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Lecture No.5 /Fluid Dynamics /2nd Year /Chemical Eng

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In this lecture , investigate the behavior of fluid in motion by using the principle of conservation of energy and principle of conservation of momentum, by applied ***Newton's second law.***

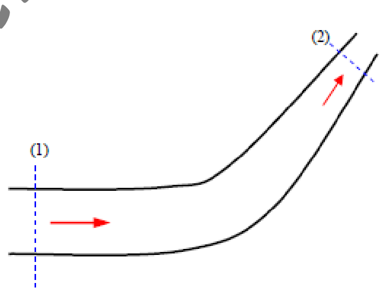
Continuity Equation:

Consider the flow of a fluid through a streamtube, as shown in Fig.1, then by steady state, the total mass (m) of fluid contained in the control volume must be invariant with time, then $\rightarrow \frac{dm}{dt} = 0$

Therefore : → Total Mass Outflow at section 2 = Total Mass Inflow at section 1

which translates into the following mathematical relation:

$$\sum_{i=1}^{M}(\rho_i V_i A_i)_{in} = \sum_{i=1}^{N}(\rho_i V_i A_i)_{out} \quad \text{-----------------------(1)}$$



Eq.1 is called Continuity Equation

i.e →m1=m2 where m mass flowrate

where *M* is the number of inlets, and *N* is the number of outlets, A cross sectional flow area. v average velocity. If the $\rho_1=\rho_2$ of fluid is constant, for incompressible fluid.

Hence for a control volume with only one-dimensional inlets and outlets,

$$\sum_{i=1}^{M}(V_i A_i)_{in} = \sum_{i=1}^{N}(V_i A_i)_{out} \rightarrow \sum_{i=1}^{M}(Q_i)_{in} = \sum_{i=1}^{N}(Q_i)_{out}$$

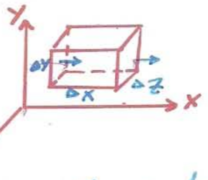
Where Q volumetric flowrate. For example, in a pipe of varying cross sectional area, the continuity equation requires that, if the density is constant, between any two sections 1 and 2 along the pipe.→ $Q = V_1 A_1 = V_2 A_2 = \text{constant}$

Eq1 can be written in term of mass velocity G as, G=ρv in SI Kg/m^2.s→m=G.A

<u>Note</u>: But Eq.1 in three-dimensional (x,y,z) differential expression of the Eq.1 must be used. Consider a small vol. element ΔxΔyΔz of fluid in Fig.2 to get the following Eq.

Fig. to get the following Eqn.:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\partial z}(\rho v_z) = 0 \quad \text{---(*)}$$

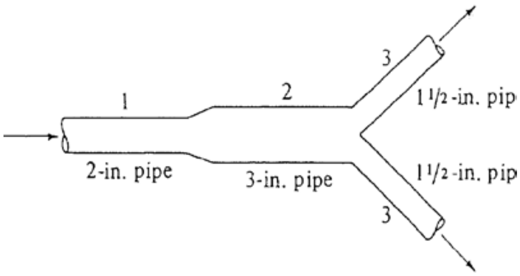


Eqn. (*) for every point in the flow steady or unsteady, Comp. or Incomp., New. or non-New.

But Cont. Eqn. for Incomp. New. or:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$ ← Cont-Eqn. in Three dimensional For Incompressible Fluid.

Ex.1A petroleum crude oil having ρ=829 Kg/m^3 is flowing through the piping arrangement shown in Fig. at a total rate of Q=1.388*10^{-3} m^3/s entering pipe1. The flow divides equally in each of pipes 3. The steel pipes are schedule 40 pipe. Calculate the following using SI units.



i. The total mass flowrate m in pipe1 & pipes3.
ii. The average velocity v in 1 & 3. Re.no.1&3
iii. The mass velocity G in 1.

Solution: From Appendix the dimensions of the pipes are as follows: 2-in. pipe (ID)=2.067 in,

$A1 = \frac{\pi}{4} d_1^2$ =0.0233ft^2=0.0233 (0.0929)=2.165*10^{-3}m^2

$1\frac{1}{2}$-in. pipe: D_3(ID) = 1.610 in., cross-sectional area

$$A_3 = 0.01414 \text{ ft}^2 = 0.01414(0.0929) = 1.313 \times 10^{-3} \text{ m}^2$$

The total mass flow rate is the same through pipes 1 and 2 and is

$$m_1 = (1.388 \times 10^{-3} \text{ m}^3/\text{s})(892 \text{ kg/m}^3) = 1.238 \text{ kg/s}$$

Since the flow divides equally in each of pipes 3,

$$m_3 = \frac{m_1}{2} = \frac{1.238}{2} = 0.619 \text{ kg/s}$$

ii. use Continuity Eq.1 m=ρvA, therefore:

$$v_1 = \frac{m_1}{\rho_1 A_1} = \frac{1.238 \text{ kg/s}}{(892 \text{ kg/m}^3)(2.165 \times 10^{-3} \text{ m}^2)} = 0.641 \text{ m/s}$$

$$v_3 = \frac{m_3}{\rho_3 A_3} = \frac{0.619}{(892)(1.313 \times 10^{-3})} = 0.528 \text{ m/s}$$

Re.no.1=ρv_1 d/μ =? Re.no.3=ρv_3 d/μ =?

$$G_1 = v_1 \rho_1 = \frac{m_1}{A_1} = \frac{1.238}{2.165 \times 10^{-3}} = 572 \frac{\text{kg}}{\text{s} \cdot \text{m}^2}$$

Ex.2 H.W .Oil is flowing through a large tank having a diameter 4m. The velocity of water relative to the tank is given as, v=5-r^2. What is the average velocity of the oil leaving by the smaller pipe having an inside diameter of 1.5m. Ans. 21.33m/s.

Ex.3 H.W. Kerosene flows through a pipe line which contracts from 400mm diameter at A to 275mm diameter at B and then forks, one branch being 150mm diameter discharging at C and the other branch 200mm diameter discharging at D. If the velocity at A is 2.25m/s and the velocity at D is 4m/s what will be the discharge at C &D and velocities at B &C.

<u>Energy of Fluid in Motion:</u>

The total energy **(E)** of a fluid in motion consists of the following components: -

a. **Internal Energy (U):** It is relate to physical state of fluid, i.e the energy of atoms and molecules resulting from their motion & configuration. It is a function of temperature.
b. **Potential Energy (PE):** This is the energy that a fluid has by virtue of its position in the Earth's field of gravity. The work required to raise a unit mass of fluid to a height ***z*** above an arbitrarily chosen datum is ***<u>zg</u>***, where g is the acceleration due to gravity. This work is equal to the potential energy of unit mass of fluid above the datum.
c. **Pressure Energy (P/ρ):** This is the energy or work required to introduce the fluid into the system without a change of volume. If *P* is the pressure and V is the volume of mass ***m*** of fluid, then ***PVlm*** is the pressure energy per unit mass of fluid. The ratio ***V/m*** is the fluid density ***ρ.*** Thus the pressure energy per unit mass of fluid is equal to ***P/ρ.***
d. **Kinetic Energy (KE):** This is the energy of fluid motion. The kinetic energy of unit mass of the fluid is $v^2/2$, where ***v*** is the velocity of the fluid relative to some fixed body.

Therefore the total E per unit mass of fluid is given by the following Eq.:

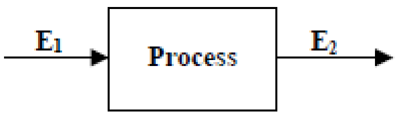
$$E = U + zg + \frac{P}{\rho} + \frac{v^2}{2}$$

where each term has the dimensions of force times distance per unit mass, ie **(ML/T^2) L/M or (L^2/T^2). If in SI unit J/Kg or m^2/s^2 or in English unit→Btu/$1b_m$ or $1b_f$.ft/$1b_m$ or ft^2/s^2**

<u>Bernoulli's Equation from Energy Fluid:</u>

Consider fluid is flowing from point1 to point2 as shown in Fig.1 Assume incompressible fluid i.e, ρ & temp. is constant, ideal fluid (frictionless or no effect viscosity), and there is no work, steady state. Therefore applied conservation energy balance Eq. as following:

E1=E2 →there is no accumulation



$$\boldsymbol{E}1 = \mathrm{U1} + \mathrm{z1g} + \frac{\mathrm{P1}}{\rho} + \frac{\mathrm{V1}^2}{2}$$

$$\boldsymbol{E}2 = \mathrm{U2} + \mathrm{z2g} + \frac{\mathrm{P2}}{\rho} + \frac{\mathrm{V2}^2}{2}$$

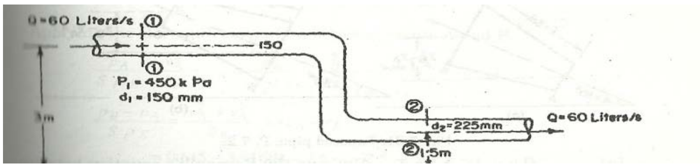
U1=U2 →constant temp. Therefore **<u>Bernoulli's Eq</u>**. is given as following:

$$\mathrm{z1g} + \frac{\mathrm{P1}}{\rho} + \frac{\mathrm{V1}^2}{2} = \mathrm{z2g} + \frac{\mathrm{P2}}{\rho} + \frac{\mathrm{V2}^2}{2}$$ ----------------**BERNOULLI'S Eq.** can be written in terms of Potential Head by dividing acc. g as following:

$$\mathrm{z1} + \frac{\mathrm{P1}}{\rho g} + \frac{\mathrm{V1}^2}{2g} = \mathrm{z2} + \frac{\mathrm{P2}}{\rho g} + \frac{\mathrm{V2}^2}{2g}$$ ----Therefore $\Delta E = \Delta zg + \frac{\Delta P}{\rho} + \frac{\Delta V^2}{2}$ is Bernoulli's Eq.

Ex.1 Oil sp.gr.=0.85 is flowing in a pipe under the conditions shown in Fig. if the total head loss from point 1 to 2 neglected (frictionless), find the pressure at point2.

Sol.



Cont Eq. Q1=Q2→ $v_1 A_1 = v_2 A_2$

Therefore, $v_1 = \frac{Q}{A1} = \frac{60*10^{-3}}{\pi/4(0.15^2)} = 3.395 m/s$

$$v_2 = \frac{Q}{A2} = \frac{60*10^{-3}}{\pi/4(0.225^2)} = 1.509 m/s$$

Applying Bernoulli's Eq. between points 1 &2 to calculate pressure at 2.

$$z1g + \frac{P1}{\rho} + \frac{V1^2}{2} = z2g + \frac{P2}{\rho} + \frac{V2^2}{2}$$

$$3 * 9.81 + \frac{0.450}{0.85*1000} + \frac{3.395^2}{2} = 1.5 * 9.81 + \frac{P2}{0.85*100} + \frac{1.509^2}{2}$$

P2=---?

Ex2.HW In a pipe of 90mm diameter water is flowing with a mean velocity of 2m/s and at a gage pressure of 350KN/m^2. Determine the total head, if the pipe is 8m above the datum line. Neglect friction. Ans.43.88m

<u>The Forces on Fluid Motion:</u>

According to Newton's second law of motion, ***<u>the net force</u>*** in x-direction (Fx) acting on a fluid element in x-direction is: -

Fx = (mass) x (acceleration in x-direction)

Fx = (m) (a)

And the following total net forces effect on an element of fluid flow in pipe:

i. force due to gravity →F_g
ii. force due to pressure →F_p
iii. force due to viscosity →F_v
iv. force due to turbulence →F_t
v. force due to compressibility →F_c
vi. force due to surface tension →F_σ

Therefore:

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x + (F_\sigma)_x$$

- If fluid in motion consider only forces effect →F_g & →F_p and neglect others forces, then this Eq. is called **<u>Euler's Eq.</u>**
- If fluid in motion consider only forces effect →F_g , →F_p &→F_v and neglect others forces, then this Eq. is called **<u>Navier Stoke's Eq.</u>**
- If fluid in motion consider only forces effect →F_g , →F_p ,→F_v,& →F_t and neglect others forces, then this Eq. is called **<u>Reynolds Stresses Eq.</u>**

<u>NOTE:</u> *In most of the problems of fluid in motion the forces due to surface tension (Fσ), and the force due to compressibility (Fc) are neglected.*

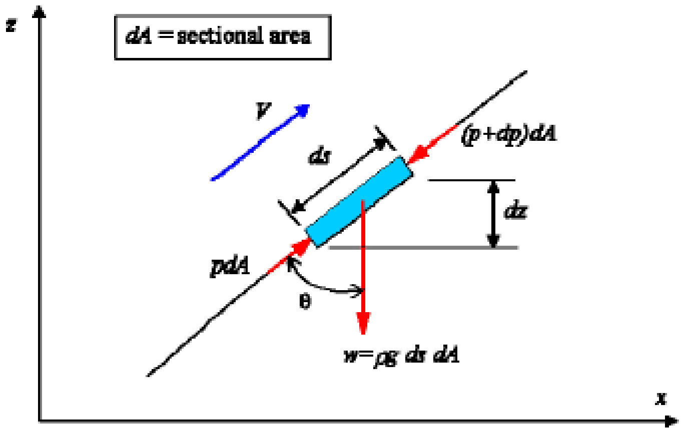
Euller's & Bernoulli's Eq. of Motion Derived from Mechanical Energy Balnce:

Let us first derive the *Bernoulli* & Euller's Eqns., which is one of the most well-known equations of motion in fluid mechanics, and yet is often misused. It is thus important to understand its limitations, and the assumptions made in the derivation.

The assumptions can be summarized as follows:

- Inviscid flow (ideal fluid, frictionless)
- Steady flow
- Along a streamline
- Constant density (incompressible flow)
- No shaft work or heat transfer.

The Bernoulli equation is based on the application of Newton's law of motion to a fluid element on a streamline.



Let us consider the motion of a fluid element of length *ds* and cross-sectional area *dA* moving at a local speed *V*, and *x* is a horizontal axis and *z* is pointing vertically upward. The forces acting on the element are the pressure forces *pdA* and *(p+dp)dA*, and the weight *w* as shown. Summing forces in the direction of motion, the *s*-direction, there results is given as:

$$pdA-(p+dp)dA-\rho g\ ds\ dA\cos\theta=\rho\ ds\ dA\ a_s$$

where a_s is the acceleration of the element in the *s*-direction. Since the flow is steady, only convective acceleration exists.

$$a_s=V\frac{dV}{ds}$$

Also, it is easy to see that cosθ=*dz/ds* On substituting and dividing the equation by **ρgdA, to obtain Euler's equation as following:**

$$\frac{dp}{\rho g}+dz+\frac{V}{g}dV=0$$

Note: This Eq. is applied for any type of fluid (incompressible & compressible)

Now if further assume that the flow is incompressible so that the density is constant, may integrate Euler's equation to get Bernoulli's Eq. as following:

$$\boxed{\frac{p}{\rho g}+z+\frac{V^2}{2g}=\text{constant}}$$

This is the **Bernoulli equation**, consisting of three **energy head** as following:

$\frac{p}{\rho g}$	Pressure head, which is the work done to move fluid against pressure
z	Elevation head, representing the potential energy; z can be measured above any reference datum
$\frac{V^2}{2g}$	Velocity head, representing the kinetic energy

- A **head** corresponds **to energy per unit weight of flow** and has dimensions of **length**.
- **Piezometric head = pressure head + elevation head**, which is the level registered by a piezometer connected to that point in a pipeline.
- **Total head = piezometric head + velocity head.**

It follows that for ideal steady flow the total energy head is constant along a streamline, but the constant may differ in different streamlines. (For the particular case of irrotational flow, the Bernoulli constant is universal throughout the entire flow field.)

Applying the Bernoulli equation to any two points on the same streamline, to get:

$$\frac{p_1}{\rho g}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\rho g}+z_2+\frac{V_2^2}{2g}$$

Ex.1 Brine of sp.gr.=1.15 is draining from the bottom of a large open tank through a 80mm pipe. The drain pipe ends at a point 10m below the surface of the brine in a tank. Considering a streamline starting at the surface of the brine in the tank and passing through the centre of the drain line to the point of discharge and assuming the friction is negligible, calculate the velocity of flow along the streamline at the point of discharge from the pipe.

Sol. Point 1 at the surface of brine in the tank.

Point 2 at the discharge brine., applying Bernoulli's Eq. between points 1&2 to get:→ Sketch the system?

$$\frac{p_1}{\rho g}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\rho g}+z_2+\frac{V_2^2}{2g}$$

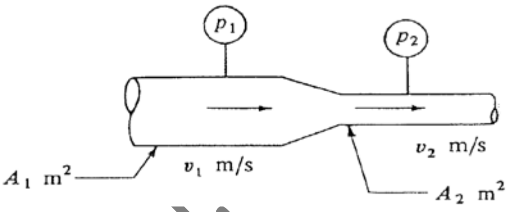
P1=P2 atm. Pressure , V1=0 (large cross sectional area) and z1-z2=10m

Therefore $V2^2=2g(z1-z2)=2g*10=196.2\rightarrow v2=14m/s$

EX.2

A liquid with a constant density ρ kg/m^3 is flowing at an unknown velocity v_1 m/s through a horizontal pipe of cross-sectional area A_1 m^2 at a pressure p_1 N/m^2, and then it passes to a section of the pipe in which the area is reduced gradually to A_2 m^2 and the pressure is p_2. Assuming no friction losses, calculate the velocity v_1 and v_2 if the pressure difference $(p_1 - p_2)$ is measured.

Sol.. The flow diagram is shown in Fig, with pressure taps to measure p_1 & p_2.



Ist step: From Cont. Eq.

$m_1 = m_2$, $\rho_1 = \rho_2$ (incompressible fluid)

therefore $$v_2 = \frac{v_1 A_1}{A_2}$$

2nd step Applying Ber. Eq. between p_1 & p_2 → $$z_1 g + \frac{v_1^2}{2} + \frac{p_1}{\rho} = z_2 g + \frac{v_2^2}{2} + \frac{p_2}{\rho}$$

$z_1 = z_2 = 0$ for horizontal line. Therefore Ber Eq. becomes:

$$0 + \frac{v_1^2}{2} + \frac{p_1}{\rho} = 0 + \frac{v_1^2 A_1^2 / A_2^2}{2} + \frac{p_2}{\rho}$$

Rearranging,

$$p_1 - p_2 = \frac{\rho v_1^2[(A_1/A_2)^2 - 1]}{2}$$

$$v_1 = \sqrt{\frac{p_1 - p_2}{\rho} \frac{2}{[(A_1/A_2)^2 - 1]}} \quad \text{(SI)}$$

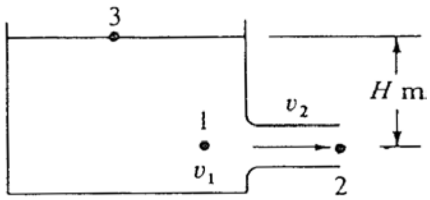
$$v_1 = \sqrt{\frac{p_1 - p_2}{\rho} \frac{2g_c}{[(A_1/A_2)^2 - 1]}} \quad \text{(English)}$$

Performing the same derivation but in terms of v_2,

$$v_2 = \sqrt{\frac{p_1 - p_2}{\rho} \frac{2}{1 - (A_2/A_1)^2}}$$

EX.3

A nozzle of cross-sectional area A_2 is discharging to the atmosphere and is located in the side of a large tank, in which the open surface of the liquid in the tank is H m above the center line of the nozzle. Calculate the velocity v_2 in the nozzle and the volumetric rate of discharge if no friction losses are assumed.



Sol. The process flow is shown in Fig.

Since A_1 is very large compared to A_2, $v_1 \cong 0$. The pressure p_1 is greater than 1 atm (101.3 kN/m^2) by the head of fluid of H m. The pressure p_2, which is at the nozzle exit, is at 1 atm. Using point 2 as a datum, $z_2 = 0$

And z1=0 , then rearranging Ber. Eq. to give:

$$z_1 g + \frac{v_1^2}{2} + \frac{p_1 - p_2}{\rho} = z_2 g + \frac{v_2^2}{2}$$

Substituting the known values,

$$0 + 0 + \frac{p_1 - p_2}{\rho} = 0 + \frac{v_2^2}{2}$$

Solving for v_2,

$$v_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho}} \qquad \text{m/s}$$

Since $p_1 - p_3 = H\rho g$ and $p_3 = p_2$ (both at 1 atm),

$$H = \frac{p_1 - p_2}{\rho g} \qquad \text{m}$$

Where H is the head of liquid with density ρ. Therefore v_2 Eq becomes as following:

$$v_2 = \sqrt{2gH}$$

The volumetric flow rate is

$$\text{flow rate} = v_2 A_2 \qquad \text{m}^3/\text{s}$$

Note: other solution by applying Ber. Between points 3 &2, get the same answer:

$$z_2 g + \frac{v_2^2}{2} + \frac{p_2 - p_3}{\rho} = z_3 g + \frac{v_3^2}{2}$$

Since $p_2 = p_3 = 1$ atm, $v_3 = 0$, and $z_2 = 0$,

$$v_2 = \sqrt{2gz_3} = \sqrt{2gH}$$

EX.3 A pipe is 15cm in dia. & it is at an elevation of 100m at section A. At section B it is an elevation of 107m & has dia. Of 30cm. When a discharge of 50 l/s of water is passed through this pipeline, pressure at A is 35KPa. neglected energy losses. Calculate pressure at B if flow is from A & B.

Modification to Bernoulli's Equation

1. **Correction of K.E term:** if v is the average velocity at the section because at any section of the pipe line the velocity is not uniform. Therefore express the velocity in terms of the average velocity, it is necessary to introduce a dimensionless coefficient as α , the value α dependent on the velocity distribution as following

$$\frac{P1}{\rho} + gz1 + \frac{v1^2}{2\alpha 1} = \frac{P2}{\rho} + gz2 + \frac{v2^2}{2\alpha 2}$$

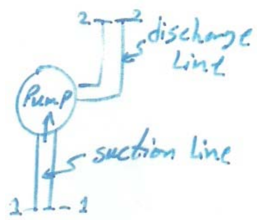
Where α is a factor called K.E correction factor ,
α=1 for Turbulent. Flow & α=0.5 for Laminar. Flow

2. **Correction for Real Fluid or Friction of fluid:** for incompressible fluids through a pipe system, there will be losses of energy due to friction separation eddying etc; Therefore the total mechanical energy will not constant but degrease in the direction of flow as a result of energy dissipation or adding a term F to the right side Ber Eq. as following:

$$\frac{P1}{\rho} + gz1 + \frac{v1^2}{2\alpha 1} = \frac{P2}{\rho} + gz2 + \frac{v2^2}{2\alpha 2} + \sum F$$

Where$\sum$ F → Total Energy losses in a pipe system due to friction has dim. Energy/mass or in term as head ($\sum$F/g) has dim (L).

3. **Correction for Pumps or Work: Pumps**: which convert mechanical energy into hydraulic energy and turbines. Therefore Ber. Eq can be written as:



$$\frac{P1}{\rho} + gz1 + \frac{v1^2}{2\alpha 1} + \eta W_s = \frac{P2}{\rho} + gz2 + \frac{v2^2}{2\alpha 2} + \sum F$$

Where W_p work of the pump in J/Kg (S.I), η→efficiency of pump, therefore ηW_s is the net work to the fluid..The work supplied to the pump is shift work ($-W_s$), the negative sign because the work added to the fluid from out system. But there are two types friction in the pump:

1- Friction by fluid motion, & 2-Mechanical friction…Since the shaft work must be discounted by these frictional force to give net mechanical energy as actually delivered to the fluid by pump $W_p = -\eta W_s$

Ber Eq between points 1 v&2 can be written in term head by dividing each term by (g) as following:

$$\frac{P1}{\rho g} + z1 + \frac{v1^2}{2g\alpha 1} + \frac{\eta W_s}{g} = \frac{P2}{\rho g} + z2 + \frac{v2^2}{2g\alpha 2} + \sum h_f$$

Where $\sum h_f = \sum F / g$ = Total head losses due to friction

Note: the power expanded by the pump ⇨ $Power = \rho Q W_p / \eta$ (in Watt).

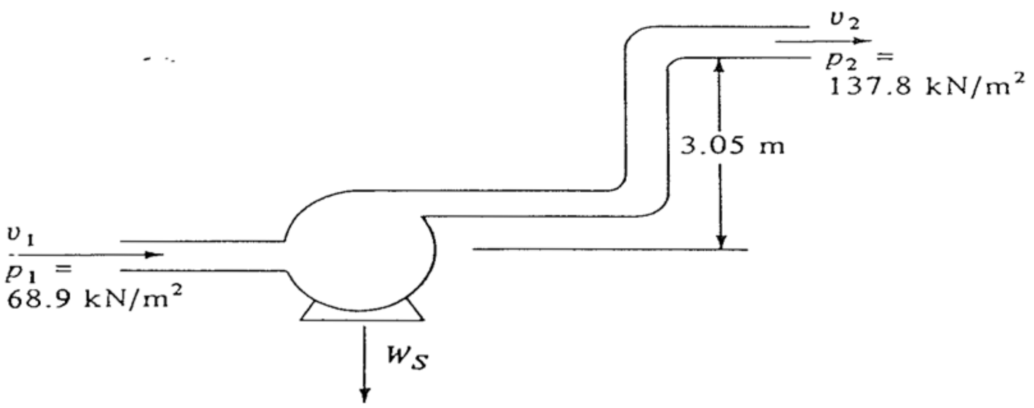
EX.4

Water with a density of 998 kg/m^3 is flowing at a steady mass flow rate through a uniform-diameter pipe. The entrance pressure of the fluid is 68.9 kN/m^2 abs in the pipe, which connects to a pump which actually supplies 155.4 J/kg of fluid flowing in the pipe. The exit pipe from the pump is the same diameter as the inlet pipe. The exit section of the pipe is 3.05 m higher than the entrance, and the exit pressure is 137.8 kN/m^2 abs. The Reynolds number in the pipe is above 4000 in the system. Calculate the frictional loss $\sum F$ in the pipe system.

Sol.

155.4 J/kg mechanical energy added to the fluid. Hence, $W_S = -155.4$, since the work done by the fluid is positive.

Setting the datum height $z_1 = 0$, $z_2 = 3.05$ m. Since the pipe is of



constant diameter, $v_1 = v_2$. Also, for turbulent flow $\alpha = 1.0$ and

$$\frac{1}{2(1)}(v_2^2 - v_1^2) = 0$$

$$z_2 g = (3.05 \text{ m})(9.806 \text{ m/s}^2) = 29.9 \text{ J/kg}$$

Since the liquid can be considered incompressible, Eq. (2.7-28) is used.

$$\frac{p_1}{\rho} = \frac{68.9 \times 1000}{998} = 69.0 \text{ J/kg}$$

$$\frac{p_2}{\rho} = \frac{137.8 \times 1000}{998} = 138.0 \text{ J/kg}$$

Using Eq. (2.7-28) and solving for $\sum F$, the frictional losses,

$$\sum F = -W_s + \frac{1}{2\alpha}(v_1^2 - v_2^2) + g(z_1 - z_2) + \frac{p_1 - p_2}{\rho} \qquad \text{(2.7-29)}$$

Substituting the known values, and solving for the frictional losses,

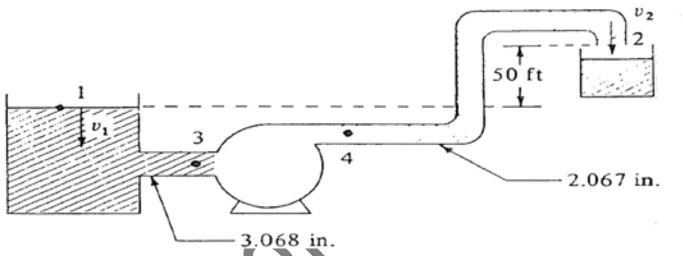
$$\sum F = -(-155.4) + 0 - 29.9 + 69.0 - 138.0$$

$$= 56.5 \text{ J/kg} \left(18.9 \frac{\text{ft} \cdot \text{lb}_f}{\text{lb}_m}\right)$$

EX.5

A pump draws 69.1 gal/min of a liquid solution having a density of 114.8 lb_m/ft^3 from an open storage feed tank of large cross-sectional area through a 3.068-in.-ID suction line. The pump discharges its flow through a 2.067-in.-ID line to an open overhead tank. The end of the discharge line is 50 ft above the level of the liquid in the feed tank. The friction losses in the piping system are $\sum F = 10.0$ ft-lb force/lb mass. What pressure must the pump develop and what is the horsepower of the pump if its efficiency is 65% ($\eta = 0.65$)? The flow is turbulent.

Sol: **1st step** Apply Cont Eq to cal. Velocity & check Re. No. $v_3 = Q/A_3 \quad v_4 = Q/A_4$



A_3=0.05134 ft^2 A_4=0.0233 ft^2

Q=69.1 gal/min.=0.1539 ft^3/s

m=Q/ρ=17.65 Ib_m/s (mass flowrate)

Hence: $v_3 = 3.0\frac{ft}{s}$ & $v_4 = 6.61\frac{ft}{s}$, $v_1 = 0$, $v_2 = v_4$

Check Re No.=$\rho v d/\mu$ →α=1 turbulent flow

2nd step Apply Ber Eq between points 1 & 2 as following:

$$\cancel{\frac{P1}{\rho}} + \cancel{gz1/g_c} + \cancel{\frac{v1^2}{2\alpha1g_c}} + \eta W_s = \cancel{\frac{P2}{\rho}} + gz2/g_c + \frac{v2^2}{2\alpha2g_c} + \sum F$$

P_1=P_2 atm. Pressure, z_1=0 at datum line, z_2=50 ft

$$\frac{gz2}{g_c} = (50)ft\frac{32.174ft/s^2}{32.174ft\ Ib_m/Ib_f s^2} = 50\frac{ft.Ib_f}{Ib_m}$$

$$\frac{v2^2}{2\alpha2g_c} = 0.678\frac{ft.Ib_f}{Ib_m}$$

$$\eta W_s = \frac{gz2}{g_c} + \frac{v2^2}{2\alpha2g_c} + \sum F = 50\frac{ft.Ib_f}{Ib_m} + 0.678\frac{ft.Ib_f}{Ib_m} + 10 = 60.678\frac{ft.Ib_f}{Ib_m}$$

Hence The shaft work W_s=$\frac{60.678\frac{ft.Ib_f}{Ib_m}}{0.65}$ = $93.3\frac{ft.Ib_f}{Ib_m}$

$$\text{pump horsepower} = \left(17.65\ \frac{lb_m}{s}\right)\left(93.3\ \frac{ft \cdot lb_f}{lb_m}\right)\left(\frac{1\ hp}{550\ ft \cdot lb_f/s}\right) = 3.00\ hp$$

3rd step Apply Ber Eq between 3&4 over the pump only, to calculate the press. The pump must developed, as following:

$$\frac{P3}{\rho} + \cancel{gz3/g_c} + \frac{v3^2}{2\alpha3g_c} + \eta W_s = \frac{P4}{\rho} + \cancel{gz4/g_c} + \frac{v4^2}{2\alpha4g_c} + \cancel{\sum F}$$ Note F=0 in a pump

$\frac{P4}{\rho} - \frac{P3}{\rho} = \frac{v3^2}{2\alpha 3 g_c} - \frac{v4^2}{2\alpha 4 g_c} + \eta W_s = 0.14 - 0.678 + 60.678 = 60.14 \frac{ft.Ib_f}{Ib_m}$ Hence

ΔP=48.0 Ib_f/in^2 (psia)=331 KPa