Lecture No.4 Flow of Compressible Fluids in Pipe

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If the pressure of the gas does change in pipe by more than about 10 per cent, it is usually satisfactory to treat the gas as an compressible fluid with consider change in temperature & density or specific volume due to change in pressure. When compressibility is taken into account, the equations of flow become very much more complex than they are for an incompressible fluid, even if the simplest possible equation of state (the *ideal gas law)*$pv = nRT$ is used to describe their behaviour. Two limiting cases of particular interest of flow in pipe are:

i. Isothermal and
ii. Adiabatic condition.

Sonic Velocity in Fluids

The speed $\boldsymbol{u_w}$ with which a small pressure wave propagates through **a** fluid can be shown [Shapiro **(1953)** to be related to the compressibility (ε) of the fluid $d\rho/dp$ by the following Eq.

$$u_w = \sqrt{\frac{\partial P}{\partial \rho}}$$

Assuming that the pressure wave propagates through the fluid polytropically, then the equation of state is

$$PV^k = \text{constant} = K$$

But v=1/ρ then

$$P = \rho^k K$$

$$\frac{\partial P}{\partial \rho} = k\rho^{k-1}K = \frac{kP}{\rho} = kPV$$

The propagation speed $\boldsymbol{u_w}$ of the pressure wave is therefore given by

$$u_w = \sqrt{kPV}$$

where P, V, are the local pressure and specific volume of the fluid through which the wave is propagating. Note that ***u,*** is relative to the gas. If the wave were to propagate isothermally, then k=1 and velocity of sound or sonic velocity becomes: $u_w = \sqrt{PV}$

If isentropically or adiabatic conditions above Eq. becomes:

$$u_w = \sqrt{\gamma PV} \quad \text{where } \gamma = C_p/C_v$$

It is common practice to state gas velocities relative to the sonic velocity as shown in:

$$N_M = \frac{U}{U_w}$$

N_M is defines as dimensionless Mach number and classified as $N_M > 1$ the fluid is called supersonic flow. If $N_M < 1$ the fluid is called subsonic flow and if $N_M = 1$, the fluid is called sonic flow.

General Equation for Flow of Ideal Gas in pipes

The general energy equation for the flow of any type of fluid through a pipe (as shown in Fig.1) has been expressed in the form (see Ch. Incompressible fluid in Lecture):

$$\frac{u\,du}{\alpha} + g\,dz + v\,dP + \delta W_s + \delta F = 0$$

------------------------(1)

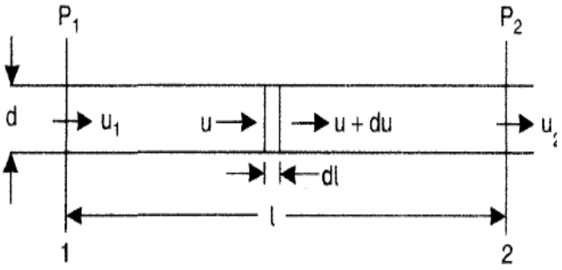

Fig.1

Assume length dl of pipe of constant A

$W_S=0$

$$\delta F = 4\left(\frac{R}{\rho u^2}\right) u^2 \frac{dl}{d}$$

Flow of Compressible fluid through pipe

--------------(2) Sub. in Eq.(1)

$$\frac{u\,du}{\alpha} + g\,dz + v\,dP + 4\frac{R}{\rho u^2}u^2\frac{dl}{d} = 0$$

--------------(3)

This equation (3) cannot be integrated directly because the velocity *u* increases as the pressure falls and is, therefore, a function of / (Figure 1). It is, therefore, convenient to work in terms of the mass velocity flow G=m/A or =ρu which remains constant throughout the length of pipe. Hence Eq.3 becomes (for turbulent flow) α=1:

$$G^2 vdv + gdZ + vdP + 4J_f G^2 v^2 \frac{dl}{d} = 0 ----- (4) dividing\ by\ v^2$$

$$G^2 \frac{dv}{v} + + \frac{dP}{v} + 4J_f G^2 \frac{dl}{d} = 0 --------- (5)$$

Now the friction factor J_f a function of the Reynolds number *Re* and the relative roughness *e/d* of the pipe surface which will normally be constant along a given pipe. Then R_e=Gd/μ . Since *G* is constant over the length of the pipe, *Re* varies only as a result of changes in the viscosity μ. Although ^ is a function of temperature, and to some extent of pressure, it is not likely to vary widely over the length of the pipe. Furthermore, the friction factor J_f is only a weak function of Reynolds number when *Re* is high, and little error will therefore arise from regarding it as constant.

Thus the integration Eq. 5 over a length l of pipe to give general Eq. for compressible fluid flow:

$$G^2 ln\frac{v_2}{v_1} + \int_{P_1}^{P_2}\frac{dP}{v} + 4J_f G^2 \frac{l}{d} = 0 ---- (6)$$

This term will be defined by Isothermal or Adiabatic

The integral will depend on the *P-v* relationship during the expansion of the gas in the pipe, and several cases are now considered (Isothermal or Adiabatic flow):

For Isothermal Flow of an Ideal Gas in Horizontal pipe:

For isothermal changes in an ideal gas: $\int_{P_1}^{P_2}\frac{dP}{v} = \frac{P_2^2 - P_1^2}{2P_1V_1} ---- (7)$ Sub in Eq.6

$$G^2 ln\frac{P_1}{P_2}+\frac{P_2^2-P_1^2}{2P_1V_1}+4J_f G^2\frac{l}{d}=0 \text{----------------}(8)$$

Eq.8 can be written in term v_m, the specific volume at the mean pressure in the pipe, is given as:

$$\frac{P_1+P_2}{2}v_m=P_1v_1$$

$$G^2 ln\frac{P_1}{P_2}+\frac{P_2-P_1}{v_m}+4J_f G^2\frac{l}{d}=0 \text{------------------------}(9)$$

Eq.9 used if the if the pressure drop in the pipe is a small proportion of the inlet pressure, the first term is negligible and the fluid may be treated as an incompressible fluid at the mean pressure in the pipe.

Note:1- It is sometimes convenient to substitute *RT/M* for *P\v* in equation 8 to give:

$$G^2 ln\frac{P_1}{P_2}+\frac{P_2^2-P_1^2}{2RT/M}+4J_f G^2\frac{l}{d}=0 ------(10)$$

2-Equations 8 and 10 are the most convenient for the calculation of gas flowrate as a function of P_1 and P_2 under isothermal conditions. Some additional refinement can be added if a compressibility factor is introduced as defined by the relation Pv = *ZRT/M,* for conditions where there are significant deviations from the *ideal gas law*.

EX.1Over a 30 m length of a 150 mm vacuum line carrying air at 295 K, the pressure falls from 0.4 kN/m^2 to 0.13 kN/m^2. If the relative roughness *e/d* is 0.003 what is the approximate flowrate? (See sol. P.144 Vol.1 5ed.).

EX.2 A flow of 50 m^3/s methane, measured at 288 K and 101.3 kN/m^2, has to be delivered along a 0.6 m diameter line, 3.0 km long with a relative roughness of 0.0001, linking a compressor and a processing unit. The methane is to be discharged at the plant at 288 K and 170 kN/m^2 and it leaves the compressor at 297 K. What pressure must be developed at the compressor in order to achieve this flowrate? (see sol. P.145 Vol.1 5ed).

Problems: 4.1,2,3,4,5,9,10,11,12

Maximum flow for Isothermal condition:

Equation 8 $P_2 \longrightarrow$ the down stream pressure for given upstream pressure P_1, if $P_1=P_2$, gives G=0 , also if $P_2=0$, gives G=0..Thus for intermediate value assume $P_2=P_w$, where $0<P_w<P_1$, the flowrate or G must be maximum.

Eq.8 dividing by G^2 to give the following: $-\ln\left(\frac{P_2}{P_1}\right)+\frac{P_2^2-P_1^2}{2P_1V_1G^2}+4J_f\frac{l}{d}=0 \text{-------}(11)$

Differentiating w.r.t P_2 for constant value of P_1:

$$\frac{P_2}{P_1v_1G^2}-\frac{2}{G^3}\left(\frac{dG}{dP_2}\right)\left(\frac{P_2^2-P_1^2}{2P_1v_1}\right)-\frac{1}{P_2}=0 ---(12)$$

The rate of flow is a maximum when d G_2/d P_2=0 , denoting conditions at the downstream end of the pipe by suffix w, when the flow is a maximum:

For this case Eq.12 can be written as: $G_w^2 = \frac{P_w^2}{P_1 v_1} ----(12)$

Since the maximum mass flowrate G_s or $G_w = \rho_s u_s = u_s/v_s$ and for isothermal conditions:

$$P_1 V_1 = P_s V_s ----(13)$$

Eq.12 can be written as:

$$u_s = \sqrt{P_s V_s} \text{---------}(14)$$

Since for isothermal conditions:

$PV = P_s V_s ----(15)$, *then combine Eq.* 14 & 15 *to give* the sonic velocity as

$$u_s = \sqrt{PV} ----(17) \textit{sonic velocity for isothermal conditions}$$

Supersonic velocities cannot be attained in a pipe of constant cross-sectional flow area.

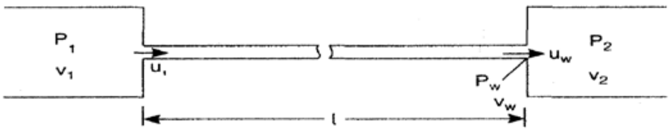

Fig. Maximum flow conditions (see notes in vol.1 p. 139)

<u>NOTE:</u> In Figure 4.9 P.163 Vol.1 values of P/P_w and wc are plotted against $8(/f/pu^2)(l/d)$. This curve gives the limiting value of P_1/P_2 for which the whole of the expansion of the gas can take place within the pipe.

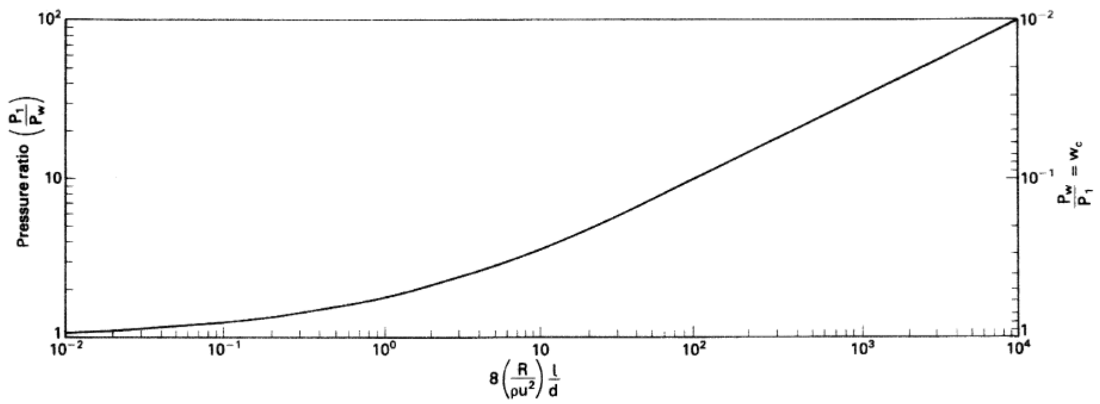

Figure 4.9. Critical pressure ratio P_1/P_w (maximum value of P_1/P_2 which can occur in a pipe) as function of $8(R/\rho u^2)(l/d)$

EX.3 (Holland 2ed. P.198) Hydrogen is to be pumped from one vessel through a pipe of length 400 m to a second vessel, which is at a pressure of 20 bar absolute. The required flow rate is 0.2 kg/s and the allowable pressure at the pipe inlet is 25 bar. The **flow** conditions are isothermal and the gas temperature **is** 25°C. If the friction factor may be assumed to have a value of 0.005, what diameter of pipe is required?

Adiabatic flow of an Ideal Gas in a Horizontal pipe:

For a fluid flowing under turbulent conditions in a pipe, W_s=0 and :

$$\delta q = dH + g\,dz + u\,du \qquad \text{------------(1)}$$

In an adiabatic process, *8q* = 0, and the equation may then be written for the flow in a pipe of constant cross-sectional area *A* to give:

$$G^2 v dv + g dz + dH = 0 ------ (2)$$

Now $dH = dU + d(Pv) = C_v dT + d(Pv) for\ an\ ideal\ gas$

$$C_p dT = C_v dT + d(Pv) for\ an\ ideal\ gas$$

$$dT = \frac{d(Pv)}{C_p - C_v}$$

$$dH = d(Pv)\left(\frac{C_v}{C_p - C_v} + 1\right) \qquad = \frac{\gamma}{\gamma - 1} d(Pv)$$

Substituting this value of dH in equation 2 and writing *g dz* = 0 for a horizontal pipe:

$$G^2 v dv + \frac{\gamma}{\gamma - 1} d(Pv) = 0 ----(3)$$

Integrating, a relation between *P* and *v* for adiabatic flow in a horizontal pipe obtained to give the following Eq. calculate downstream pressure P_2:

$$\frac{G^2 v^2}{2} + \frac{\gamma}{\gamma - 1} Pv = \frac{G^2 v_1^2}{2} + \frac{\gamma}{\gamma - 1} P1v1 = constant\ K ----(4)$$

<u>This Eq.4 it is used to calculate downstream pressure P_2 in term of G, it can be written in term velocities u as following:</u>

$$\frac{\gamma}{\gamma - 1}(P_2 V_2 - P_1 V_1) + \frac{(u_2^2 - u_1^2)}{2} = 0 --- (5)$$

<u>Eq. 4 or 5 can be derived to calculate v_2 downstream specific volume at the end of the pipe, (Note see derivation in Vol.1 p.147 5ed.) , then written as following:</u>

$$8J_f \frac{l}{d} = \left[\frac{\gamma - 1}{2\gamma} + \frac{P_1}{v_1 G^2}\right]\left[1 - (\frac{v_1}{v_2})^2\right] - \frac{\gamma + 1}{\gamma} ln \frac{v_2}{v_1} --- (6)$$

For adiabatic conditions, sonic velocity calculate as shown above equation (u_w or u_s):

$$u_w = \sqrt{\gamma PV}$$

Ex. Problem 4.6 (vol.1) Nitrogen at 12 MN/m2 pressure is fed through a 25 mm diameter mild steel pipe to a synthetic ammonia plant at the rate of 0.4 kg/s. What will be the drop in pressure over a 30 m length of pipe assuming isothermal expansion of the gas at 300 K? What is the average quantity of heat per unit area of pipe surface that must pass through the walls in order to maintain isothermal conditions? What would be the pressure drop in the pipe if it were perfectly lagged?

Note: see problems in Vol.1 5ed. 4.7

<u>Converging-Diverging Nozzles for Gas Flow (Laval Nozzles):</u>

Converging-diverging nozzles, as shown in Figure, sometimes known as **<u>Laval nozzles</u>**, are used for the expansion of gases where the pressure drop is large. Because the flow rate is large for high-pressure differentials, there is little time for heat transfer to take place between the gas and surroundings and the expansion is effectively isentropic. The specific volume *V2* at a downstream pressure *P2,* is given by:

$$v_2 = v_1 \left(\frac{P_1}{P_2}\right)^{1/\gamma} = v_1 \left(\frac{P_2}{P_1}\right)^{-1/\gamma}$$

If gas flows under turbulent conditions from a reservoir at a pressure PI, through a horizontal nozzle, the velocity of flow *u-2,* at the pressure *PI* is given by:

$u_2^2 \quad r^2$

$$u_2^2 = \frac{2\gamma}{\gamma - 1} P_1 v_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}\right]$$

Since :$m = \rho u A\ (Cont. Eq)\ \ Hence\ A_2 = \frac{m}{\rho_2 u_2}\ \ this\ the\ required\ cross -$ $sectional\ area\ for\ flow\ when\ the\ pressure\ has\ fallen\ to\ P2\ can\ be\ found$

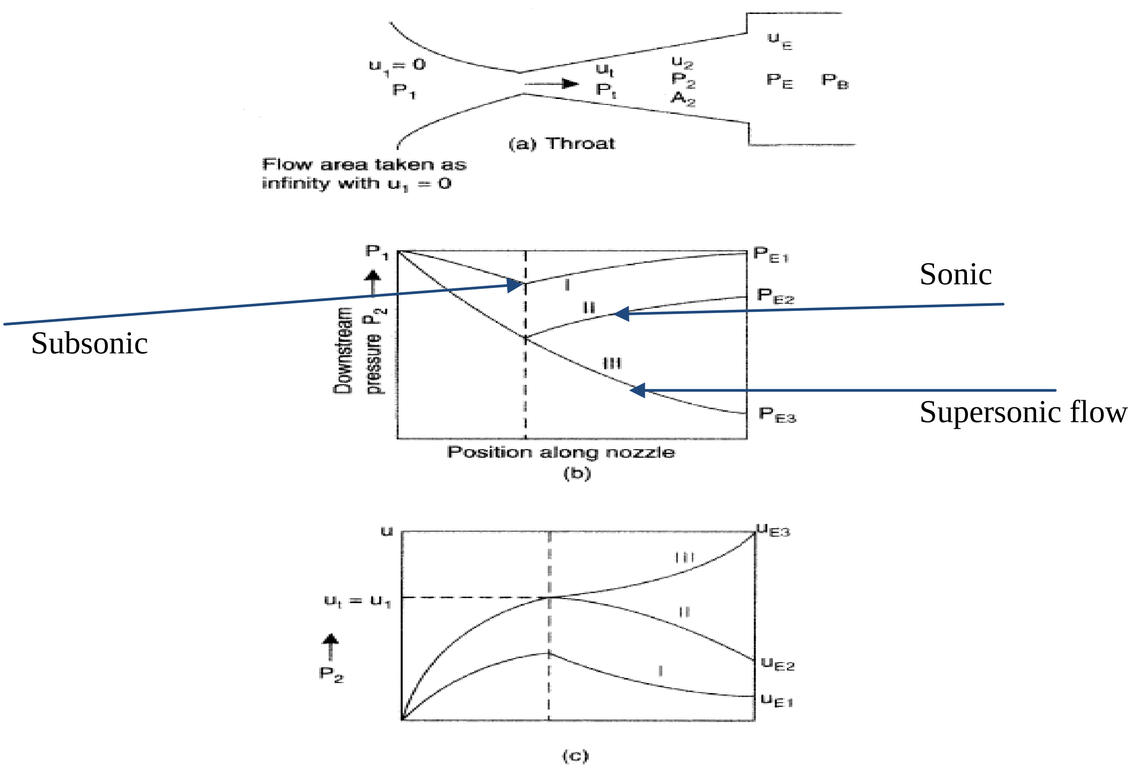

Figure 4.5. Flow through converging-diverging nozzles

Note: See notes in Vol.1 p.132 more explanation for this flow.

The pressure and area for flow:

The area required at any point depends upon the ratio of the downstream to the upstream pressure P_2/P_1 and it is helpful to establish the minimum value of A_2. A_2 may be expressed in terms of P_2 and w[= (P_2/P_1)] using above equations.

$$A_2^2 = \frac{G^2 V_1(\gamma-1)}{2P_1V_1}\frac{W^{-\frac{2}{\gamma}}}{1-W^{\frac{\gamma-1}{\gamma}}}$$

For a given rate of flow m, A_2 decreases from an effectively infinite value at pressure p_1 at the inlet a minimum value given by:

$$\frac{dA_2^2}{dw} = 0$$

$$(1 - w^{(\gamma-1)/\gamma})\frac{-2}{\gamma}w^{-1-2/\gamma} - w^{-2/\gamma}\frac{1-\gamma}{\gamma}w^{-1/\gamma} = 0$$

$$w = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$$

Eqn. of A_2 can be written in terms as following:

$$G^2 = \frac{2\gamma}{\gamma-1}\left(\frac{P_2}{P_1}\right)^{2/\gamma}\frac{P_1}{v_1}\left[1-\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}\right]$$

And the value of flowrate can be shown to have a maximum value of Gmax.=$\sqrt{\gamma p_2 v_2}$

Ex.(se Holland Textbook p.125) Nitrogen contained in a large tank at a pressure $\boldsymbol{P} = \boldsymbol{200000}$ Pa and atemperature of 300 K flows steadily under adiabatic conditions into a second tank through a converging nozzle with a throat diameter of 15 mm. The pressure in the second tank and at the throat of the nozzle is $\boldsymbol{P}$, = 140000 Pa. Calculate the mass flow rate, M, of nitrogen assuming frictionless flow and ideal gas behaviour. Also calculate the gas speed at the nozzle and establish that the flow is subsonic. The relative molecular mass of nitrogen is 28.02 and the ratio of the specific heat capacities y is 1.39.

Gas Compression and Compressors:

Compressors are devices for supplying energy or pressure head to a gas. For the most part, compressors like pumps can be classified into:

Centrifugal

and **Positive displacement types.**

Centrifugal compressors impart a high velocity to the gas and the resultant kinetic energy provides the work for compression. Positive displacement compressors include rotary and reciprocating compressors although the latter are the most important for high pressure applications.

the shaft work of compression W required to compress unit mass of gas from pressure **P1** to pressure ***P2***

$$W = \int_1^2 V\mathrm{d}P$$

When ideal gases are compressed under reversible adiabatic conditions they obey equation **6.32,** which can be written as

so that the specific volume is given by

$$P_1V_1^{\gamma} = PV^{\gamma}$$

$$W = \left(\frac{\gamma}{\gamma-1}\right)P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$$

This Eqn gives theoretical adiabatic work of compression from pressures P_1 to P_2.

Compression is often done in several stages with the gas being cooled between stages. For two-stage compression from P1 to *Pz* to ***P3,*** with the gas cooled to the initial temperature T1 at constant pressure, above Eqn Becomes:

$$W = \left(\frac{\gamma}{\gamma-1}\right)P_1V_1\left\{\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right] + \left[\left(\frac{P_3}{P_2}\right)^{(\gamma-1)/\gamma} - 1\right]\right\}$$

In the case of compression from pressure P1 to pressure *Pz* through n stages each having the same pressure ratio $(P2/Pl)^{1/n}$**,** the compression work is given by

$$W = \left(\frac{n\gamma}{\gamma-1}\right)P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right]$$

When ideal gases are compressed under reversible adiabatic conditions the temperature rise from T1 to ***T2*** is given by equation

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\gamma/(\gamma-1)}$$

So far only reversible adiabatic compression of an ideal gas has been considered. For the irreversible adiabatic compression of an actual gas, the shaft work W required to compress the gas from state 1 to state 2 can be

$$W = H_2 - H_1$$

where H is the enthalpy per unit mass of gas.

The actual work of compression is greater than the theoretical work because of clearance gases, back leakage and friction.

Note: If the compression gas under isothermal condition for an ideal, then the work done per cycle is given as following:

$$\boldsymbol{W = P_1 v_1 ln \frac{P_2}{P_1}}$$

The relation between pressure and volume of gas under compression for single stage as shown in following Figures (Note Ch.8 in vol.1 p.312):

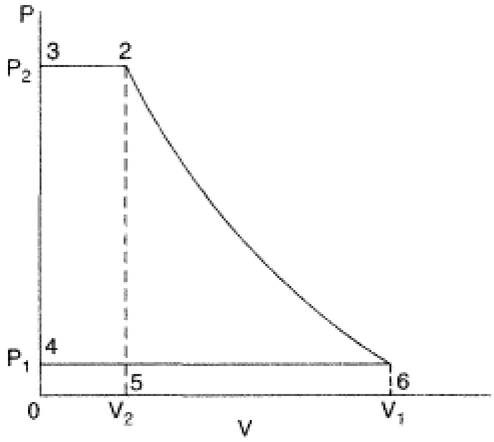

Figure 8.39. Single-stage compression cycle — no clearance

Point 1 represents the initial condition of the gas *(P\ and V\)*.
Line 1-2 represents the compression of gas to pressure *P2,* volume *V2.*
Line 2-3 represents the expulsion of the gas at a constant pressure *P2.*
Line 3-4 represents a sudden reduction in the pressure in the cylinder from *PI* to PI. As the whole of the gas has been expelled, this can be regarded as taking place instantaneously.
Line 4-1 represents the suction stroke of the piston, during which a volume *V* of gas is admitted at constant pressure, .P_1.

Clearance volume through Compression Gas & Find Work:

The theoretical work of compressor is less than actual work, because clearance of gas. In practice, it is not possible to expel the whole of the gas from the cylinder at the end of the compression; the volume remaining in the cylinder after the forward stroke of the piston is termed the ***clearance volume***. The volume displaced by the piston is termed the ***swept volume***, and therefore the total volume of the cylinder is made up of the clearance volume plus the swept volume. The ***clearance c* is** defined as the ratio of the clearance volume to the swept volume. A typical cycle for a compressor with clearance is shown in Fig.

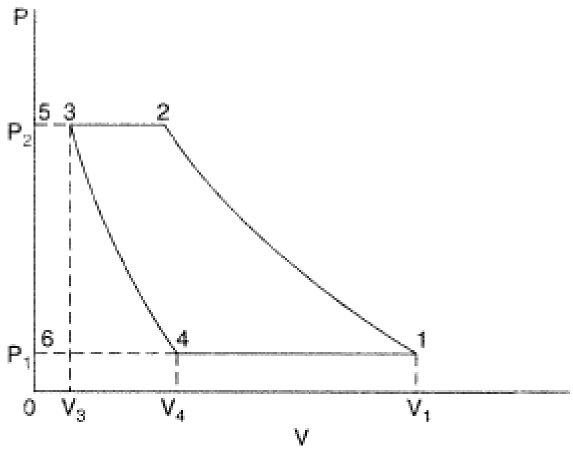

Figure 8.40. Single-stage compression cycle — with clearance

Line 1 -2 represents the compression of the gas to a pressure *P2* and volume *V2.*
Line 2-3 represents the expulsion of gas at constant pressure *PI*, so that the volume remaining in the cylinder is V_3.
Line 3-4 represents an expansion of this residual gas to the lower pressure P_1 and volume V_4 during the return stroke.

Line 4-1 represents the introduction of fresh gas into the cylinder at constant pressure P_1 The work done on the gas during each stage of the cycle is as follows.

Compression	$-\int_{V_1}^{V_2} P\,dV$
Expulsion	$P_2(V_2 - V_3)$
Expansion	$-\int_{V_3}^{V_4} P\,dV$
Suction	$-P_1(V_1 - V_4)$

The total work done during the cycle is equal to the sum of these four components, is given as following:

W $$= P_1(V_1 - V_4)\frac{\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]$$

Thus, theoretically, the clearance volume does not affect the work done per unit mass of gas, since $V_1 - V_4$ is the volume admitted per cycle. V_4 can be calculated in terms V_3 as following:

For isentropic conditions:

$$V_4 = V_3\left(\frac{P_2}{P_1}\right)^{1/\gamma}$$

and: $$V_1 - V_4 = (V_1 - V_3) + V_3 - V_3\left(\frac{P_2}{P_1}\right)^{1/\gamma}$$
$$= (V_1 - V_3)\left[1 + \frac{V_3}{V_1 - V_3} - \frac{V_3}{V_1 - V_3}\left(\frac{P_2}{P_1}\right)^{1/\gamma}\right]$$

Now (V1 — V3) is the swept volume, *Vs*, say; and V3/(V1 — V3) is the clearance *c*.

$$V_1 - V_4 = V_s\left[1 + c - c\left(\frac{P_2}{P_1}\right)^{1/\gamma}\right]$$

Therefore the total work done on the fluid per cycle is:

$$P_1 V_s \frac{\gamma}{\gamma - 1}\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma} - 1\right]\left[1 + c - c\left(\frac{P_2}{P_1}\right)^{1/\gamma}\right]$$

The factor [1 + *c* — c(P2/P1)$^{1/\gamma}$ **is called the theoretical volumetric efficiency** and is a measure of the effect of the clearance on an isentropic compression. The actual volumetric efficiency will be affected, in addition, by the inertia of the valves and leakage past the piston.

Temperature increase through compression gas, therefore it is required cooling between stages as shown in Fig.(see Vol.1 p.317, 5ed):

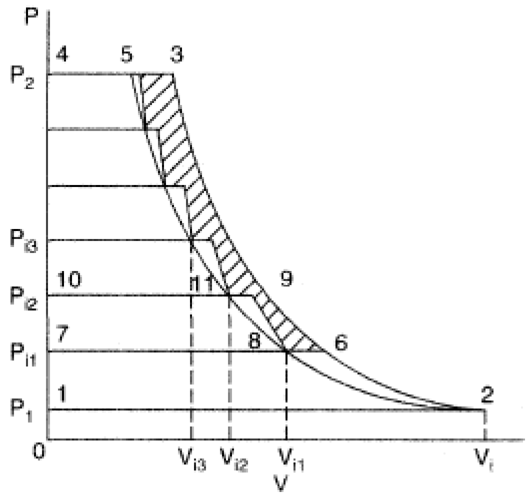

Figure 8.41. Multistage compression cycle with interstage cooling

Ex.(Holland p.130)

Calculate the theoretical work required to compress 1 kg of a diatomic ideal gas initially at a temperature of 200 K adiabatically from a pressure of 10000 Pa to a pressure of 100000 Pa in (i) a single stage, (ii) a compressor with two equal stages and (iii) a compressor with three equal

Calculations

(i) For a single stage compression,

$$W=\left(\frac{\gamma}{\gamma-1}\right)P_1V_1\left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}-1\right]$$

From given values

$$\frac{P_2}{P_1}=10$$

Therefore

$$\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/\gamma}=10^{0.2857}=1.931$$

Equation of state

$$PV=R'T=\frac{RT}{\text{RMM}}$$

Therefore

$$P_1V_1=\frac{RT_1}{\text{RMM}}$$

$$=\frac{8314.3\ \text{J/(kmol K)}\times 200\ \text{K}}{28.0\ \text{kg/kmol}}$$

$$=5.939\times 10^4\ \text{J/kg}$$

Also

$$\frac{\gamma}{\gamma-1}=3.5$$

Substituting these values into equation 6.89

$$W\,(1\ \text{stage})=(3.5)(5.939\times 10^4\ \text{J/kg})(1.931-1)$$

$$W\,(1\ \text{stage})=1.935\times 10^5\ \text{J/kg}=\underline{193.5\ \text{kJ/kg}}$$

(ii) For adiabatic compression of an ideal gas in n equal stages

stages. The relative molecular mass of the gas is 28.0 and the ratio of specific heat capacities γ is 1.40.

$$W = \left(\frac{n\gamma}{\gamma - 1}\right) P_1 V_1 \left[\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} - 1\right]$$

For $n = 2$

$$\left(\frac{P_2}{P_1}\right)^{(\gamma-1)/n\gamma} = 10^{0.1429} = 1.389$$

Since

$$\frac{n\gamma}{\gamma - 1} = 7.0$$

and as before

$$P_1 V_1 = 5.939 \times 10^4 \text{ J/kg}$$

it follows that

$$W \text{ (2 stages)} = (7.0)(5.939 \times 10^4 \text{ J/kg})(1.389 - 1)$$

$$W \text{ (2 stages)} = 1.617 \times 10^5 \text{ J/kg} = \underline{161.7 \text{ kJ/kg}}$$

(iii) Repeating the above calculation for $n = 3$ gives

$$W \text{ (3 stages)} = \underline{152.8 \text{ kJ/kg}}$$

Compressor efficiencies

The efficiency quoted for a compressor is usually either an **isothermal efficiency** or an **isentropic efficiency**.

1-The isothermal efficiency is the ratio of the work required for an ideal isothermal compression to the energy actually expended in the compressor.

2-The isentropic efficiency is defined in a corresponding manner on the assumption that the whole compression is carried out in a single cylinder.

Since the energy expended in an isentropic compression is greater than that for an isothermal compression, the isentropic efficiency is always the greater of the two. Clearly the efficiencies will depend on the heat transfer between the gas undergoing compression and the surroundings and on how closely the process approaches a reversible compression.

Ex. (Vol.1 P.319)

A single-acting air compressor supplies 0.1 m^3/s of air measured at, 273 K and 101.3 kN/m^2 which is compressed to 380 kN/m^2 from 101.3 kN/m^2. If the suction temperature is 289 K, the stroke is 0.25 m, and the speed is 4.0 Hz, what is the cylinder diameter? Assuming the cylinder clearance is 4 per cent and compression and re-expansion are isentropic ($\gamma = 1.4$), what are the theoretical power requirements for the compression?

Ex (Vol.1 P.320)

Air at 290 K is compressed from 101.3 kN/m^2 to 2065 kN/m^2 in a two-stage compressor operating with a mechanical efficiency of 85 per cent. The relation between pressure and volume during the compression stroke and expansion of the clearance gas is $PV^{1.25}$ = constant. The compression ratio in each of the two cylinders is the same, and the interstage cooler may be assumed 100 per cent efficient. If the clearances in the two cylinders are 4 per cent and 5 per cent respectively, calculate:

(a) the work of compression per kg of air compressed;
(b) the isothermal efficiency;
(c) the isentropic efficiency ($\gamma = 1.4$), and
(d) the ratio of the swept volumes in the two cylinders.

Tutorial No.4

Flow of Compressible Fluids in Pipe

Vol.1 5ed (Ch.4+Ch.8)

4.1. A gas, having a molecular weight of 13 kg/kmol and a kinematic viscosity of 0.25 cm^2/s, is flowing through a pipe 0.25 m internal diameter and 5 km long at the rate of 0.4 m^3/s and is delivered at atmospheric pressure. Calculate the pressure required to maintain this rate of flow under isothermal conditions.

The volume occupied by 1 kmol at 273 K and 101.3 kN/m^2 is 22.4 m^3.

What would be the effect on the required pressure if the gas were to be delivered at a height of 150 m (i) above and (ii) below its point of entry into the pipe?

4.2. Nitrogen at 12 MN/m^2 is fed through a 25 mm diameter mild steel pipe to a synthetic ammonia plant at the rate of 1.25 kg/s. What will be the drop in pressure over a 30 m length of pipe for isothermal flow of the gas at 298 K?

Absolute roughness of the pipe surface = 0.005 mm.
Kilogram molecular volume = 22.4 m^3.
Viscosity of nitrogen = 0.02 mN s/m^2.

4.3. Hydrogen is pumped from a reservoir at 2 MN/m^2 pressure through a clean horizontal mild steel pipe 50 mm diameter and 500 m long. The downstream pressure is also 2 MN/m^2 and the pressure of this gas is raised to 2.6 MN/m^2 by a pump at the upstream end of the pipe. The conditions of flow are isothermal and the temperature of the gas is 293 K. What is the flowrate and what is the effective rate of working of the pump?

Viscosity of hydrogen = 0.009 mN s/m^2 at 293 K.

4.4. In a synthetic ammonia plant the hydrogen is fed through a 50 mm steel pipe to the converters. The pressure drop over the 30 m length of pipe is 500 kN/m^2, the pressure at the downstream end being 7.5 MN/m^2. What power is required in order to overcome friction losses in the pipe? Assume isothermal expansion of the gas at 298 K. What error is introduced by assuming the gas to be an incompressible fluid of density equal to that at the mean pressure in the pipe? $\mu = 0.02$ mN s/m^2.

4.5. A vacuum distillation plant operating at 7 kN/m^2 at the top has a boil-up rate of 0.125 kg/s of xylene. Calculate the pressure drop along a 150 mm bore vapour pipe used to connect the column to the condenser. The pipe length may be taken as equivalent to 6 m, $e/d = 0.002$ and $\mu = 0.01$ mN s/m^2.

4.6. Nitrogen at 12 MN/m^2 pressure is fed through a 25 mm diameter mild steel pipe to a synthetic ammonia plant at the rate of 0.4 kg/s. What will be the drop in pressure over a 30 m length of pipe assuming isothermal expansion of the gas at 300 K? What is the average quantity of heat per unit area of pipe surface that must pass through the walls in order to maintain isothermal conditions? What would be the pressure drop in the pipe if it were perfectly lagged?

4.7. Air, at a pressure of 10 MN/m^2 and a temperature of 290 K, flows from a reservoir through a mild steel pipe of 10 mm diameter and 30 m long into a second reservoir at a pressure P_2. Plot the mass rate of flow of the air as a function of the pressure P_2. Neglect any effects attributable to differences in level and assume an adiabatic expansion of the air. $\mu = 0.018$ mN s/m^2, $\gamma = 1.36$.

4.8. Over a 30 m length of 150 mm vacuum line carrying air at 293 K the pressure falls from 1 kN/m^2 to 0.1 kN/m^2. If the relative roughness e/d is 0.002, what is the approximate flowrate?

4.9. A vacuum system is required to handle 10 g/s of vapour (molecular weight 56 kg/kmol) so as to maintain a pressure of 1.5 kN/m^2 in a vessel situated 30 m from the vacuum pump. If the pump is able to maintain a pressure of 0.15 kN/m^2 at its suction point, what diameter pipe is required? The temperature is 290 K, and isothermal conditions may be assumed in the pipe, whose surface can be taken as smooth. The ideal gas law is followed.

Gas viscosity = 0.01 mN s/m^2.

4.10. In a vacuum system, air is flowing isothermally at 290 K through a 150 mm diameter pipeline 30 m long. If the relative roughness of the pipewall e/d is 0.002 and the downstream pressure is 130 N/m^2, what will the upstream pressure be if the flow rate of air is 0.025 kg/s?

Assume that the ideal gas law applies and that the viscosity of air is constant at 0.018 mN s/m^2.

What error would be introduced if the change in kinetic energy of the gas as a result of expansion were neglected?

4.11. Air is flowing at the rate of 30 kg/m^2s through a smooth pipe of 50 mm diameter and 300 m long. If the upstream pressure is 800 kN/m^2, what will the downstream pressure be if the flow is isothermal at 273 K? Take the viscosity of air as 0.015 mN s/m^2 and the kg molecular volume as 22.4 m^3. What is the significance of the change in kinetic energy of the fluid?

4.12. If temperature does not change with height, estimate the boiling point of water at a height of 3000 m above sea-level. The barometer reading at sea-level is 98.4 kN/m^2 and the temperature is 288.7 K. The vapour pressure of water at 288.7 K is 1.77 kN/m^2. The 'molecular mass' of air is 29 kg/kmol.

4.13. A 150 mm gas main is used for transferring a gas (molecular mass 13 kg/kmol and kinematic viscosity 0.25 cm^2/s) at 295 K from a plant to a storage station 100 m away, at a rate of 1 m^3/s. Calculate the pressure drop, if the pipe can be considered to be smooth.

If the maximum permissible pressure drop is 10 kN/m^2, is it possible to increase the flowrate by 25%?

8.1. A three-stage compressor is required to compress air from 140 kN/m^2 and 283 K to 4000 kN/m^2. Calculate the ideal intermediate pressures, the work required per kilogram of gas, and the isothermal efficiency of the process. Assume the compression to be adiabatic and the interstage cooling to cool the air to the initial temperature. Show qualitatively, by means of temperature-entropy diagrams, the effect of unequal work distribution and imperfect intercooling, on the performance of the compressor.

8.3. A single-stage double-acting compressor running at 3 Hz is used to compress air from 110 kN/m^2 and 282 K to 1150 kN/m^2. If the internal diameter of the cylinder is 20 cm, the length of stroke 25 cm and the piston clearance 5%, calculate (a) the maximum capacity of the machine, referred to air at the initial temperature and pressure, and (b) the theoretical power requirements under isentropic conditions.

8.4. Methane is to be compressed from atmospheric pressure to 30 MN/m^2 in four stages.

Calculate the ideal intermediate pressures and the work required per kilogram of gas. Assume compression to be isentropic and the gas to behave as an ideal gas. Indicate on a temperature-entropy diagram the effect of imperfect intercooling on the work done at each stage.

8.6. In a single-stage compressor:

$$\begin{aligned} \text{Suction pressure} &= 101.3\ \text{kN/m}^2. \\ \text{Suction temperature} &= 283\ \text{K}. \\ \text{Final pressure} &= 380\ \text{kN/m}^2. \end{aligned}$$

If each new charge is heated 18 K by contact with the clearance gases, calculate the maximum temperature attained in the cylinder, assuming adiabatic compression.

8.9. A single-acting air compressor supplies 0.1 m^3/s of air (at STP) compressed to 380 kN/m^2 from 101.3 kN/m^2 pressure. If the suction temperature is 288.5 K, the stroke is 250 mm, and the speed is 4 Hz, find the cylinder diameter. Assume the cylinder clearance is 4% and compression and re-expansion are isentropic ($\gamma = 1.4$). What is the theoretical power required for the compression?

8.10. Air at 290 K is compressed from 101.3 to 2000 kN/m^2 pressure in a two-stage compressor operating with a mechanical efficiency of 85%. The relation between pressure and volume during the compression stroke and expansion of the clearance gas is $PV^{1.25}$ = constant. The compression ratio in each of the two cylinders is the same and the interstage cooler may be taken as perfectly efficient. If the clearances in the two cylinders are 4% and 5% respectively, calculate:

(a) the work of compression per unit mass of gas compressed;
(b) the isothermal efficiency;
(c) the isentropic efficiency ($\gamma = 1.4$);
(d) the ratio of the swept volumes in the two cylinders.