquid from the bottom tray goes to a reboiler where it is partly vaporized, the vapour is fed back to the tower and the liquid part is continously withdrawn as the bottom product. The bottom product is rich in the L.V.C. (B) and has a small amount of (A) in it.
* Transport of the M.V.C (A) occurs from the liquid to the vapour, while transport of the L.V.C (B) occurs from the vapour to the liquid phase. Thus, a distill^n. column involves counter-diffusion of the components (not necessarily E.M.D). As the vapour flows up, it becomes richer in (A), similarly, the liquid becomes richer in the L.V.C (B) as it flows down the column.

* In the section of the column above the feed point, the conc. of the M.V.C. is Larger than that in the feed. This means that the vap. is enriched or purified by discarding the L.V.C. (B) into the down flowing liquid. So the section above the feed tray is called the "rectifying or enriching" section.
For the section below, the feed tray, the M.V.C. is removed or stripped out of the liquid, so this is called the "stripping section".

* Variables, parameters and factors involved in the design of a trayed distill^n. tower.

- The flow rate, composition and state of the feed.
- The required degree of seperation.
- The reflux ratio and the condition of the reflux
- The operating pressure and the allowable press. drop across the column.
- Tray type and column internals.

Material and Energy Balance Equations

Reflux ratio can be defined as :-

$$R = \frac{L_o}{D} \Longrightarrow \boxed{L_o = D \cdot R} \text{ ----- } (1)$$

* Now consider envelope (1) in fig.(1), enclosing the condenser and reflux :-

- Total material balance :-

$$V_1 = L_o + D = D \cdot R + D \Longrightarrow \boxed{V_1 = D(R+1)} \text{ .... } (2)$$

- Component (A) balance :-

$$V_1 y_1 = L_o x_o + D \cdot x_D \text{ ----- } (3)$$

- Energy balance :-

$$V_1 \cdot H_{V_1} = L_o H_{L_o} + D \cdot H_D + Q_C \text{ --- } (4)$$

From equ^{ns} (2) and (4) :-

$$D(R+1) H_{V_1} = L_o H_{L_o} + D \cdot H_D + Q_C$$

$$\therefore \boxed{Q_C = D\left[(R+1) H_{V_1} - R \cdot H_{L_o} - H_D\right]} \text{ .... } (5)$$

↳ Condenser heat load equ^n.

* for envelope (2), enclosing apart of the rectifying and the condenser.

- Total material balance

$$V_{n+1} = L_n + D \text{ ---- } (6)$$

- Component (A) balance:-

$$V_{n+1} \cdot y_{n+1} = L_n x_n + D \cdot x_D \quad --- \text{(7)}$$

- Energy-balance:-

$$V_{n+1} \cdot H_{V_{n+1}} = L_n H_{L_n} + D \cdot H_D + Q_C \quad -- \text{(8)}$$

* For envelope (3), enclosing a part of stripping section and the reboiler.

- Total material balance:-

$$\bar{L}_m = \bar{V}_{m+1} + W \quad --- \text{(9)}$$

- Component (A) balance:-

$$\bar{L}_m \cdot x_m = \bar{V}_{m+1} \cdot y_{m+1} + W \cdot x_w \quad --- \text{(10)}$$

- Energy balance:-

$$\bar{L}_m H_{L_m} + Q_B = \bar{V}_{m+1} \cdot H_{V_{m+1}} + W \cdot H_w \quad ---- \text{(11)}$$

(*) Envelope (4) the entire column:-

- Total material balance:-

$$F = D + W \quad ----- \text{(12)}$$

- Component (A) balance:-

$$F \cdot Z_F = D \cdot x_D + W \cdot x_w \quad --- \text{(13)}$$

- Energy balance:-

$$F \cdot H_F + Q_B = D \cdot H_D + W \cdot H_w + Q_C \quad ---- \text{(14)}$$

* All the above equations can be solved algebraically to determine the number of ideal trays required.

Determination of the number of trays using Mc Cabe-Thiele-method "Graphical-method"

The Mccabe-Thiele method involves a graphical solution of the material balance equations, together with the equilibrium relation or equilibrium data.

Assumptions :-

1- The most important assumption, is that the molar rate of over flow of the liquid from one tray to another is constant over any section.

$L_o = L_1 = L_2 \ldots\ldots L_m = L$ = constant in rectifying section.

$\bar{L}_m = \bar{L}_{m+1} = \ldots\ldots \bar{L}_N = \bar{L}$ = " " stripping section

2- Heat loss from the column is negligible.

It is to be noted that if there is heat loss or gain, there will be accompanying condensation or vaporization within the column, and the flow rates will vary along the column as a result.

The major steps of the graphical construction in this method are :-

* The equil^m. curve using the available data.
* The operating lines for rectifying and stripping sections.
* The steps between the equil^m. and operating lines to find out The number of ideal plates and trays.

* The Rectifying section :-

From eq. 7 :-

$$V.y_{n+1} = L.x_n + D.x_D \quad \text{------} \quad (15)$$

From eq. 1 :-

$$R = \frac{L_o}{D} = \frac{L}{D}$$

$$\therefore (R+1) = \frac{V}{D} \qquad \text{from eq}^n. (2)$$

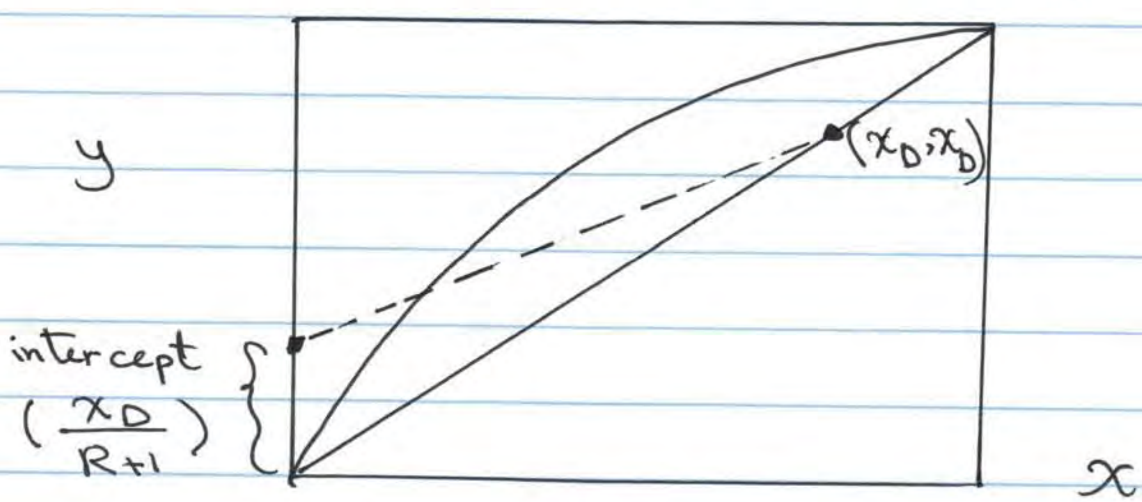
$$y_{n+1} = \frac{L}{V} x_n + \frac{D}{V} x_D$$

$$= \frac{L/D}{L/D} x_n + \frac{x_D}{V/D}$$

$$\therefore \boxed{y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1}} \quad \text{------} \quad (16)$$

Eq. (16) is straight line eq. with slope $= \frac{R}{R+1}$ and intercept $\left(\frac{x_D}{R+1}\right)$ on the y-axis.

To plote the line use point (x_D, x_D) on the diagonal and the inter cept term, rather than using the slope.



(*) The stripping section :-

From eq. (10) :-

$$\bar{L} \cdot x_m = \bar{V} \cdot y_{m+1} + W \cdot x_w \quad \text{------} (17)$$

putting $\bar{V} = \bar{L} - W \longrightarrow T.M.B$ $\qquad \bar{L} = \bar{V} + W$

$$\therefore y_{m+1} = \frac{\bar{L}}{\bar{V}} x_m - \frac{W}{\bar{V}} x_w$$

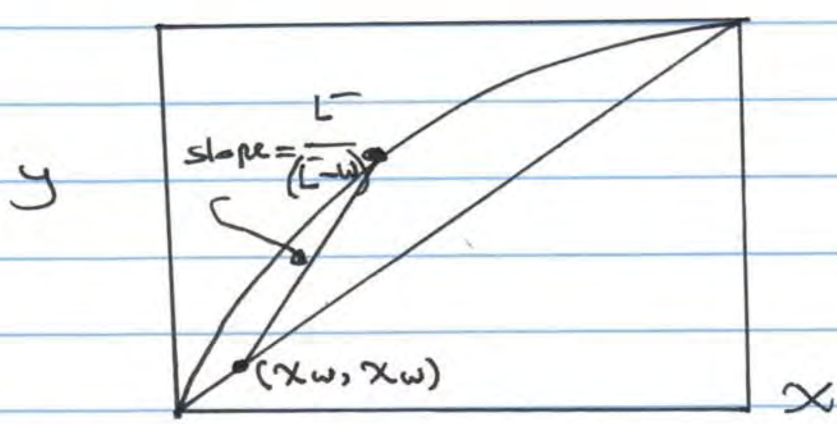
$$\boxed{y_{m+1} = \frac{\bar{L}}{\bar{L} - W} x_m - \frac{W}{\bar{L} - W} x_w} \quad \text{---} (18)$$

This is the eq. for the stripping section, passes through point (x_w, x_w), with slope equal to $\bar{L}/(\bar{L} - w)$.

Similar to reflux ratio defined in a rectified section, we may define a quantity called "boil-up-ratio" (R_v), for this section.

$$R_v = \frac{\text{moles of vap. leaving the reboiler per hour.}}{\text{moles of liq. drawn as the bott. product per hour.}}$$

$$\boxed{R_v = \frac{\bar{V}}{W}} \quad \text{------} (19)$$



Ex.(1) :- (How to draw operating lines for both sections)

A mixture of benzene and toluene containing 40 mol % benzene is to be seperated continuously in a tray tower at a rate of 200 kmol / hr. The top product should have 95 mol % of benzene and the bottom must not contain more than 4 mol % of it. The reflux is a saturated liquid, and the reflux ratio of 2.0 is maintained.

The feed is a saturated liquid (at its bubble point).

Obtain and plot the operating lines for both sections.

What is the boil-up-ratio?

The (V.L.E) data at 101.3 KPa are :-

x:	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y:	0	0.21	0.38	0.51	0.63	0.72	0.79	0.85	0.91	0.96	1.0

Sol. $F = 200$ kmol / hr, $Z_f = 0.4$, $x_D = 0.95$, $x_w = 0.04$, $R = 2$

Total M.B :-

$F = D + W \Rightarrow 200 = D + W \Rightarrow W = 200 - D$

Benz. M.B :-

$F * Z_f = D \cdot x_D + W \cdot x_w$

$200 * 0.4 = D * 0.95 + W * 0.04$

$80 = 0.95 D + 0.04 (200 - D)$

$\therefore$ $\underline{D = 79.12}$ kmol / hr , $\underline{W = 120.9}$ kmol / h

$R = \frac{L_o}{D} = 2 \Rightarrow L_o = 158.24$ kmol / hr

$V_1 = D (R + 1) \Rightarrow V_1 = 237.3$ kmol / hr

$\boxed{V_1 = V}$ Feed is liquid, and vap. rate remain constant in Rect. section.

In stripp. section :-

$\bar{L} = L_o + 200$

$\bar{L} = 158.2 + 200 = 358.2 \frac{kmol}{hr}$ and $V = \bar{V} = 237.3 \frac{kmol}{hr}$

operating lines :-

(a) rectifying section

$$y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1}$$

$$y_{n+1} = \frac{2}{3} x_n + \frac{0.95}{3}$$

$$\boxed{y_{n+1} = 0.667\, x_n + 0.317} \text{ --- } (*)$$

(b) Stripping section :-

$$y_{m+1} = \frac{\bar{L}}{\bar{L}-W} x_m - \frac{W}{\bar{L}-W} x_w$$

$$y_{m+1} = \frac{358.2}{358.2-120.9} x_m - \frac{120.9}{358.2-120.9} * 0.04$$

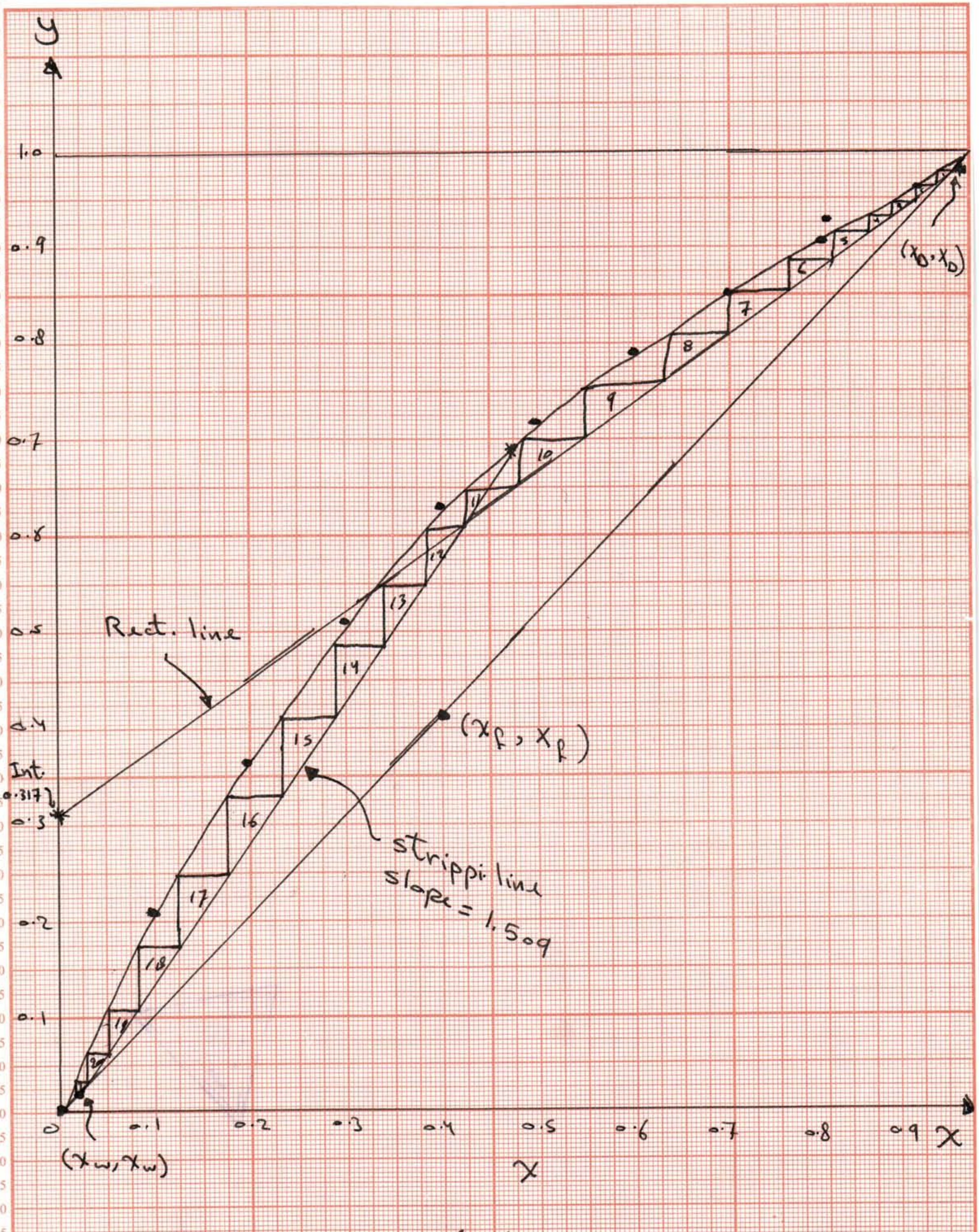
$$\boxed{y_{m+1} = 1.509\, x_m - 0.0204} \text{ --- } (**)$$

Now plot equilibrium data and both operating lines.

The boil-up ratio $R_V = \frac{\bar{V}}{W} = \frac{237.3}{120.9}$

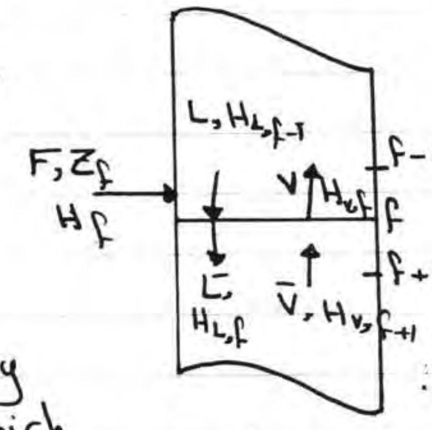
$$R_V = 1.963$$

∴ From Fig.: The number of stages :- 21


y
1.0
0.9
0.8
0.7
0.6
0.5
0.4
Int.
0.317
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
x
Rect. line
strippi line
slope = 1.509
(x_f, x_f)
(x_w, x_w)
(x_D, x_D)

The Feed Line :-

As the feed enters the column, the liquid and vapour flow rates undergo step changes depending up on the state of the feed (how much liq., and vap.).



Feed -plate

We shall write the material and energy balance equations over the plate to which the feed is introduced, this plate will be written by suffix "f".

T.M.B :-

$$F + L + \bar{V} = \bar{L} + V \quad ------ (20)$$

Energy - Balance :-

$$F \cdot H_F + L \cdot H_{L,f-1} + \bar{V} \cdot H_{V,f+1} = \bar{L} \cdot H_{L,f} + V \cdot H_{V,f} \quad ---- (21)$$

Assume :-
$$\left. \begin{aligned} H_{L,f-1} &= H_{L,f} = H_L \\ H_{V,f+1} &= H_{V,f} = H_V \end{aligned} \right\} \quad ---- (22)$$

Now subsitute (22) in (21) :-

$$F \cdot H_F + L \cdot H_L + \bar{V} \cdot H_V = \bar{L} \cdot H_L + V \cdot H_V$$

$$(\bar{L} - L) H_L = (\bar{V} - V) H_V + F \cdot H_F \quad ------ (23)$$

From eq. (20) and (23) :-

q = Amount of liq. in feed

$$\boxed{\frac{\bar{L} - L}{F} = \frac{H_F - H_V}{H_L - H_V} = \frac{H_V - H_F}{\underbrace{H_V - H_L}_{-\lambda}} = q} \quad --- (24)$$

Super heated or sub cooled

on the basis of eq. (24), we may define :-

$$q = \frac{\text{Heat required to convert 1 mole feed to sat. vap.}}{\text{molar heat of vaporiz}^n \text{ of the sat. liquid.}}$$

Now if :-

a) $q = 1$ liquid at bubble pt. (sat. liq.)

b) $q = 0$ vapour at dew. pt (sat. vap.)

c) if the feed is a two-phase mixture (liq. + vap.), then (q) represent the fraction of liquid in it.

∴ $(1-q)$ gives a measure of the "quality" of feed.

Simplyfing and recombination equations :-

$$\boxed{y = \frac{q}{q-1}\, x - \frac{z_f}{q-1}} \quad \text{.... (25) Feed line equation}$$

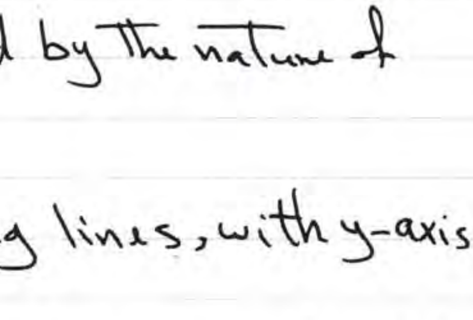
sub cooled or super heated

$\frac{q}{q-1}$ = slope of the line, governed by the nature of the feed

$-\frac{z_f}{q-1}$ = intersection of the operating lines, with y-axis

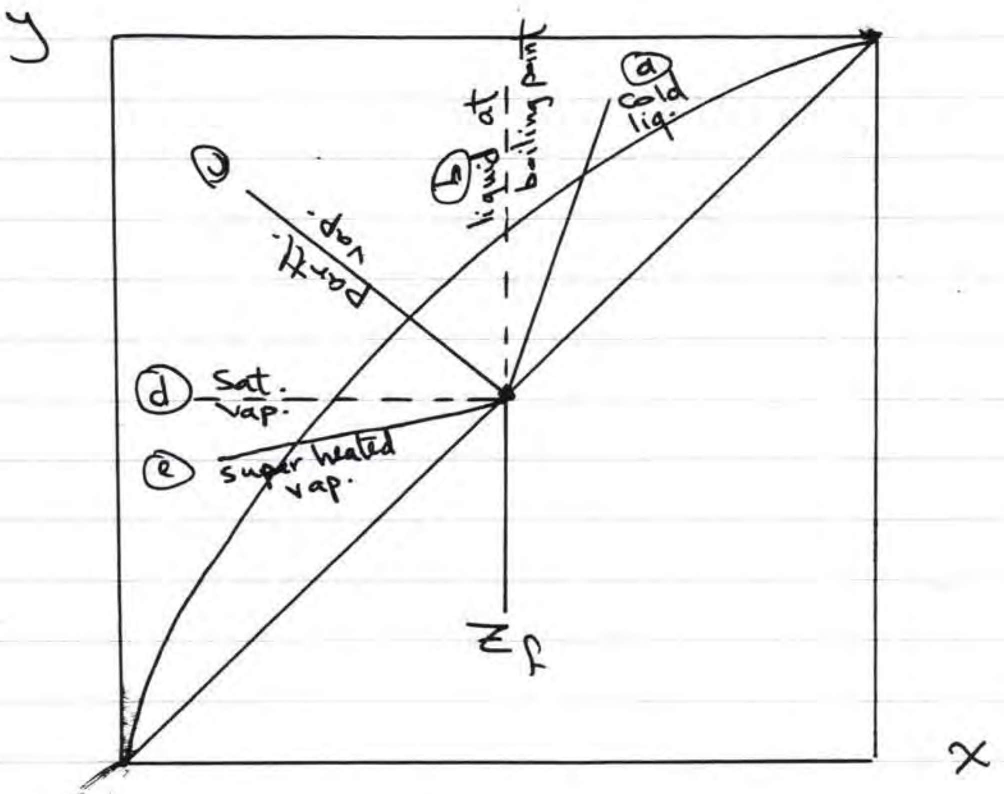
$\frac{z_f}{q}$ = " " " " " x-axis

Notes :- The point of intersection of upper operating line (Rect. line) and feed line, can be joined with (x_w, x_w) to draw stripping section line.



Feed Conditions :-

	(q)	shape	slope
a) Cold feed (liquor)	$q > 1$	/	(+ve)
b) Feed at boiling point	$q = 1$	|	(∞)
c) Feed partly vapour	$0 < q < 1$	\	(−ve)
d) Feed saturated vapour	$q = 0$	—	(0)
e) Feed superheated vapour	$q < 0$	/	(+ve)



Effect of the feed condition at fixed reflux ratio.

Eqn of q. line | point of q line

(a) $y = \frac{q}{q-1}x - \frac{Z_f}{q-1}$ — $(Z_f, Z_f), \left(\frac{Z_f}{q}, 0\right)$

(b) $x = Z_f$ — vertical through (Z_f).

(c) $y = -\left(\frac{1-f_v}{f_v}\right)x + \frac{Z_f}{f_v}$ — $(Z_f, Z_f), \left(0, \frac{Z_f}{f_v}\right)$.

(d) $y = Z_f$ — horiz. through (Z_f).

(e) $y = \frac{q}{q-1}x - \frac{Z_f}{q-1}$ — $(Z_f, Z_f), \left(0, \frac{Z_f}{1-q}\right)$

(H.W) Try to obtain the above relations.

ⓐ If the feed is saturated liquid, $q = 1$, the slope of the feed line is (∞), and feed line will be vertical line through (Z_f, Z_f).

ⓑ If the feed is a saturated vapour, $q = 0$, the slope of the feed line is zero, and feed line will be horizental line through (Z_f, Z_f).

ⓒ If the feed is a mixture of liquid and vapor or superheated vap. or subcooled liquid, then the slope of the feed line can be calculated using equation (24), where :-

$$\boxed{H_L = C_{P_s} \cdot M_{av.} \cdot (T - T_o) + \Delta H_s} \quad \text{Enthalpy of liquid.} \qquad (26)$$

H_L = molar enthalpy of solution at temp. T, kJ/kmol.
C_{P_s} = Specific heat of solution, kJ/kg.K.
$M_{av.}$ = average molecular weight of solution.
T_o = ref. Temp.
ΔH_s = heat of solution at ref. Temp. T_o, kJ/kmol.

$$\boxed{H_V = \sum y_i^* * M_i \left[C_{P_i} (T - T_o) + \lambda_i \right]} \quad \text{Enthalpy of vapour.} \qquad (27)$$

H_V = Enthalpy of the vapour, kJ/kmol
λ = heat of vaporization, kJ/kg.

$H_F = \sum$ Latent heat + Sensible heat

$$\boxed{H_F = \sum (x_i * M_i * \lambda_i) + (x_i * C_{P_i} * (T - T_o))} \qquad (28)$$

H_F = Enthalpy of the feed.

Ex.(2) :- (How to draw feed line for different feed conditions)

A mixture of benzene and toluene containing 58% mole benzene is to be seperated in a continuous column operating at 1 atm total press. Draw the feed line for the following feed condition :-

ⓐ Saturated vapour. (at dew point).
ⓑ Saturated liquid. (at bubble point).
ⓒ 65 mass % vapour. (partial vaporiza)
ⓓ vapour at 120 °C. (super-heated)
ⓔ liquid at 50°C. (subcooled)

Given that:-

$Cp_{(B)L} = 146.5 \quad kJ/kmol.K$

$Cp_{(B)V} = 97.6 \ kJ/kmol.K$

$M_B \lambda_{(B)} = 30,770 \quad kJ/kmol.$

$Cp_{(T)L} = 170 \ kJ/kmol.K$

$Cp_{(T)V} = 124.3 \quad kJ/kmol.K$

$M_T \lambda_{(T)} = 32,120 \quad kJ/kmol.$

ref. Temp. = 90 °C

Sol. :-

Using eq. (25) to draw feed line.

$$y = \frac{q}{q-1} x - \frac{z_f}{q-1}$$

(a) For saturated vapour. $\therefore q = 0$

Slope $= \frac{0}{0-1} \Rightarrow$ horizental line through point F (z_f, z_f).

(b) For saturated liquid. $\therefore q = 1$

slope $= \frac{q}{q-1} = \infty \Rightarrow$ vertical line through point F.

(c) For 65% vap. $\Rightarrow \therefore$ feed contain 35% liquid.

$\therefore q = 0.35 \Rightarrow$ slope $= \frac{0.35}{0.35 - 1} = -0.538$

intercept $= \frac{-z_f}{q-1} \Rightarrow$ inter. $= \frac{-0.58}{0.35 - 1} = 0.892$

We have (F) point and intercept.

(d) For vapour at 120°C $\Rightarrow$ super heated vapour.

$\therefore$ Eq. (24) is to be used.

$$q = (H_v - H_f) / (H_v - H_L)$$

$\Delta H_s = 0$ (pure component)

$H_L = Cp_{sol.} \cdot M_{av} (T - T_o) + \cancel{\Delta H_s}^{\,0}$

For the solution at its bubble pt ($T = 90\,^\circ C$), take $T_o = 90$: then :-

$\boxed{H_L = 0}$ for $\Delta T = 0$

(*) Enthalpy of vapour :- for $x = 0.58$, $y^* = 0.78$ (من الرسم)

$$H_V = y^*_B \cdot M_B [Cp_B \cancel{(T - T_o)}^{\,0} + \lambda_B] + y^*_T M_T [\cancel{Cp_T (T - T_o)}^{\,0} + \lambda_T]$$

$H_V = 0.78 * 30770 + (1 - 0.78) * 32120$

$H_V = 31{,}067$ KJ/kmol.

(*) Feed enthalpy :-

We assume that 0.58 kmol benzene and 0.42 kmol of toluene are vaporized seperately at $90\,^\circ c$, the vapours are heated to $120\,^\circ c$ and then mixed

$$H_F = 0.58 * 30770 + 0.58 * 97.6 (120 - 90)] + [0.42 * 32120 + 0.42 (124.3)(120 - 90)]$$

$H_F = 34601$

Now :- $q = \dfrac{H_V - H_F}{H_V - H_L}$