

الجامعة التكنولوجية

قسم الهندسة الكيمائية

المرحلة الثالثة

انتقال كتلة

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$$q = \frac{31067 - 34601}{31067 - 0} = -0.114$$

$$\text{intercept} = \frac{-Z_F}{q-1} = \frac{-0.58}{-0.114-1} = 0.521$$

② The Feed is subcooled at 50°C

We assume that 0.58 kmol of (B) and (0.42) kmol of (T) are cooled from ref. Temp. ($T_0 = 90^\circ\text{C}$) to ($T = 50^\circ\text{C}$) and then mixed to get 1 kmol of feed.

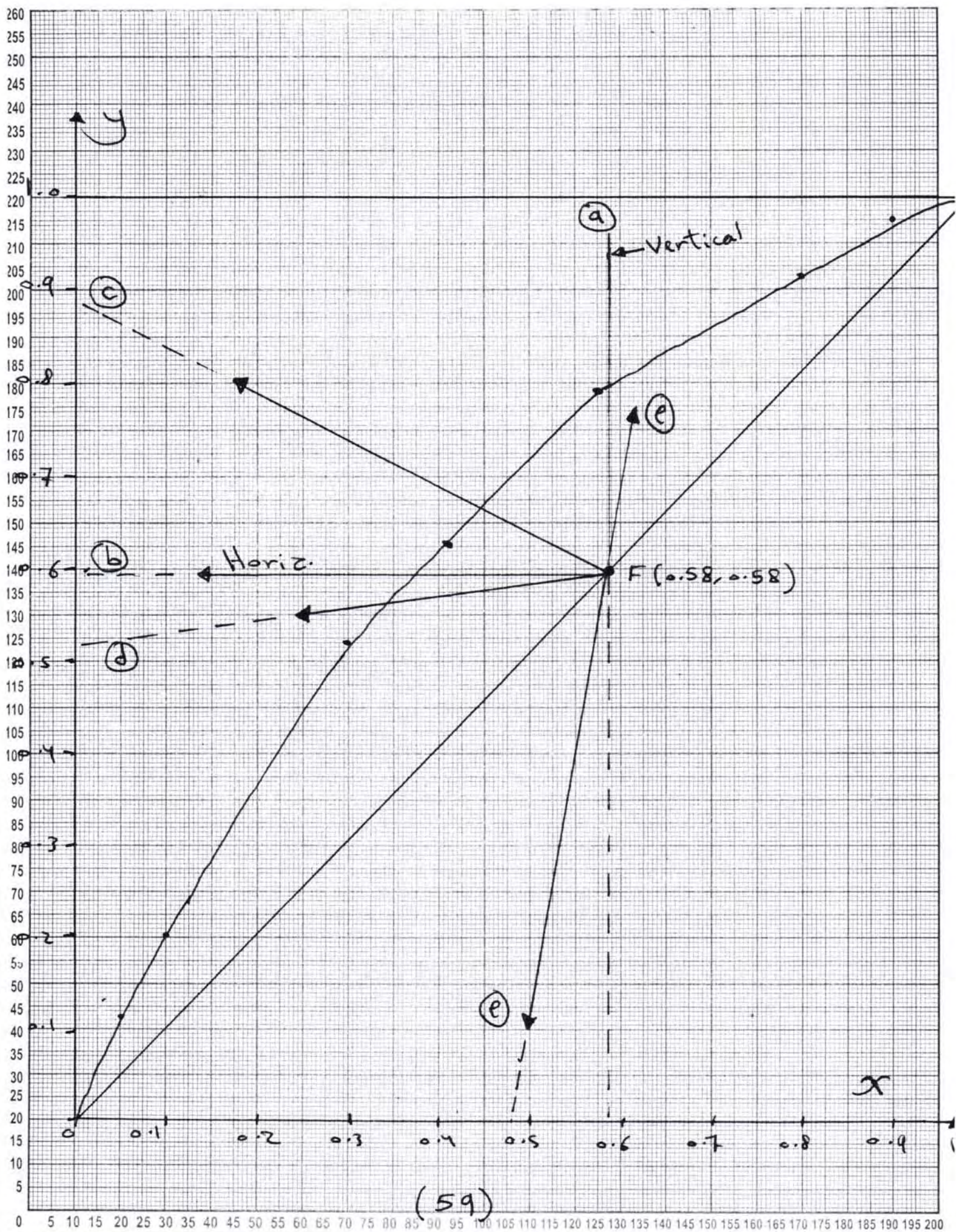
$$\therefore H_F = 0.58 \times 146.5 \times (50 - 90) + 0.42 \times 170 \times (50 - 90)$$

$$H_F = -6240 \text{ kJ/kmol.}$$

$$\therefore q = \frac{31067 - (-6240)}{31067 - 0} = 1.2$$

$$\text{intercept with } x\text{-axis} = \frac{Z_F}{q} = \frac{0.58}{1.2} = 0.484$$

Note:- ① For liq. feed, it is introduced just above the feed tray. But a vapour feed is introduced just below it.
 ② If the feed is a mixture of liq. and vap., it is desirable that it is separated into the vap., and the liq. phases first. The liq. part should enter the column just above feed tray, and the vap. part just below it. However, this is not always done in practice, and a mixed feed is often introduced as a whole over the feed tray.



(59)

Ex. (3) = Determination of No. of ideal trays.

A stream of aqueous methanol having 45 mol% CH_3OH is to be separated into a top product having 96 mol% methanol and a bottom liquid with 4% methanol. The feed is at its bubble point and the operating pressure is 101.3 kPa. A reflux ratio of 1.5 is used.

(a) Determine the No. of ideal trays.

(b) Find the No. of real trays if overall tray eff. is 40%. On which real tray should the feed introduced.

The equilib^m. Data for the system.

X:	0	0.02	0.04	0.06	0.08	0.1	0.2	0.3	...	0.7	0.8	0.9	1
y:	0	0.134	0.23	0.304	0.365	0.41	0.58	0.66	...	0.87	0.92	0.96	1
T:	100	96.4	93.5	91.2	89.3	87.7	84.4	78.0	...	69.3	67.6	66	64

Solution: $x_D = 0.96$, $x_W = 0.04$, $Z_F = 0.45$

Plot equil^m. data, the points D(0.96, 0.96), W(0.04, 0.04) and F(0.45, 0.45) are located on the x-y plane.

(a) The upper operating line is drawn through the point (D) with the intercept with the y-axis

$$\text{Intercept} = \frac{x_D}{R+1} = \frac{0.96}{1.5+1} = 0.384$$

\therefore Feed is a saturated liquid, then feed line is vertical line through point (F).

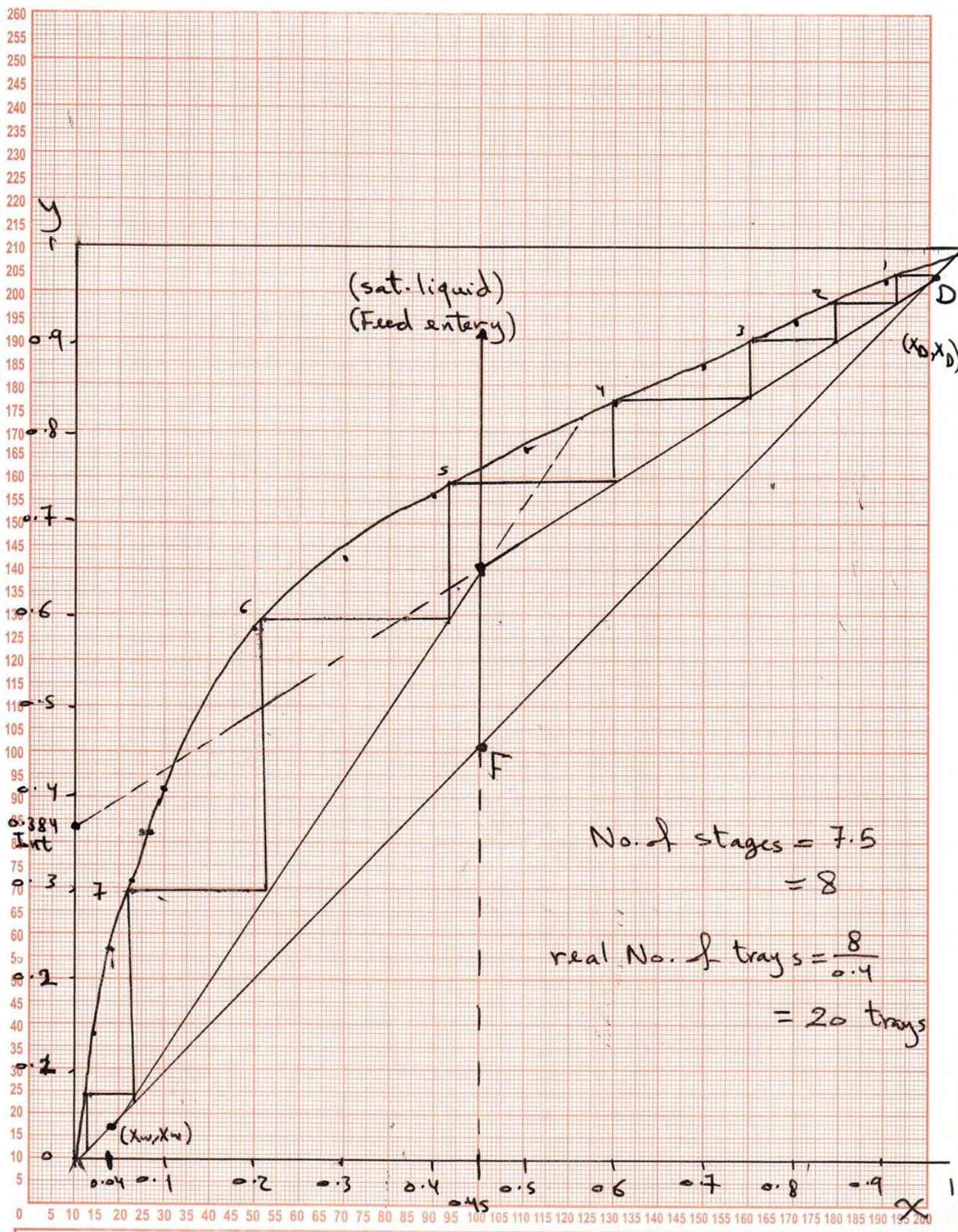
* The upper operating line meets the feed line at point (M).

* The point (M) and (W) are joined to get the lower or stripping section operating line.

① No. of ideal plates = 7.5 From the figure
 ≈ 8

No. of real plates = $8 / 0.4$
= 20 trays plus the reboiler
which is assumed to act like an ideal stage.

* Feed enters in a place between (4-5)



(2)

Analytical Determination of The No. of Ideal Stages.

(a) Total Reflux \Rightarrow Give (N_{\min})

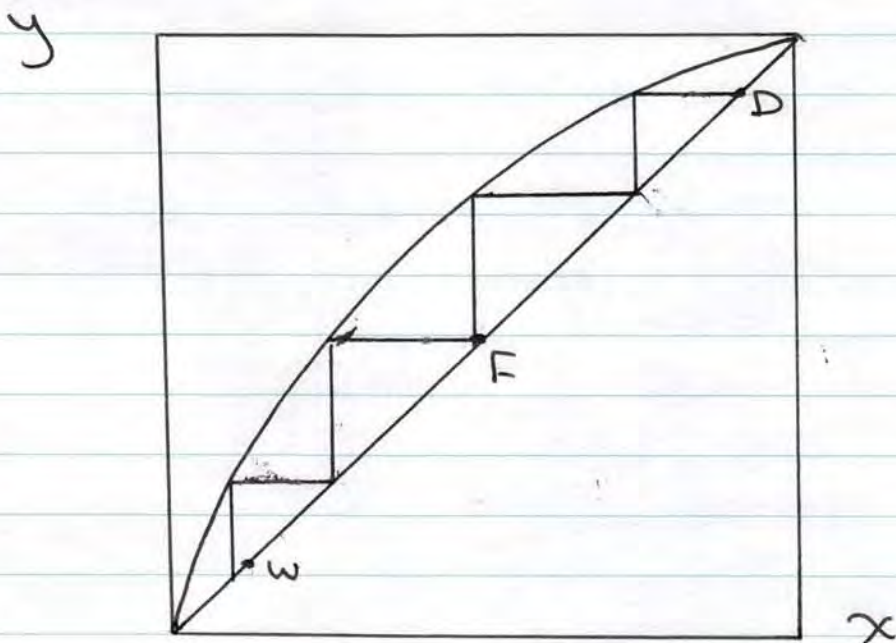
$\therefore R = \frac{L_0}{D}$ and at total (R) where no distillate is accumulated then $D = 0$

$\therefore R = \infty$ and No. product is drawn from reboiler

At total reflux, the slope of the rectifying section operating line is $\left(\frac{R}{R+1}\right)$ for $R \rightarrow \infty$. The slope of the line is unity and pass through the point (x_D, x_D) on the diagonal. Therefore, the operating line coincides with the diagonal. So does the stripping section line.

The No. of ideal stages is obtained by stair case construction between equilibrium line and the diagonal.

Total reflux is very often used during the startup of the column till steady state condition reached.



No. of plates at total R .
(min^m. No. of plates)

Fenske's equation 2-

This equation can be used to calculate theoretically the "Minimum. No. of Trays" if the relative volatility remains constant (α_{avg})

$$y_n = \frac{R}{R+1} x_{n+1} + \frac{x_D}{R+1} \quad \text{at } R = \infty$$

$\therefore y_n = x_{n+1}$ for rectifying section
Same $y_m = x_{m+1}$ \rightarrow stripping section

$$\alpha = \frac{y_A/x_A}{y_B/x_B} \Rightarrow \boxed{\frac{y_{AW}}{y_{BW}} = \alpha_0 \left(\frac{x_{AW}}{x_{BW}} \right)}$$

Vap. rise from the reboiler (y_{AW}) have a relation with liquid fall in stage one, joining in operating line.

$$x_{A1} = y_{AW} \quad , \quad x_{B1} = y_{BW}$$

$$\frac{x_{A1}}{x_{B1}} = \frac{y_{AW}}{y_{BW}}$$

$$\boxed{\frac{x_{A1}}{x_{B1}} = \alpha_0 \left(\frac{x_{AW}}{x_{BW}} \right)}$$

Same for stage one:-

$$\alpha_1 = \frac{y_{A1}/x_{A1}}{y_{B1}/x_{B1}}$$

$$\frac{x_{A2}}{x_{B2}} = \alpha_0 \alpha_1 \left(\frac{x_{Aw}}{x_{Bw}} \right)$$

continue to stage (n)

$$\left(\frac{x_A}{x_B} \right)_D = \alpha_0 \alpha_1 \alpha_2 \dots \alpha_n \left(\frac{x_A}{x_B} \right)_w$$

↳ $\alpha_{avg.}$ = avg. volatility of the M.V.C

$$N_{min.} = \frac{\text{Log} \left[\frac{x_D(1-x_w)}{x_w(1-x_D)} \right]}{\text{Log} \alpha_{avg.}} - 1$$

Fenske's equation (at total Reflux)

(b) Minimum Reflux Ratio (R_m) for (N_{∞})

(1) Graphically :-

Can be defined as that ratio at which an infinite No. of stages are needed to obtain the desired overhead and bottom products.

The determination of (R_m) is based on identifying the "Pinch point".

For a particular reflux ratio (R_1), DE_1 is the enriching section operating line with slope $= (R/R+1)$. It intersects the feed line at point (M_1), WM_1 is the stripping section operating line.

As the reflux ratio (R_1) decreases to (R_2) the slope of U.D.L decreases, but intercept increases, the point (E_1) moves to (E_2). DM_2 is the U.D.L and WM_2 is the S.D.L, and they intersect the feed line at M_2 . Then the driving force is less and the No. of theo. trays will be more.

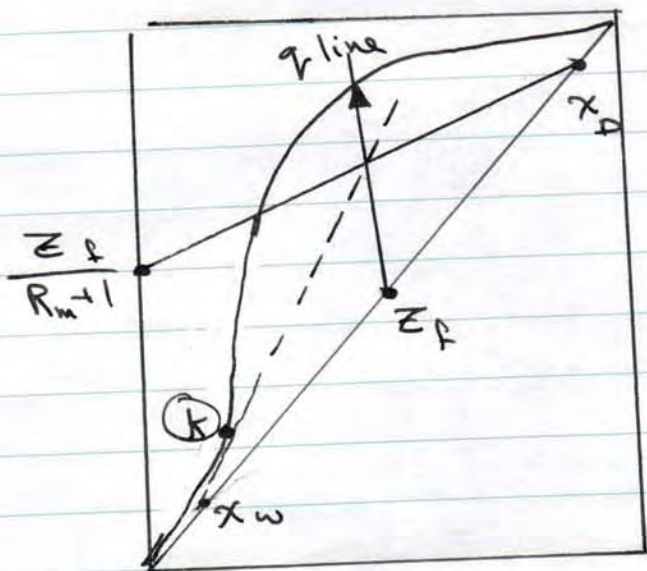
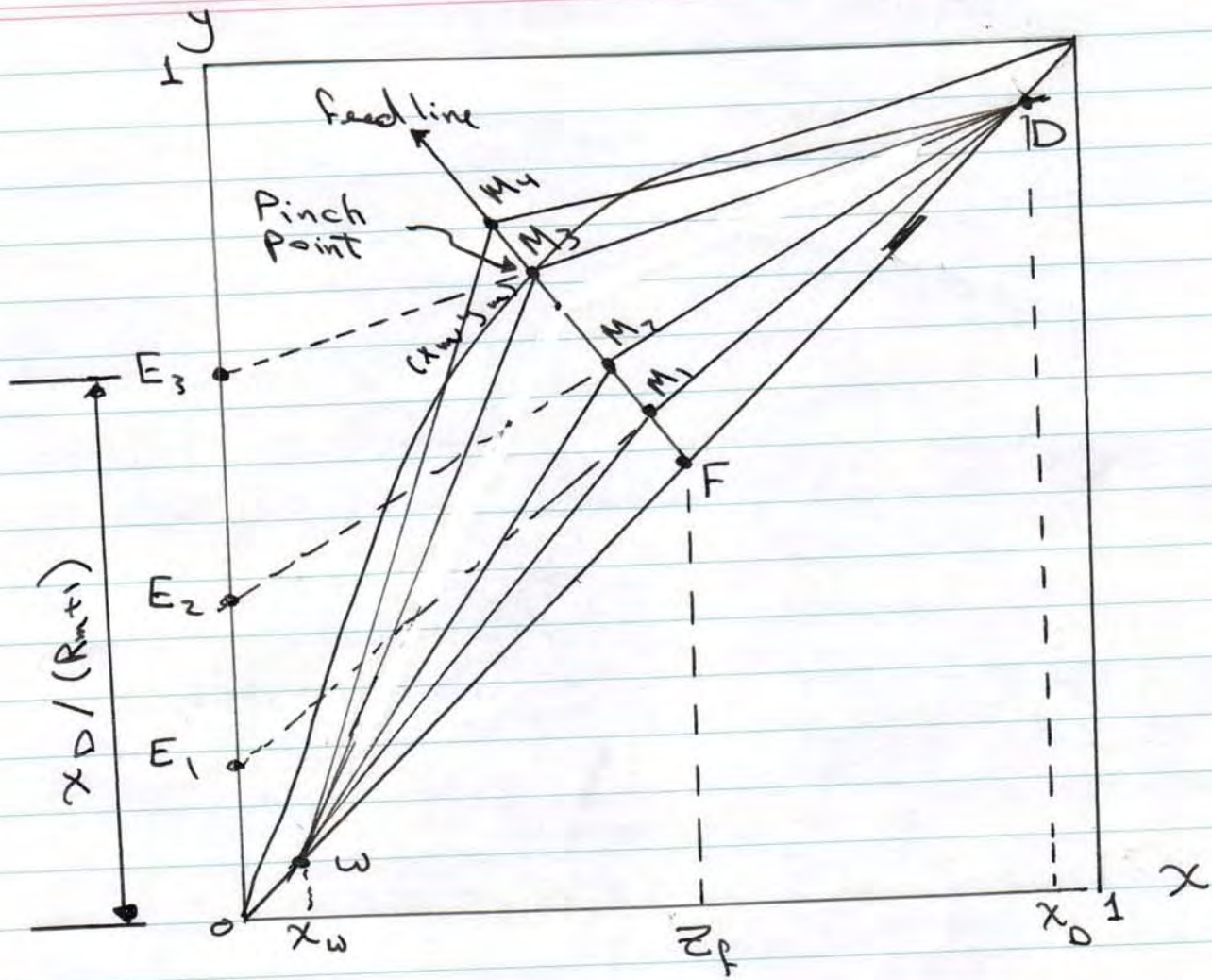
If the reflux ratio is gradually reduced, to (E_3), then line DE_3 intersect the feed line at (M_3) which lies on the equilibrium curve.

Then driving force will be zero at M_3 , it is the "Pinch point".

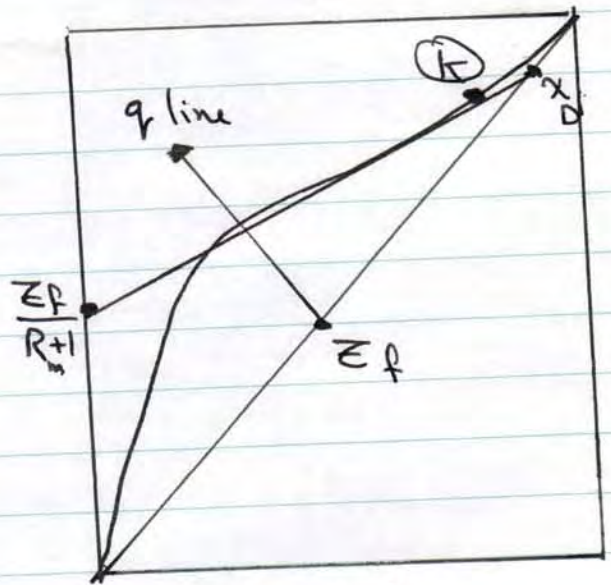
The No. of Theo. plates will be infinite.

The operating line (DM_3) corresponds to the min. reflux ratio.

For further reduced in (R) the operating line will intersect the feed line at a point above the equilibrium curve, this is impossible.



Pinch Point for stripping section



Pinch Point for Rectifying section

② Under Wood Method For Calculating R_{min}

A mathematical method to determine (R_{min}) for ideal solutions at constant α .

$$y_m = \frac{R_m}{R_m + 1} x_m + \frac{x_D}{R_m + 1} \quad \text{--- (1) U.D.L}$$

$$y_m = \frac{q}{q-1} x_m - \frac{z_f}{q-1} \quad \text{--- (2) Feed.L}$$

$$y_m = \frac{\alpha \cdot x_m}{1 + (\alpha - 1) x_m} \quad \text{--- (3) Equil.^m L}$$

From eq. (1) and (2) :-

$$\frac{R_m}{R_m + 1} x_m + \frac{x_D}{R_m + 1} = \frac{q}{q-1} x_m - \frac{z_f}{q-1}$$

$$x_m = \frac{x_D (q-1) + z_f (R_m + 1)}{R_m + q}$$

$$y_m = \frac{R_m z_f + q x_D}{R_m + q}$$

Sub. in eq. (3) :-

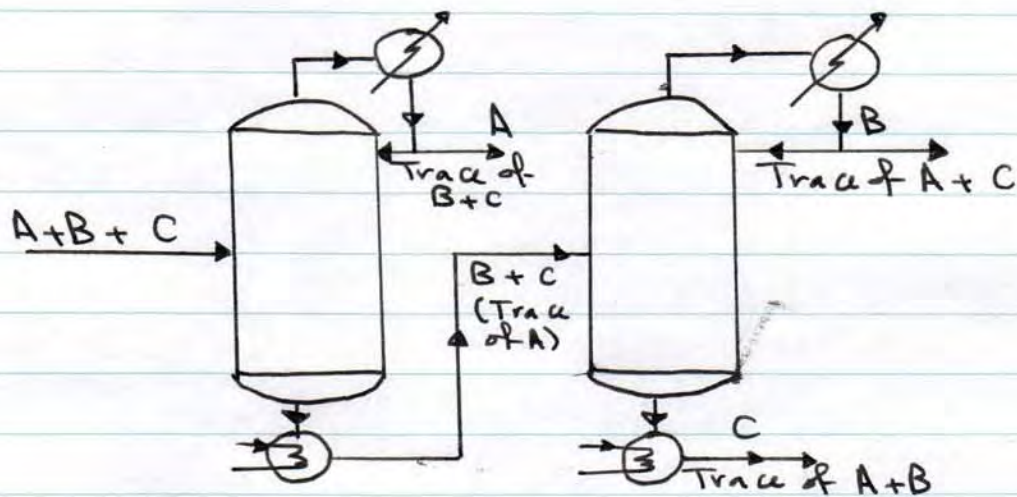
$$\frac{R_m Z_f + q x_D}{R_m (1 - Z_f) + q (1 - x_D)} = \alpha \frac{x_D (q - 1) + Z_f (R_m + 1)}{(R_m + 1) (1 - Z_f) + (q - 1) (1 - x_D)}$$

Underwood equation

$$R_m = \frac{1}{\alpha - 1} \left[\frac{x_D}{Z_f} - \frac{\alpha (1 - x_D)}{(1 - Z_f)} \right] \quad \text{For Sat. liquid}$$

$$R_m = \frac{1}{\alpha - 1} \left[\frac{\alpha \cdot x_D}{Z_f} - \frac{1 - x_D}{1 - Z_f} \right] - 1 \quad \text{For Sat. vapour.}$$

- Multi component Distillation -



Key-Component :-

The two components of the feed whose concentrations are specified in the distillate and in the bottom product are called the key-component.

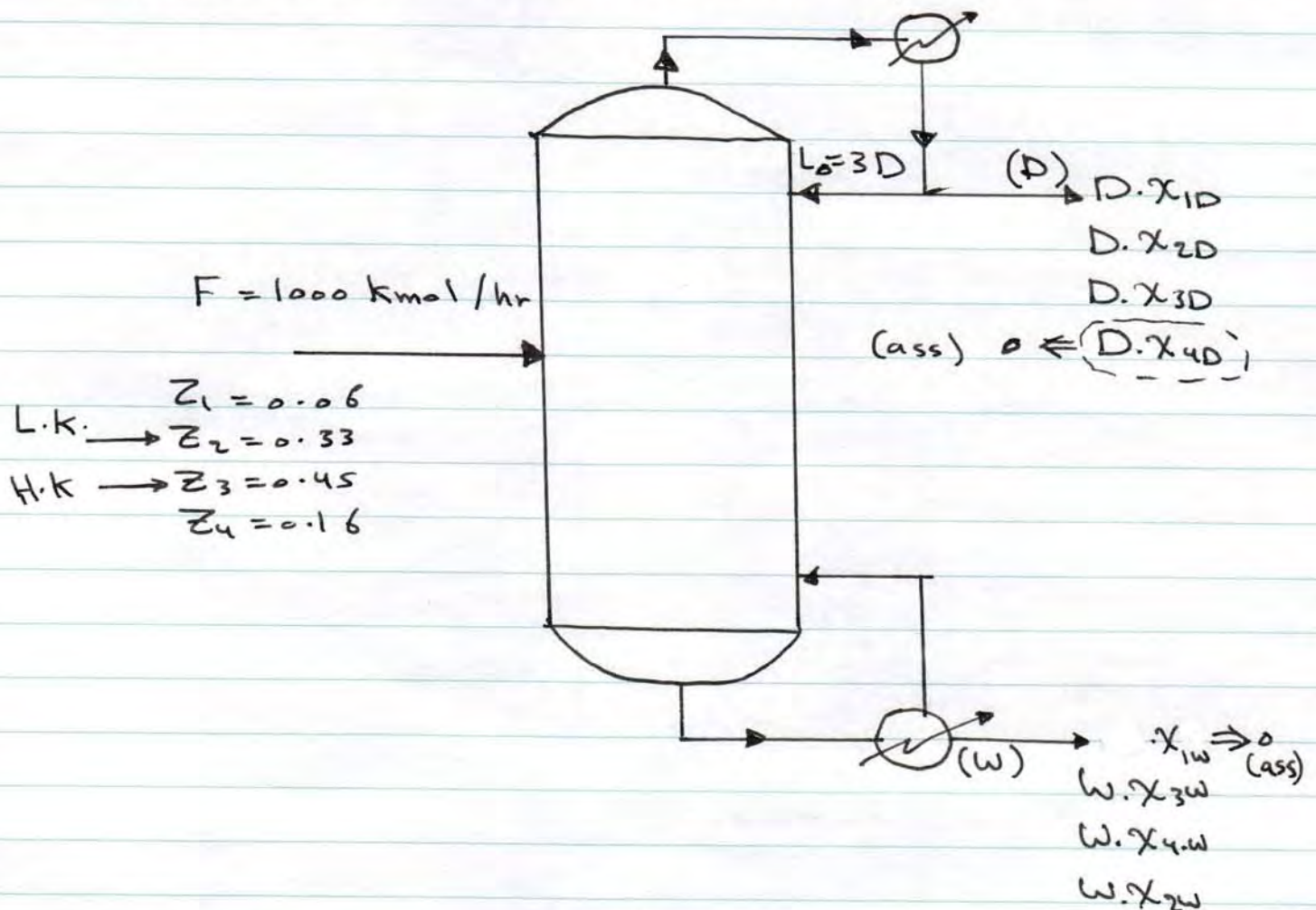
The more volatile of the two keys that concentrates in the distillate is the light key (LK), the less volatile one is the heavy key (HK). All other components which get distributed between the distillate and the bottoms are (non-keys). A non-key more volatile than the (LK) is called a light nonkey (LNK), a non key less volatile than the heavy key is a heavy non-key (HNK).

In some separation problems, there may be one or more components having volatilities intermediate between the (LK) and (HK), these are called (intermediate-keys) or (distributed keys).

Ex. (1) :- (Distribution of the keys and non-keys)

It is required to separate a saturated quaternary mixture containing propane ($Z_1 = 0.06$), n-butane ($Z_2 = 0.33$), n-pentane ($Z_3 = 0.45$) and n-hexane ($Z_4 = 0.16$), at a rate of 1000 kmol/hr . The total press. = 1 atm . The reflux ratio is $R = 3.0$. It is desired to recover 99% of the butane in the distillate and 99.5% of the pentane in the bottoms. Calculate:-

- The flow rates and composition of the distillate and the bottom product.
- the condensation rate and the boil-up rate, if (R) assumed to be at bubble point.



Sol. :- There are eight unknowns, $[D, W, X_{1D}, X_{2D}, X_{3D}, X_{1W}, X_{2W}, X_{3W}]$.

(LK) is the butane

(HK) is the pentane

(LNK) is the propane

(HNK) is the hexane

For (L.K) :-

We want 99% of comp. (2) to be recovered in (D),

$$0.99 * 1000 * 0.33 = D * X_{2D}$$

$$D * X_{2D} = 326.7 \text{ kmol/hr}$$

$$\therefore W * X_{2W} = F * Z_2 - D * X_{2D}$$

$$= (1000 * 0.33) - 326.7$$

$$W * X_{2W} = 3.3 \text{ kmol/hr}$$

L.K.C

For (H.V)

99.5% to be recovered of (3).

$$0.995 * 1000 * 0.45 = W * X_{3W}$$

$$W * X_{3W} = 447.75 \text{ kmol/hr}$$

$$D * X_{3D} = (1000 * 0.45) - 447.75$$

$$D * X_{3D} = 2.25 \text{ kmol/hr}$$

H.K.C

For (L.N.K)

$$F * Z_1 = D * X_{1D} + W * X_{1W}$$

$$1000 * 0.06 = D * X_{1D} + \text{Zero}$$

$$D * X_{1D} = 60 \text{ kmol/hr}$$

$$\text{Total (D)} = D * X_{1D} + D * X_{2D} + D * X_{3D} + D * X_{4D}$$

$$D = 60 + 326.7 + 2.25 \Rightarrow D = 388.95 \text{ kmol/hr}$$

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Now (x_{1D}, x_{2D}, x_{3D}) can be found $(x_{1D} = \frac{D \cdot x_{1D}}{D}, \dots)$

$$F = D + W$$

$$1000 = 388.95 + W \Rightarrow W = 611.05 \text{ kmol/hr}$$

$$\sum x_{iw} = 1$$

$$x_{1w} + x_{2w} + x_{3w} + x_{4w} = 1$$

$$0 + \frac{3.3}{611.05} + \frac{447.75}{611.05} + x_{4w} = 1$$

$$x_{4w} = 0.2618$$

Component	(Z)	x_{iD}	x_{iw}
Propane (1)	0.06	0.1543	0.0000
butane (2)	0.33	0.84	0.0055
pentane (3)	0.45	0.0057	0.7327
hexane (4)	0.16	0.0000	0.2618

⑥ rate of condensation = $R \cdot D$

$$= 3 \times 388.95 = 1166.85 \text{ kmol/hr}$$

⑦ at bubble point, then the vaporization rate in the reboiler = rate of condensation

$$\Rightarrow \text{boil-up} = 1166.85 \text{ kmol/hr}$$

Approximate Methods Calculation

- The (FUG) Technique.

The most important approximate methods of calculation collectively called the Fenske-Underwood-Gilliland (FUG) method.

* Fenske Equation :-

For A (LK) and B (HK) in multicomponent mixture :-

$$N_{\min} = \frac{\text{Log} \left[\frac{(x_{AD} \cdot x_{BW})}{x_{BD} \cdot x_{AW}} \right]}{\text{Log} (\alpha_{AB})_{\text{avg}}} - 1 \quad \text{---} \quad (*)$$

N_{\min} = min. number of trays including the reboiler

$$\alpha_{AB}|_{\text{avg}} = \frac{\alpha_{A|LK}}{\alpha_{B|HK}} \quad \text{and} \quad \alpha|_{\text{avg}} = (\alpha_{A \text{ Top}} \cdot \alpha_{B \text{ Bott.}})^{0.5}$$

- For fractional recovery of the key component instead of concentration then :-

$$N_{\min} = \frac{\text{Log} \left[\frac{P_{A,D} \cdot P_{B,W}}{(1-P_{A,D})(1-P_{B,W})} \right]}{\text{Log} (\alpha_{AB})_{\text{avg}}} \quad \text{---} \quad (**)$$

Once the (N_{\min}) is determined, it is easy to calculate the distribution of a non-key or (intermediate), by substituting ($P_{B,W}$) by any other component, or using eq. (*).

Ex. (2) :- [The min. Number of trays and distribution of the non-key]

A feed mixture containing six component, $Z_1 = 0.032$, $Z_2 = 0.068$, $Z_3 = 0.17$, $Z_4 = 0.3$, $Z_5 = 0.32$ and $Z_6 = 0.11$, is to be separated by distillation so that 98.5% of comp. (3) goes to distillate and 98% of comp. (5) goes to bottom product.

- 1- Determine the min. number of trays required.
- 2- Composition of the top and bottom products.

Given that: $\alpha_{15} = 3.15$, $\alpha_{25} = 2.75$, $\alpha_{35} = 2.35$
 $\alpha_{45} = 1.4$, $\alpha_{55} = 1.0$, $\alpha_{65} = 0.75$
 Feed flow rate 1000 kmol/hr.

Sol. :- Comp. (3) is the L.K
 comp. (5) = H.K

- For the (L.K) :- 98.5% to be recov.

$$D \cdot X_{3D} = 0.985 * 1000 * 0.17 = 167.45$$

$$W \cdot X_{3W} = F \cdot Z_3 - D \cdot X_{3D}$$

$$= 1000 * 0.17 - 167.45$$

$$W \cdot X_{3W} = 2.55 \text{ kmol/hr}$$

- For the (H.K): 98% to be recov.

$$W \cdot X_{5W} = 0.98 * 1000 * 0.32 = 313.6$$

$$D \cdot X_{5D} = 1000 * 0.98 - 313.6 = 6.4$$

$$N_{\min} = \frac{\log \left[\frac{X_{3D} \cdot X_{5W}}{X_{5D} \cdot X_{3W}} \right]}{\log \alpha_{35}} = \frac{\log \left[\frac{D \cdot X_{3D} \cdot W \cdot X_{5W}}{D \cdot X_{5D} \cdot W \cdot X_{3W}} \right]}{\log (2.35)}$$

$$N_{\min} = 9.45$$

(75)

or ($N_{min.}$) can be found by using eq. (**):-

$$f_{3D} = 0.985, \quad f_{5W} = 0.98$$

$$N_{min} = \frac{\log [(0.985 * 0.98) / (1 - 0.985)(1 - 0.98)]}{\log 2.35}$$

$$N_{min.} = 9.45$$

② To find the distribution of comp. (1) consider pair (1-3) and using Fenske - eq.

$$N_{min} = 9.45 = \frac{\log \left[\frac{x_{1D} \cdot x_{3W}}{x_{3D} \cdot x_{1W}} \right]}{\log \alpha_{13}} = \frac{\log \left[\frac{D \cdot x_{1D} \cdot W \cdot x_{3W}}{D \cdot x_{3D} \cdot W \cdot x_{1W}} \right]}{\log \alpha_{13} \leftarrow \frac{\alpha_1}{\alpha_3}}$$

$$9.45 = \frac{\log \left[\frac{D \cdot x_{1D} * 2.55}{W \cdot x_{1W} * 167.45} \right]}{\log (3.15 / 2.35)}$$

$$\frac{D \cdot x_{1D}}{W \cdot x_{1W}} = 1046.6 \Rightarrow D \cdot x_{1D} = 1046.6 W \cdot x_{1W}$$

From mat. balance :-

$$F \cdot z_1 = D \cdot x_{1D} + W \cdot x_{1W}$$

$$1000 * 0.032 = 1046.6 \cdot W \cdot x_{1W} + W \cdot x_{1W}$$

$$W \cdot x_{1W} = 0.0305 \quad \text{and} \quad D \cdot x_{1D} = 31.969$$