الجامعة التكنولوجية

قسم الهندسة الكيمياوية

المرحلة الاولى

الفيزياء

أ.م. د. فلك اسامة

In its natural state, an average of 5.71×10^6 kg of water flowed per second over Niagara Falls, falling 51.0 m. If all the work done by gravity could be converted into electric power as the water fell to the bottom, how much power would the falls generate?

Variables

height of falls	$h = 51.0$ m
magnitude of acceleration due to gravity	$g = 9.80$ m/s^2
potential energy	PE
mass of water over falls per unit time	$m/t = 5.71 \times 10^6$ kg/s
power	P
work done by gravity	W

What is the strategy?

1. Use the definition of power as the rate of work done to define an equation for the power of the falls.

2. Use the fact that work done by gravity equals the negative of the change in gravitational potential energy to solve for the power.

Physics principles and equations

Power is the rate at which work is performed.

$$P = \frac{W}{\Delta t}$$

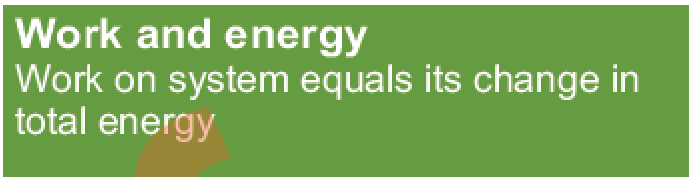
Change in gravitational *PE*

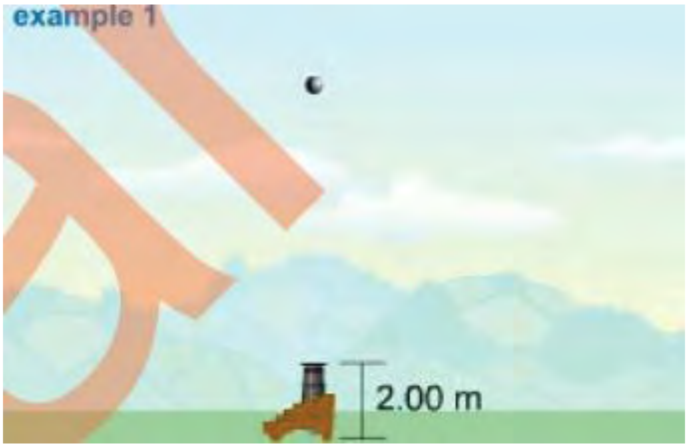
$\Delta PE = mg\Delta h$

Work done by gravity

$W = -\Delta PE$

kg cannonball straight up. The barrel of the cannon is 2.00 m long, and it exerts an average force of 6,250 N while the cannonball is in the cannon. We will ignore air resistance. Can we determine the cannonball's velocity when it has traveled 125 meters upward? As you may suspect, the answer is "yes". The cannon does 12,500 J of work on the cannonball, the product of the force (6,250 N) and the displacement (2.00 m). (We assume the cannon does no work on the cannonball after it leaves the cannon.) At a height of 125 meters, the cannonball's increase in *PE* equals $mg\Delta h$, or 3,920 J. Since a total of 12,500 J of work was done on the ball, the rest of the work must have gone into raising the cannonball's *KE*: The change in *KE* is 8,580 J. Applying the definition of kinetic energy, we determine that its velocity at 125 m is 73.2 m/s. We could further analyze the cannonball's trip if we were so inclined. At the peak of its trip, all of its energy is potential since its velocity (and *KE*) there are zero. The *PE* at the top is 12,500 J. Again applying the formula $mg\Delta h$, we can determine that its peak height above the cannon is about 399 m.

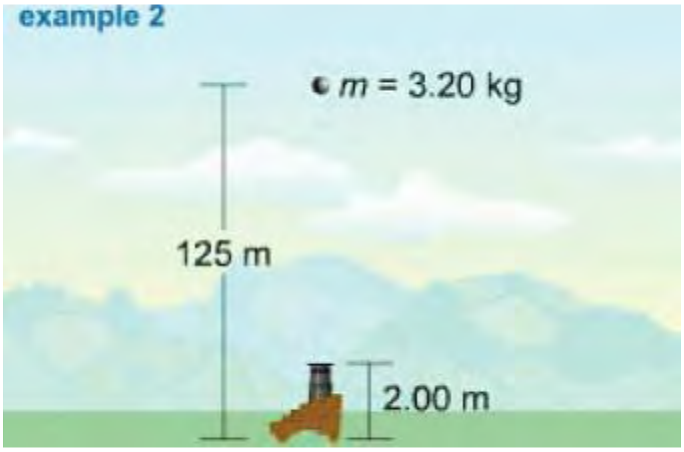




The cannon supplies 6,250 N of force along its 2.00 m barrel. How much work does the cannon do on the cannonball?

$W = (F \cos \theta)\Delta x = F\Delta x$

$W = (6250 \text{ N})(2.00 \text{ m}) = 12{,}500 \text{ J}$



What is the cannonball's velocity at 125 m? Its mass is 3.20 kg.

$W = \Delta PE + \Delta KE$

$W = mg\Delta h + \Delta KE$

$12{,}500 \text{ J} = (3.20 \text{ kg})(9.80 \text{ m/s}^2)(125 \text{ m})$
$+ \Delta KE$

$\Delta KE = 8{,}580 \text{ J}$

$\frac{1}{2} mv^2 = 8{,}580 \text{ J}$

$v^2 = 2(8{,}580 \text{ J})/(3.20 \text{ kg})$

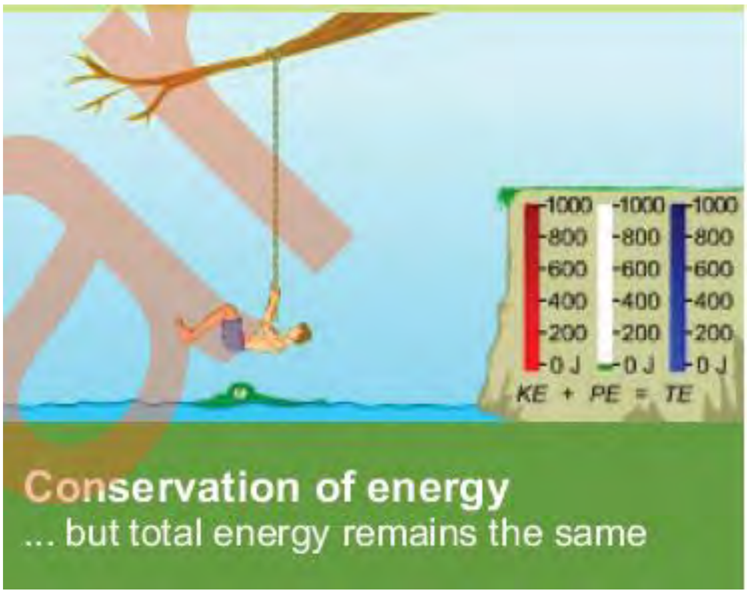
$v = 73.2 \text{ m/s}$

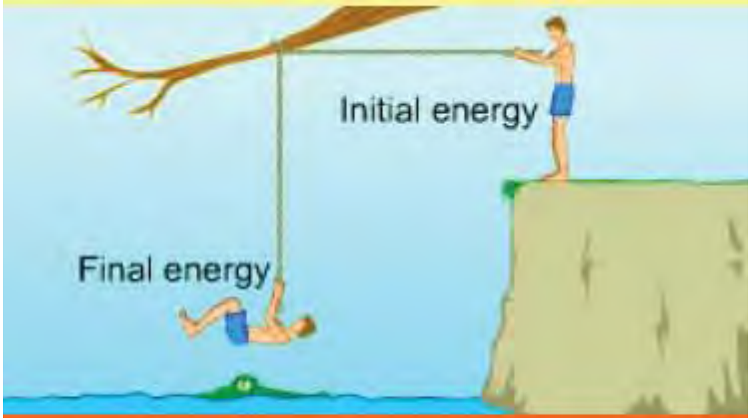
Conservative and non-conservative forces Earlier, when discussing potential energy, we mentioned that we would explain conservative forces later. The concept of potential energy only applies to conservative forces. Gravity is an example of a *conservative force*. It is conservative because the total work it does on an object that starts and finishes at the same point is zero. For example, if a 20 kg barbell is raised 2.0 meters, gravity does □40 J of work, and when the barbell is lowered 2.0 meters back to its initial position, gravity does +40 J of work. When the barbell is returned to its initial position, the sum of the work done by gravity on the one that has no interactions with its environment. The particles within the system may interact with one another, but no net external force or field acts on an isolated system. Only external forces can change the total energy of a system. If a giant spring lifts a car, you can say the spring has increased the energy of the car. In this case, you are considering the spring as supplying an external force and not as part of the system. If you include the spring in the system, the increase in the energy of the car is matched by a decrease in the potential energy contained of the spring, and the total energy of the system remains the same. For the law of conservation of energy to apply, there can be no non-conservative forces like friction within the system. The law of conservation of energy can be expressed mathematically, as shown in Equation 1. The equation states that an isolated system's total energy at any final point in time is the same as its total energy at an initial point in time. When considering mechanical energy, we can state that the sum of the kinetic and potential energies at some final moment equals the sum of the kinetic and potential energies at an initial moment.

In the case of the boy on the rope, if you know his mass and height on the riverbank, you can calculate his gravitational potential energy. In this example, rather than saying his *PE* equals zero on the ground, we say it equals zero at the bottom of the arc. This simplifies matters. Using the law of conservation of energy, you can then determine what his kinetic energy, and therefore his speed, will be when he reaches the bottom of the arc, nearest to the water, since at that point all his energy is kinetic.

Let's leave the boy swinging for a while and switch to another example: You drop a weight. When the weight hits the ground it will stop moving. At this point, the weight has neither kinetic energy nor potential energy because it has no motion and its height off the Earth's surface is zero. Does the law of conservation of energy still hold true? Yes, it does, although we need to broaden the forms of energy included in the discussion. With careful observation you might note that the ground shakes as the weight hits it (more energy of motion). The weight and the ground heat up a bit (thermal energy). The list can continue: energy of the motion of flying dirt, the energy of sound and so on. The amount of mechanical energy does decline, but when you include all forms of energy, the overall energy stays constant. There is a caveat to the law of conservation of energy. Albert Einstein demonstrated that there is a relationship between mass and energy. Mass can be converted into energy, as it is inside the Sun or a nuclear reactor, and energy can be converted into mass. It is the sum of mass and energy that remains constant. Our current focus is on much less extreme situations. Using the principle of conservation of energy can have many practical benefits, as automotive engineers are now demonstrating. When it comes to energy and cars, the focus is often on how to cause the car to accelerate, how fast they will reach say a speed of 100 km/h. Of course, cars also need to slow down, a task assigned to the brakes. As conventional cars brake, the energy is typically dissipated as heat as the brake pads rub on the rotors. Innovative new cars, called hybrids, now capture some of the kinetic energy and convert it to chemical energy stored in batteries or mechanical energy stored in flywheels. The engine then recycles that energy back into kinetic energy when the car needs to accelerate, saving gasoline

Conservation of energy
PE transforms to *KE* ...



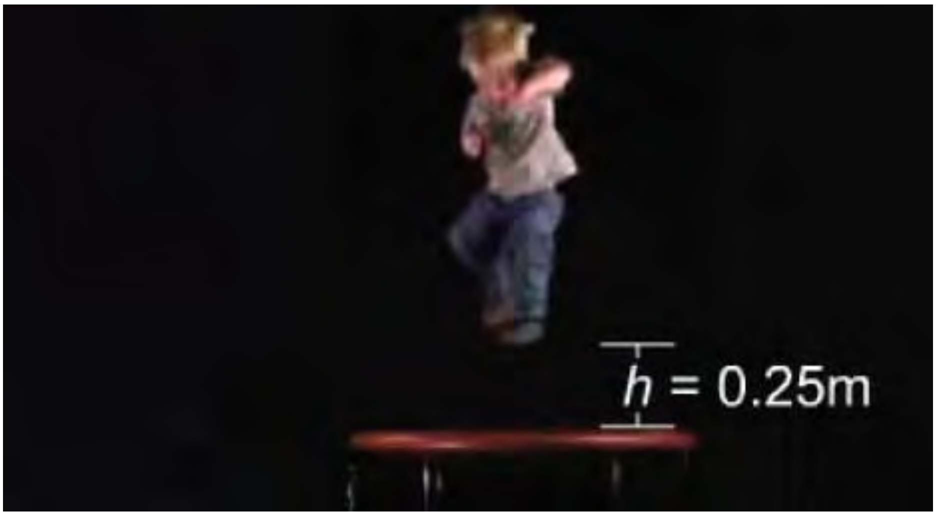


Conservation of energy

$$E_f = E_i$$

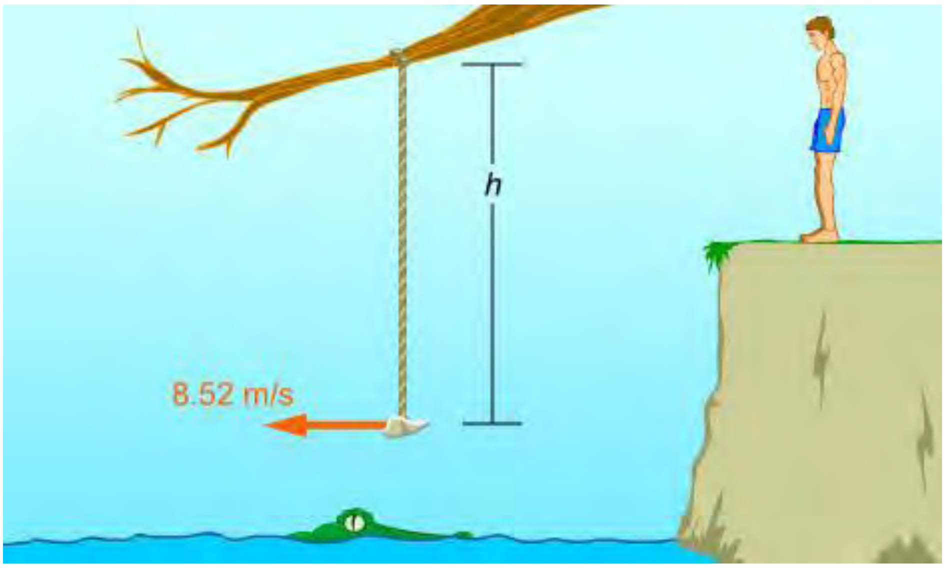
$$PE_f + KE_f = PE_i + KE_i$$

E = total energy
KE = kinetic energy
PE = potential energy



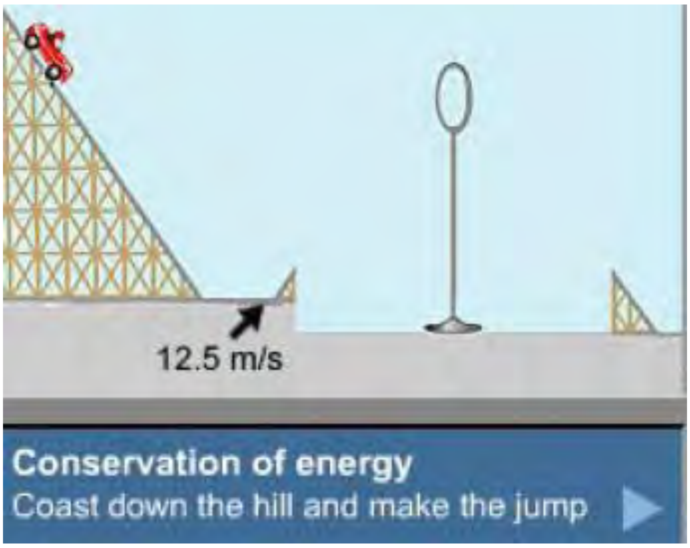
Sam is at the peak of his jump. Calculate Sam's speed when he reaches the trampoline's surface.

Sam is jumping up and down on a trampoline. He bounces to a maximum height of 0.25 m above the surface of the trampoline. How fast will he be traveling when he hits the trampoline? We define Sam's potential energy at the surface of the trampoline to be zero.



A boy releases a pork chop on a rope. The chop is moving at a speed of 8.52 m/s at the bottom of its swing. How much higher than this point is the point from which the pork chop is released? Assume that it has no initial speed when starting its swing.

The law of conservation of energy states that the total energy in an isolated system remains constant. In the simulation on the right, you can use this law and your knowledge of potential and kinetic energies to help a soapbox derby car make a jump. A soapbox derby car has no engine. It gains speed as it rolls down a hill. You can drag the car to any point on the hill. A gauge will display the car's height above the ground. Release the mouse button and the car will fly down the hill. In this interactive, if the car is traveling 12.5 m/s at the bottom of the ramp, it will successfully make the jump through the hoop. Too slow and it will fall short; too fast and it will overshoot. You can use the law of conservation of energy to figure out the vertical position needed for the car to nail the jump.



Friction and conservation of energy

In this section, we show how two principles we have discussed can be combined to solve a typical problem. We will use the principle of conservation of energy and how work done by an external force affects the total energy of a system to determine the effect of friction on a block sliding down a plane. Suppose the 1.00 kg block shown to the right slides down an inclined wooden plane. Since the block is released from rest, it has no initial velocity. It loses 2.00 meters in height as it slides, and it slides 6.00 meters along the surface of the inclined plane. The force of kinetic friction is 2.00 N. You want to know the block's speed when it reaches the bottom position.

To solve this problem, we start by applying the principle of conservation of energy. The block's initial energy is all potential, equal to the product of its mass, *g* and its height (*mgh*). At a height of 2.00 meters, the block's *PE* equals 19.6 J. The potential energy will be zero when the block reaches the bottom of the plane. Ignoring friction, the *PE* of the block at the top equals its *KE* at the bottom.

Now we will factor in friction. The force of friction opposes the block's motion down the inclined plane. The work it does is negative, and that work reduces the energy of the block. We calculate the work done by friction on the block as the force of friction times the displacement along the plane, which equals □12.0 J. The block's energy at the top (19.6 J) plus the □12.0 J means the block has 7.6 J of kinetic energy at the bottom. Using the definition of kinetic energy, we can conclude that the 1.00 kg block is moving at 3.90 m/s. You can also calculate the effect of friction by determining how fast the block would be traveling if there were no friction. All 19.6 J of *PE* would convert to *KE*, yielding a speed of 6.26 m/s. Friction reduces the speed of the block by approximately 38%.

Review of forces, work and energy

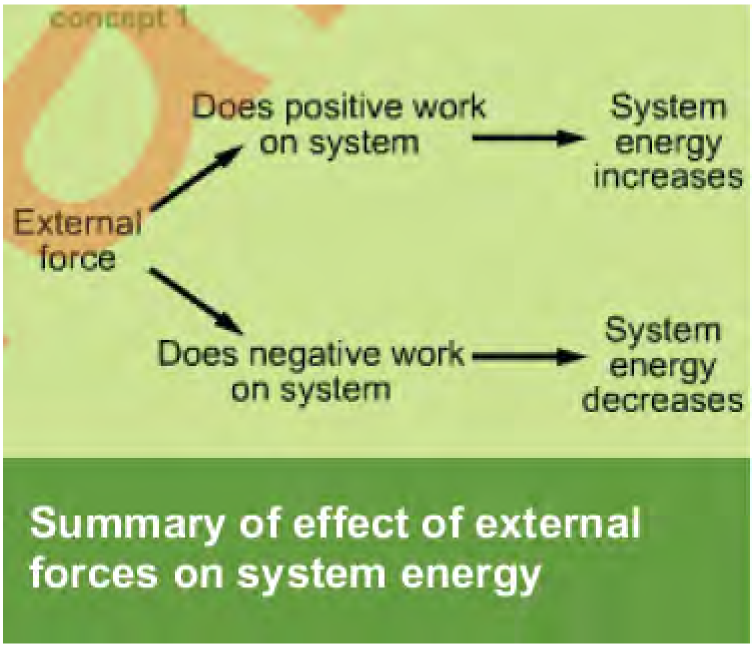
In this chapter we have discussed the work done when a force is exerted on a particle (the work-kinetic energy theorem). We have discussed work and energy with respect to a system of objects (potential energy). We have also covered conservative and non-conservative forces.

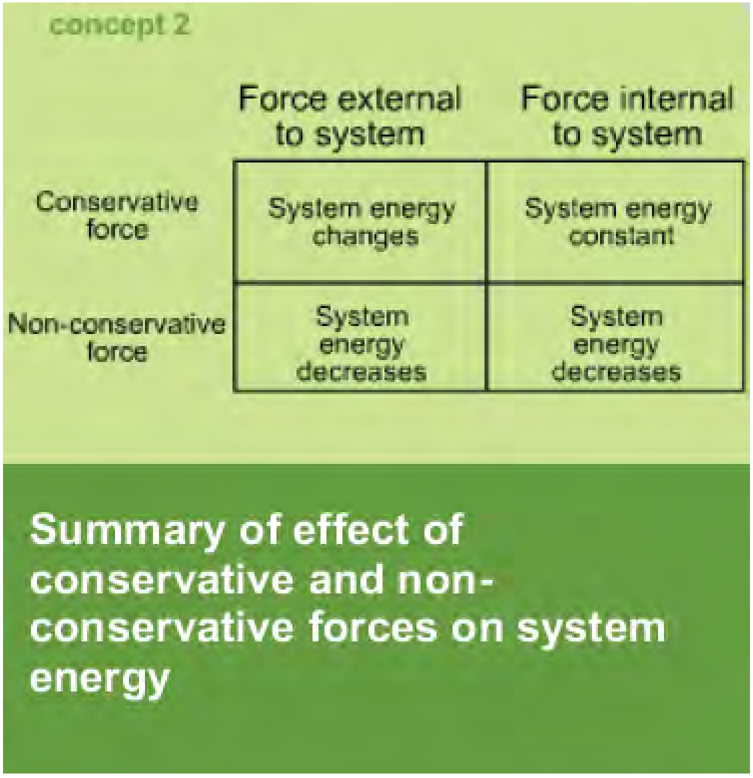
We further categorized forces by stating that some are *external forces*, forces from a source outside the objects that make up the system. For example, we talked about a foot applying force to a soccer ball, and a painter hoisting up a paint bucket. In both these examples, the foot and painter are considered external to the system, which consists either of a single particle (the ball) or multiple objects (the bucket and the Earth). In contrast, other forces are *internal forces* in a system. In the bucket/Earth system, for example, the force of gravity is an internal force. It arises from the objects that make up the system.

In this section, we review and summarize the effect on mechanical energy from all these types of forces: external and internal, conservative and non-conservative. We want to consider how the work done by these various types of forces affects the mechanical energy of a system. We will start with external forces, and consider the effect of the work done by an external force on the total energy of a system. Any net external force acting on a particle or system changes the system's energy. Positive work done on a system by an external force increases the system's total energy, and negative work done on a system by an external force decreases its total energy. This is illustrated in the diagram in Concept 1. If the system consists of one particle, then the work equals the change in kinetic energy. This is the work-kinetic energy theorem. A single particle cannot have potential energy, so positive work on the particle increases its *KE*, and negative work done on the particle (or work done by the particle) decreases its *KE*. The work done on any system by an external force changes the system's total mechanical energy. Let's consider a system that consists of an apple and the Earth. Positive work may increase the *PE* (you lift the apple upwards at a constant rate), or *KE* (you run faster and faster with the apple held at a constant height), or both (you throw an initially stationary apple skyward).

You need to be careful of the sign of the work done by considering whether the force is in the direction of the displacement (positive work) or the opposite direction (negative work). If you throw a ball, you increase its energy, and when you catch it, you decrease its energy. Non-conservative forces decrease the mechanical energy of a system. (There are scenarios where they can be considered as increasing the mechanical energy, but we will ignore them here.) If you slide a block down a plane, the non-conservative forces of kinetic friction and air resistance act in the opposite direction of the block's displacement. This means they do negative work, and reduce the mechanical energy of the system.

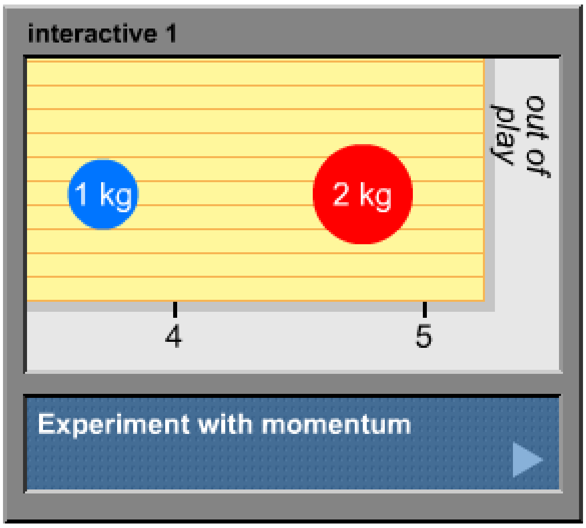
Now let's consider the effect of internal forces on the energy of a system. We will start with internal conservative forces. These forces do not change the total mechanical energy of a system. Consider a system consisting of a block, an inclined plane, and the Earth. The block is sliding down the plane. The force of gravity is conservative, and the decrease in gravitational potential energy as the block slides down the plane is matched by an increase in kinetic energy. This is the law of conservation of energy. The conservative force of gravity does not change the total mechanical energy of the system. We will foreshadow thermodynamics here. The force of friction will increase the temperature of the block and plane. It increases the *internal*





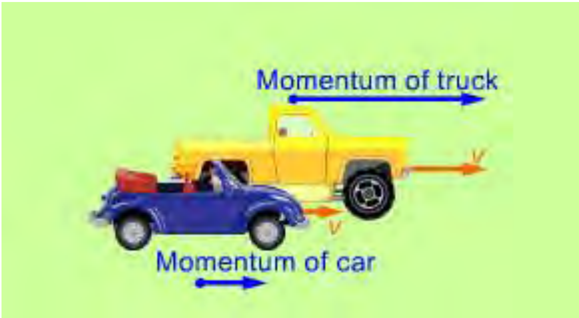
Introduction

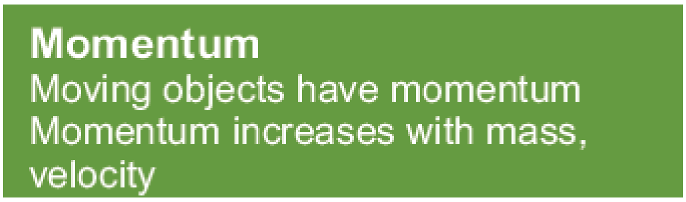
"The more things change, the more they stay the same" is a well-known French saying. However, though witty and perhaps true for many matters on which the French have great expertise, this saying is simply not good physics. Instead, a physicist would say: "Things stay the same, period. That is, unless acted upon by a net force." Perhaps a little less *joie de vivre* than your average Frenchman, but nonetheless the key to understanding momentum. What we now call momentum, Newton referred to as “quantity of motion.” The linear momentum of an object equals the product of its mass and velocity. (In this chapter, we focus on linear momentum. Angular momentum, or momentum due to rotation, is a topic in another chapter.) Momentum is a useful concept when applied to collisions, a subject that can be a lot of fun. In a collision, two or more objects exert forces on each other for a brief instant of time, and these forces are significantly greater than any other forces they may experience during the collision. At the right is a simulation □ a variation of shuffleboard □ that you can use to begin your study of momentum and collisions. You can set the initial velocity for both the blue and the red pucks and use these velocity settings to cause them to collide. The blue puck has a mass of 1.0 kg, and the red puck a mass of 2.0 kg. The shuffleboard has no friction, but the pucks stop moving when they fall off the edge. Their momenta and velocities are displayed in output gauges. Using the simulation, answer these questions. First, is it possible to have negative momentum? If so, how can you achieve it? Second, does the collision of the pucks affect the sum of their velocities? In other words, does the sum of their velocities remain constant? Third, does the collision affect the sum of their momenta? Remember to consider positive and negative signs when summing these values. Press PAUSE before and after the collisions so you can read the necessary data. For an optional challenge: Does the collision conserve the total kinetic energy of the pucks? If so, the collision is called an elastic collision. If it reduces the kinetic energy, the collision is called an inelastic collision.

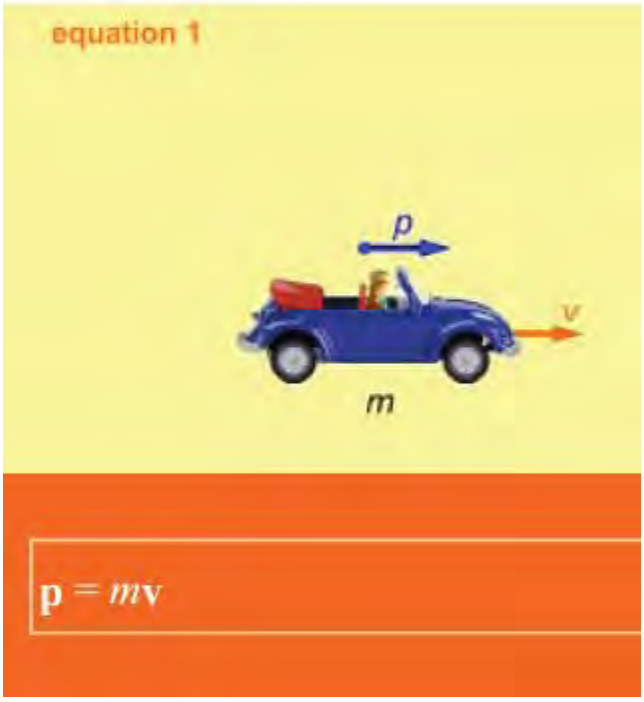


Momentum *Momentum (linear):* Mass times velocity.

An object’s linear momentum equals the product of its mass and its velocity. A fast moving locomotive has greater momentum than a slowly moving ping-pong ball. The units for momentum are kilogram·meters/second (kg·m/s). A ping-pong ball with a mass of 2.5 grams moving at 1.0 m/s has a momentum of 0.0025 kg·m/s. A 100,000 kg locomotive moving at 5 m/s has a momentum of 5×105 kg·m/s. Momentum is a vector quantity. The momentum vector points in the same direction as the velocity vector. This means that if two identical locomotives are moving at the same speed and one is heading east and the other west, they will have equal but **opposite** momenta, since they have equal but oppositely directed velocities.

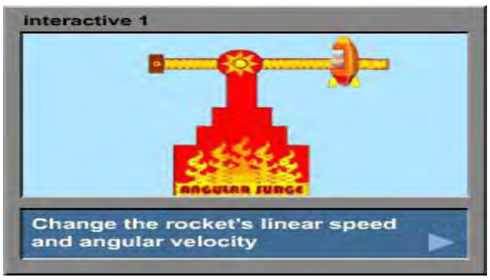






If you feel as though you spend your life spinning around in circles, you may be pleased to know that an entire branch of physics is dedicated to studying that kind of motion. This chapter is for you! More seriously, this chapter discusses motion that consists of rotation about a fixed axis. This is called *pure rotational motion*. There are many examples of pure rotational motion: a spinning Ferris wheel, a roulette wheel, or a music CD are three instances of this type of motion. In this chapter, you will learn about rotational displacement, rotational velocity, and rotational acceleration: the fundamental elements of what is called *rotational kinematics*. You will also learn how to relate these quantities using equations quite similar to those used in the study of linear motion. The simulation on the right features the “Angular Surge,” an amusement park ride you will be asked to operate in order to gain insight into rotational kinematics. The ride has a rotating arm with a “rocket” where passengers sit. You can move the rocket closer to or farther from the center by setting the distance in the simulation. You can also change the rocket’s period, which is the amount of time it takes to complete one revolution. By changing these parameters, you affect two values you see displayed in gauges: the rocket’s angular velocity and its linear speed. The rocket’s angular velocity is the change per second in the angle of the ride’s arm, measured from its initial position. Its units are radians per second. For instance, if the rocket completes one revolution in one second, its angular velocity is 2π radians (360°) per second. This simulation has no specific goal for you to achieve, although you may notice that you can definitely have an impact on the passengers!

What you should observe is this: How do changes in the period affect the angular velocity? The linear speed? And how does a change in the distance from the center (the radius of the rocket’s motion) affect those values, if at all? Can you determine how to maximize the linear speed of the rocket? To run the ride, you start the simulation, set the values mentioned above, and press GO. You can change the settings while the ride is in motion.



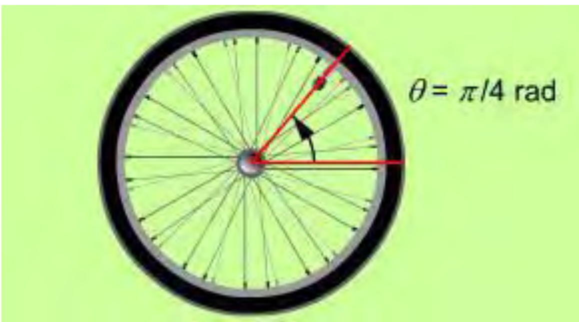
Angular position *Angular position:* The amount of rotation from a reference position, described with a positive or negative angle.

When an object such as a bicycle wheel rotates about its axis, it is useful to describe this motion using the concept of angular position. Instead of being specified with a linear coordinate such as x, as linear position is, angular position is stated as an angle.

In Concept 1, we use the location of a bicycle wheel's valve to illustrate angular position. The valve starts at the 3 o'clock position (on the positive x axis), which is zero radians by convention. As the illustration shows, the wheel has rotated one-eighth of a turn, or □/4 radians (45°), in a counterclockwise direction away from the reference position. In other words, angular position is measured from the positive x axis. Note that this description of the wheel's position used radians, not degrees; this is because radians are typically used to describe angular position. The two lines we use to measure the angle radiate from the point about which the wheel rotates.

The *axis of rotation* is a line also used to describe an object's rotation. It passes through the wheel's center, since the wheel rotates about that point, and it is perpendicular to the wheel. The axis is assumed to be stationary, and the wheel is assumed to be rigid and to maintain a constant shape. Analyzing an object that changes shape as it rotates, such as a piece of soft clay, is beyond the scope of this textbook. We are concerned with the wheel's rotational motion here: its motion around a fixed axis. Its linear motion when moving along the ground is another topic.

As mentioned, angular position is typically measured with *radians* (rad) instead of degrees. The formula that defines the radian measure of an angle is shown in Equation 1. The angle in radians equals the arc length s divided by the radius r. As you may recall, 2□ radians equals one revolution around a circle, or 360°. One radian equals about 57.3°. To convert radians to degrees, multiply by the conversion factor 360°/2□. To convert degrees to radians, multiply by the reciprocal: 2□/360°. The Greek letter □

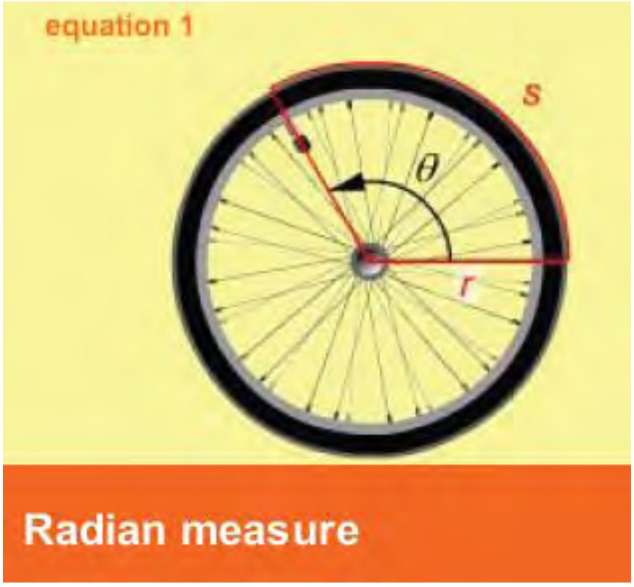


Angular position

Rotation from 3 o'clock position

- Counterclockwise rotation: positive
- Clockwise rotation: negative

Units are radians

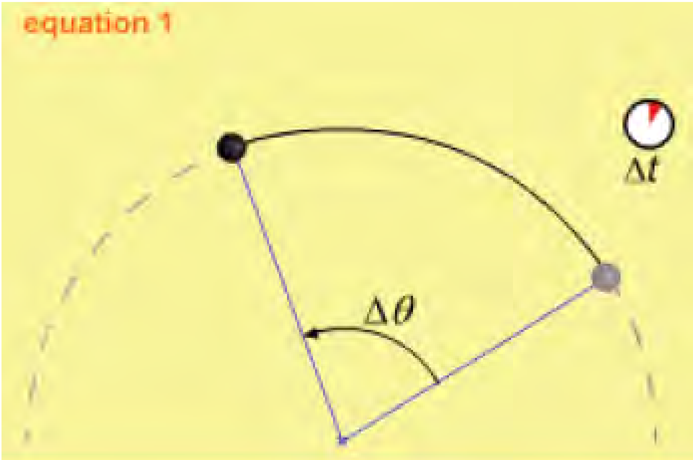


Angular velocity: Angular displacement per unit time.

In Concept 1, a ball attached to a string is shown moving counterclockwise around a circle. Every four seconds, it completes one revolution of the circle. Its angular velocity is the angular displacement 2π radians (one revolution) divided by four seconds, or $\pi/2$ rad/s. The Greek letter ω (omega) represents angular velocity. As is the case with linear velocity, angular velocity can be discussed in terms of average and instantaneous velocity. *Average angular velocity* equals the total angular displacement divided by the elapsed time. This is shown in the first equation in Equation 1. *Instantaneous angular velocity* refers to the angular velocity at a precise moment in time. It equals the limit of the average velocity as the increment of time approaches zero. This is shown in the second equation in Equation 1. The sign of angular velocity follows that of angular displacement: positive for counterclockwise rotation and negative for clockwise rotation. The magnitude (absolute value) of angular velocity is *angular speed*.

Angular velocity

Angular displacement per unit time



Angular velocity

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

$\bar{\omega}$ = average angular velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$$

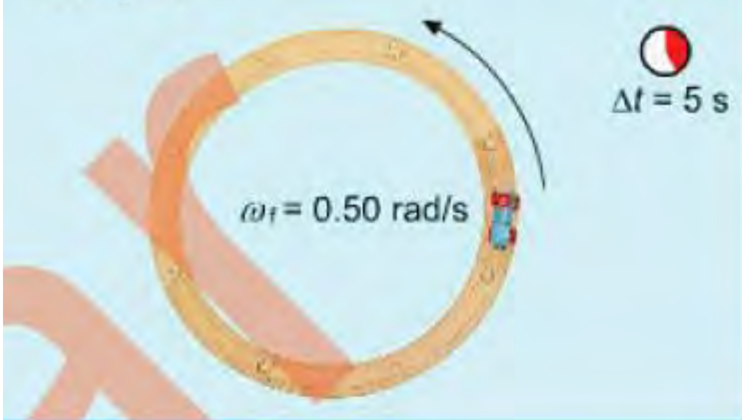
α = instantaneous angular acceleration

ω = angular velocity

Δt = elapsed time

Units: rad/s^2

example 1



The toy train starts from rest and reaches the angular velocity shown in 5.0 seconds. What is its average angular acceleration?

$$\bar{\alpha} = \frac{\Delta \omega}{\Delta t}$$

$$\bar{\alpha} = \frac{0.50 \text{ rad/s} - 0.00 \text{ rad/s}}{5.0 \text{ s}}$$

$$\bar{\alpha} = 0.10 \text{ rad/s}^2$$

Over the course of 1.00 hour, what is (a) the angular displacement, (b) the angular velocity and (c) the angular acceleration of the minute hand?

Think about the movement of the minute hand over the course of an hour. Be sure to consider the direction!

Variables

elapsed time	$\Delta t = 1.00$ h
angular displacement	$\Delta\theta$
angular velocity	ω
angular acceleration	α

What is the strategy?

1. Calculate the angular displacement.

2. Convert the elapsed time to seconds.

3. Use the angular displacement and time to determine the angular velocity and angular acceleration.

Physics principles and equations

Definition of angular velocity

for angular displacement, angular velocity and angular acceleration instead of linear displacement, velocity and acceleration. As with the linear motion equations, these equations hold true when there is constant acceleration. We also show these equations below along with their linear counterparts. To apply the equations in physics problems, the first step is to identify the known values and which values are being asked for. Sketching a diagram of the situation may help you with this. The next step is to find an equation that includes both the known and the unknown (asked-for) values. Your goal is to find an equation, if possible, that has only one unknown value: the one you want to find. When applying the rotational equations, remember that positive displacement and velocity represent counterclockwise motion, and negative displacement and velocity indicate clockwise motion. Let's now work an example problem. Imagine you have just turned on the blender shown on the right. You let it run for 5.0 seconds. During this time period its blade has a constant angular acceleration of 44 radians per second squared. What is the angular displacement of the blade during this time? This problem implicitly tells you that the initial angular velocity is zero, since the blender has just been turned on. The second equation above includes time, initial angular velocity and acceleration. It also contains the value you seek to calculate: the angular displacement. This makes it the right equation to use. It does not include the value for final angular velocity, which is fine because you are not told that value, nor are you asked to calculate it. The details of the calculation appear on the right. The angular displacement is 550 radians. Because the value is positive, the motion is counterclockwise. Here is a table of the rotational motion variables and the equations that relate them, along with their linear counterparts.

The blender is turned on and runs for 5.0 seconds with a constant angular acceleration of 44 rad/s². What is the angular displacement of a blade?

$$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$$

$\omega_i = 0$ rad/s

$\Delta\theta = 0 \text{ rad} + \tfrac{1}{2}(44 \text{ rad/s}^2)(5.0 \text{ s})^2$

$\Delta\theta = 0 + (22)(25) \text{ rad}$

$\Delta\theta = 550 \text{ rad}$

	linear	**rotational**
position	x	θ
displacement	Δx	$\Delta\theta$
velocity	$v = \Delta x/\Delta t$	$\omega = \Delta\theta/\Delta t$
acceleration	$a = \Delta v/\Delta t$	$\alpha = \Delta\omega/\Delta t$
	$v_f = v_i + at$	$\omega_f = \omega_i + \alpha t$
	$\Delta x = v_i t + \tfrac{1}{2}at^2$	$\Delta\theta = \omega_i t + \tfrac{1}{2}\alpha t^2$
	$v_f^2 = v_i^2 + 2a\Delta x$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$
	$\Delta x = \tfrac{1}{2}(v_i + v_f)t$	$\Delta\theta = \tfrac{1}{2}(\omega_i + \omega_f)t$

The carousel accelerates from rest for two revolutions at a constant angular acceleration of 0.11 rad/s^2. What is its final angular velocity?

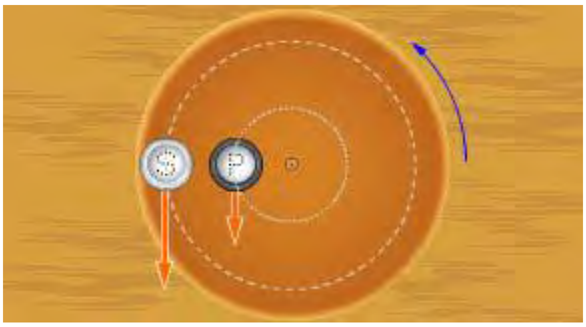
Variables

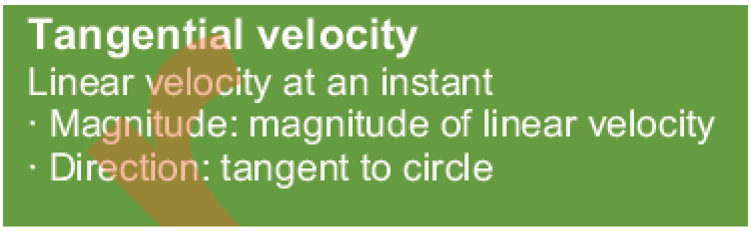
Since the carousel is starting up, the initial angular velocity must be zero. The angular displacement is given in revolutions, which must be converted to radians.

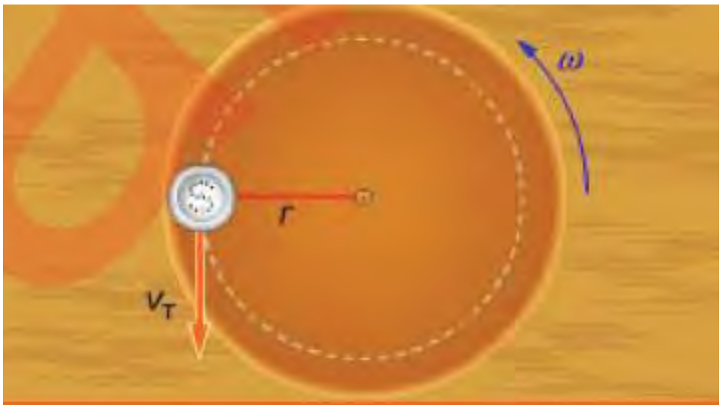
initial angular velocity	$\omega_i = 0$ rad/s
final angular velocity	ω_f
angular acceleration	$\alpha = 0.11$ rad/s^2
angular displacement	$\Delta\theta = (2.0)(2\pi \text{ rad})$

Tangential velocity: The instantaneous linear velocity of a point on a rotating object.

Concepts such as angular displacement and angular velocity are useful tools for analyzing rotational motion. However, they do not provide the complete picture. Consider the salt and pepper shakers rotating on the lazy Susan shown to the right. The containers have the same angular velocity because they are on the same rotating surface and complete a revolution in the same amount of time. However, at any instant, they have different **linear** speeds and velocities. Why? They are located at different distances from the axis of rotation (the center of the lazy Susan), which means they move along circular paths with different radii. The circular path of the outer shaker is longer, so it moves farther than the inner one in the same amount of time. At any instant, its linear speed is greater. Because the direction of motion of an object moving in a circle is always tangent to the circle, the object's linear velocity is called its tangential velocity. To reinforce the distinction between linear and angular velocity, consider what happens if you decide to run around a track. Let's say you are asked to run one lap around a circular track in one minute flat. Your angular velocity is 2π radians per minute. Could you do this if the track had a radius of 10 meters? The answer is yes. The circumference of that track is $2\pi r$, which equals approximately 63 meters. Your pace would be that distance divided by 60 seconds, which works out to an easy stroll of about 1.05 m/s (3.78 km/h). What if the track had a radius of 100 meters? In this case, the one-minute accomplishment would require the speed of a world-class sprinter capable of averaging more than 10 m/s. (If the math ran right past you, note that we are again multiplying the radius by 2□ to calculate the circumference and dividing by 60 seconds to calculate the tangential velocity.) Even though the angular velocity is the same in both cases, 2□ radians per minute, the tangential speed changes with the radius. As you see in Equation 1, tangential speed equals the product of the distance to the axis of rotation, *r*, and the angular velocity, □. The units for tangential velocity are meters per second. The direction of the velocity is always tangent to the path of the object. Confirming the direction of tangential velocity can be accomplished using an easy home experiment. Let's say you put a dish on a lazy Susan and then spin the lazy Susan faster and faster. Initially, the dish moves in a circle, constrained by static friction. At some point, though, it will fly off. The dish will always depart in a straight line, tangent to the circle at its point of departure. The tangential speed equation can also be used to restate the equation for centripetal acceleration in terms of angular velocity. Centripetal acceleration equals *v*2/*r*. Since *v* = *r*□, centripetal acceleration also equals □2*r*. We derive the equation for tangential speed using the diagram below. To understand the derivation, you must recall that the arc length □*s* (the distance along the circular path) equals the angular displacement □□ in radians times the radius *r*. Also recall that the instantaneous speed *v*T equals the displacement divided by the elapsed time for a very small increment of time.





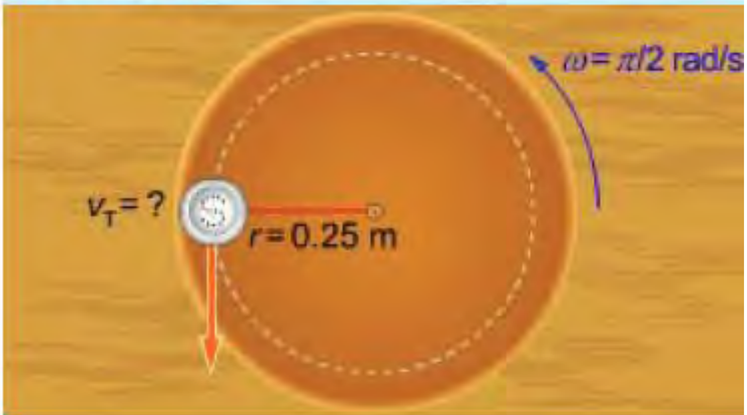


Tangential velocity

$$v_T = r\omega$$

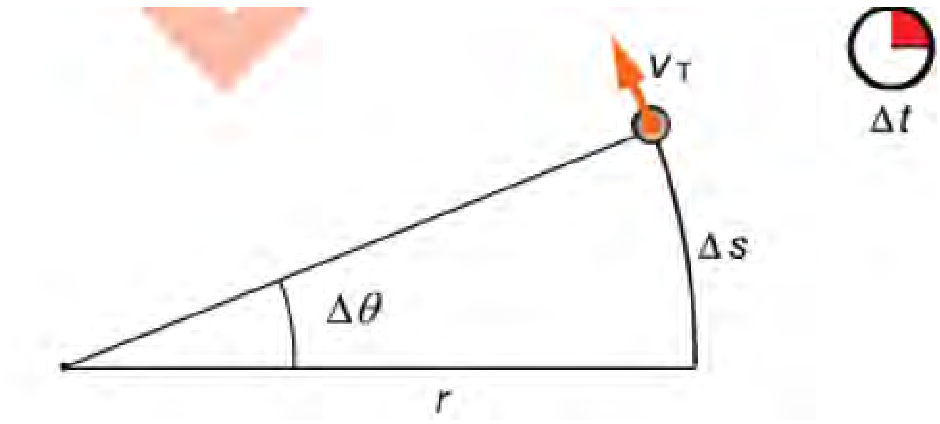
v_T = tangential speed
r = distance to axis
ω = angular velocity
Direction: tangent to circle

example 1

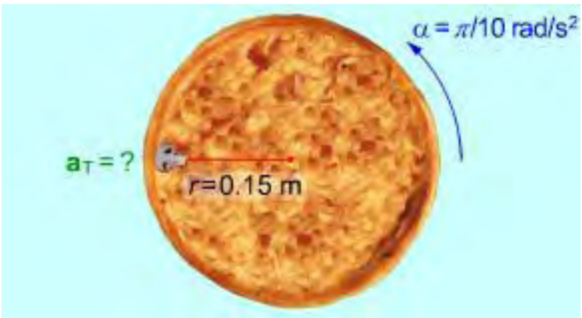


At the instant shown, what is the salt shaker's tangential velocity?

$v_T = r\omega$
$v_T = (0.25\text{ m})(\pi/2\text{ rad/s})$
$\mathbf{v}_T = 0.39$ m/s, pointing down



always toward the center of the circle. Now imagine that the car speeds up as it circles the track. It now completes a lap more quickly, so its angular velocity is increasing, which means it has positive angular acceleration (when it is moving counterclockwise; it is negative in the other direction). The car now has tangential acceleration (its linear speed is changing), and this can be calculated by multiplying its angular acceleration by the track's radius. The equation for tangential acceleration is derived below from the equations for tangential velocity and angular acceleration. We begin with the basic definition of linear acceleration and substitute the tangential velocity equation. The result is an expression which contains the definition of angular acceleration. We replace this expression with □, angular acceleration, which yields the equation we desire.



What is the tangential acceleration of the mushroom slice at this instant?

$a_T = r\alpha$

$a_T = (\pi/10 \text{ rad/s}^2)(0.15 \text{ m})$

$\mathbf{a}_T = 0.047 \text{ m/s}^2$, pointing down

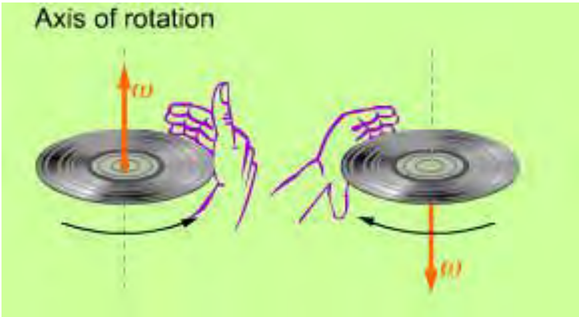
Step	Reason
1. $a_T = \lim_{\Delta t \to 0} \frac{\Delta v_T}{\Delta t}$	definition of linear acceleration
2. $\Delta v_T = r\Delta\omega$	tangential velocity equation
3. $a_T = \lim_{\Delta t \to 0} \frac{r\Delta\omega}{\Delta t} = r\left(\lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t}\right)$	substitute equation 2 into equation 1
4. $a_T = r\alpha$	definition of angular acceleration

Tangential and centripetal acceleration

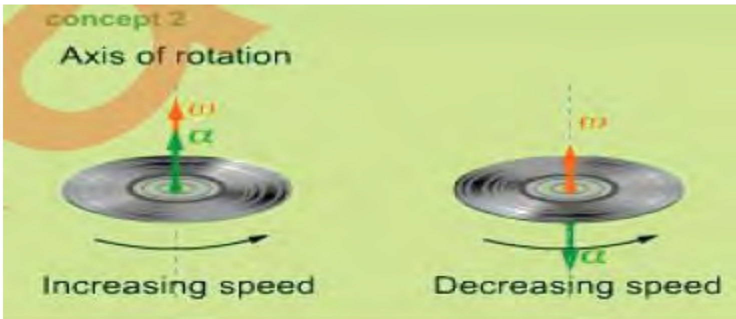
In Concept 1, a toy train is shown going around a circular track at steadily increasing speed. How can we calculate its overall acceleration at any moment? The train has both centripetal and tangential acceleration. The overall acceleration can be broken into these two components. The acceleration perpendicular to the direction of motion, directed toward the center of the circle, is the centripetal acceleration. Its magnitude at any instant is calculated using the equation for centripetal acceleration from a previous chapter: speed squared divided by the radius. The acceleration parallel to the velocity vector is the tangential acceleration, which is perpendicular to the centripetal acceleration. Since the train is increasing in speed, it has non-zero tangential acceleration. (This is not uniform circular motion.) The overall acceleration equals the vector sum of the centripetal and tangential accelerations. The two vectors are perpendicular, so they form two legs of a right triangle. The Pythagorean theorem can be used to calculate the magnitude of the overall acceleration, as the first formula in Equation 1 shows. The direction of the overall acceleration, measured from the centripetal acceleration vector (or the radius line), can be calculated using trigonometry. You see that formula in Equation 1 as well.

Vectors and angular motion

Although we have not stressed this fact, angular velocity and angular acceleration are both vectors. In this section, we discuss the direction in which they point, using the *right-hand rule* to determine their direction. To apply this rule to angular velocity, curl your right hand around the axis of rotation, wrapping your fingers in the direction of the motion. This is illustrated to the right, where the hand wraps around the axis that passes through the center of the record. Your thumb then points in the direction of the angular velocity vector, which lies along the axis of rotation. The direction of the angular acceleration vector depends on whether the object in question is speeding up or slowing down. When an object speeds up, the angular acceleration vector points in the same direction as the angular velocity vector, reflecting the change in the velocity vector. When an object slows down, the angular acceleration vector points in the direction **opposite** to the angular velocity vector, again reflecting the change in the angular velocity vector. You may have noticed that we have not mentioned angular displacement. This is because it is not treated as a vector.



Angular velocity vector
Along axis of rotation
Magnitude proportional to angular speed
Direction determined by right-hand rule



Angular acceleration vector
Speeding up: same direction as angular velocity vector
Slowing down: opposite direction

example 1

The motorcycle rider speeds up as she starts her ride. What is the direction of the angular velocity vector? The angular acceleration vector?

Angular velocity vector: up

Angular acceleration vector: up

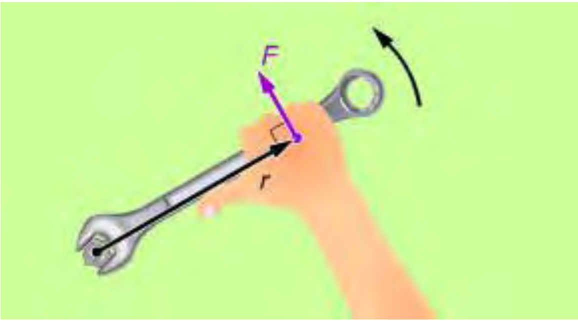
Interactive summary problem: 11.6 seconds to liftoff

You are again operating the Angular Surge ride at a local amusement park. The ride begins with the arm in the launch position for the rocket. The motor starts the ride by providing a constant positive angular acceleration for the first 11.6 seconds. The ride has a rocket on a rotating arm, and you can control the arm's angular acceleration. You can also control the distance of the rocket from the axis of rotation. Your goal is to set both these values so that 11.6 seconds after startup, the rocket has completed one or more complete revolutions **and** has a tangential velocity of 13.0 m/s. If you do this correctly, the rocket will blast off. You can position the rocket from four to 10 meters from the Torque

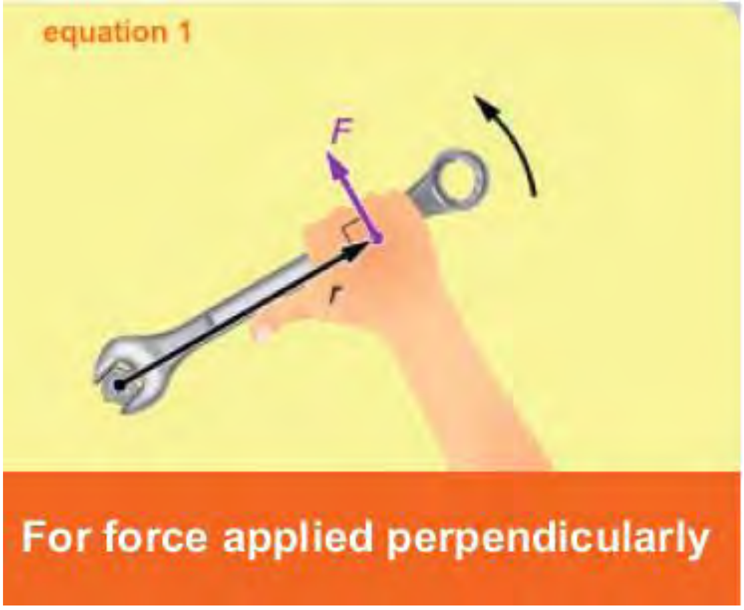
Torque: A force that causes or opposes rotation.

A net force causes linear acceleration: a change in the linear velocity of an object. A net torque causes angular acceleration: a change in the angular velocity. For instance, if you push hard on a wrench like the one shown in Concept 1, you will start it and the nut rotating. We will use a wrench that is loosening a nut as our setting to explain the concept of torque in more detail. In this section, we discuss two of the factors that determine the amount of torque. One factor is how much force *F* is exerted and the other is the distance *r* between the axis of rotation and the location where the force is applied. We assume in this section that the force is applied perpendicularly to the line from the axis of rotation and the location where the force is applied. (If this description seems cryptic, look at Concept 1, where the force is being applied in this manner.) When the force is applied as stated above, the torque equals the product of the force *F* and the distance *r*. In Equation 1, we state this as an equation. The Greek letter □ (tau) represents torque. Your practical experience should confirm that the torque increases with the amount of force and the distance from the axis of rotation. If you are trying to remove a “frozen” nut, you either push harder or you get a longer wrench so you can apply the force at a greater distance.

The location of a doorknob is another classic example of factoring in where force is applied. A torque is required to start a door rotating. The doorknob is placed far from the axis of rotation at the hinges so that the force applied to opening the door results in as much torque as possible. If you doubt this, try opening a door by pushing near its hinges. The wrench and nut scenario demonstrates another aspect of torque. The angular acceleration of the nut is due to a **net** torque. Let's say the nut in Concept 1 is stuck: the force of static friction between it and the bolt creates a torque that opposes the torque caused by the force of the hand. If the hand pushes hard enough and at a great enough distance from the nut, the torque it causes will exceed that caused by the force of static



Torque
Causes or opposes rotation
Increases with:
· amount of force
· distance from axis to point of force



product in Equation 1.

Children are sophisticated about torques, whether they know it or not. They understand that torques can be added. For example, if two children sit on the same side of a seesaw, their torques combine to create a larger net torque than that supplied by one child alone. If they sit on opposite sides, the net torque is less than either child's torque alone. Children also learn that they can adjust the amount of torque they apply by moving toward or away from the axis of rotation. This means two children with different weights can balance each other, since both torques are a function of their weights and their distances from the axis of rotation. The heavier child slides closer to the axis, and the net torque is zero.

r = position vector
F = force
θ = angle between **r** and **F**
Units: newton-meters (N·m)

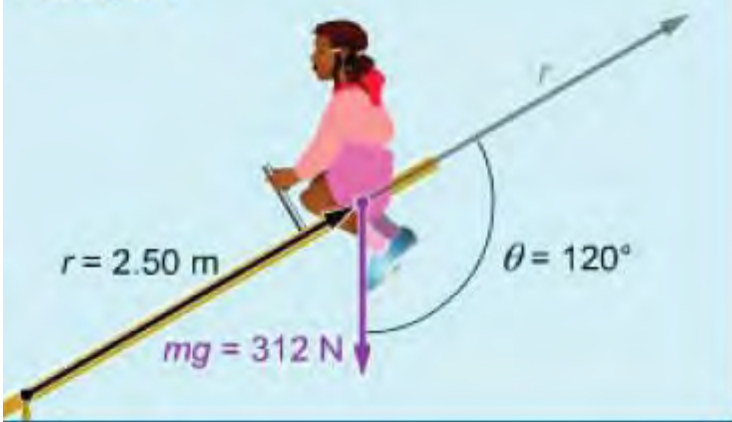
equation 2

Calculating torque using lever arm

$$\tau = F\,(r \sin \theta)$$

$r \sin \theta$ = lever arm

example 1



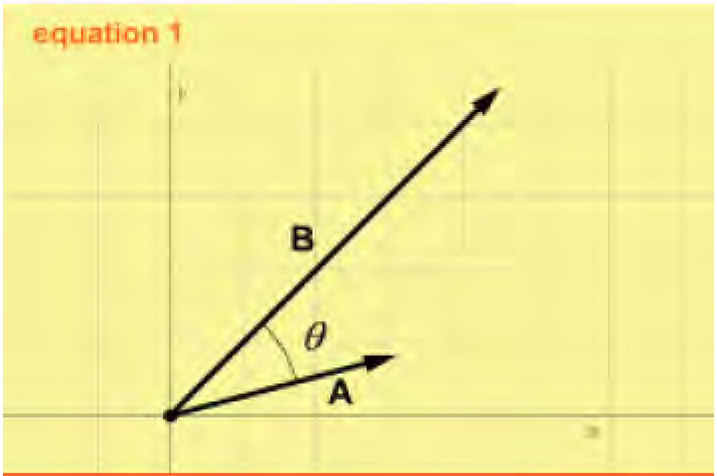
What is the torque exerted by the girl?

$\tau = rF \sin \theta$

$\tau = (2.50\text{ m})(312\text{ N})(\sin 120°)$

$\tau = -675\text{ N·m}$ (clockwise)

Cross product: A vector whose magnitude equals the product of the magnitudes of two vectors and the sine of the smaller angle between them. Its direction is determined by the right-hand rule.



Magnitude of A × B

$$AB \sin\theta$$

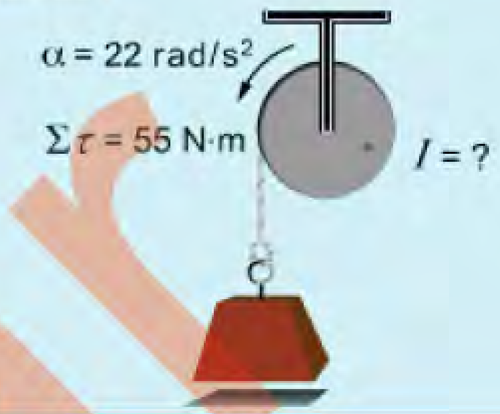
A = magnitude of vector **A**
B = magnitude of vector **B**
θ = smaller angle between **A** and **B**

Several physics properties, including torque, are calculated using the cross product. The cross product is a way to multiply two vectors. The result is a vector that is sometimes called their *vector product*. To determine the magnitude of the cross product, multiply the product of the magnitudes of the two vectors by the sine of the angle between them. This formula is shown on the right. As the diagram shows, placing vectors tail-to-tail will allow you to determine the correct angle. The angle used is the smaller angle between the two vectors. A technique called the *right-hand rule* will help you determine the direction of the vector that results from the cross product. (Right-hand rules are also frequently used in the study of electricity and magnetism.) How to apply the right-hand rule is shown on the right (you and your classmates may

$$\Sigma\tau = I\alpha$$

$\Sigma\tau$ = net torque
I = moment of inertia
α = angular acceleration
Units for I: kg·m^2

example 1



What is the moment of inertia of the pulley?

$\Sigma\tau = I\alpha$
$I = \Sigma\tau/\alpha$
$I = (55\ \text{N·m})/(22\ \text{rad/s}^2)$
$I = 2.5\ \text{kg·m}^2$

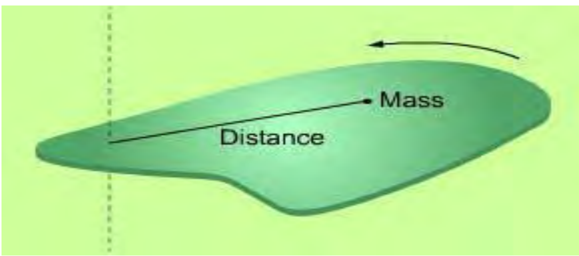
Calculating the moment of inertia

If you were asked whether the same amount of torque would cause a greater angular acceleration with a Ferris wheel or a bicycle wheel, you would likely answer: the bicycle wheel. The greater mass of the Ferris wheel means it has a greater moment of inertia. It accelerates less with a given torque. But more than the amount of mass is required to determine the moment of inertia; the distribution of the mass also matters.

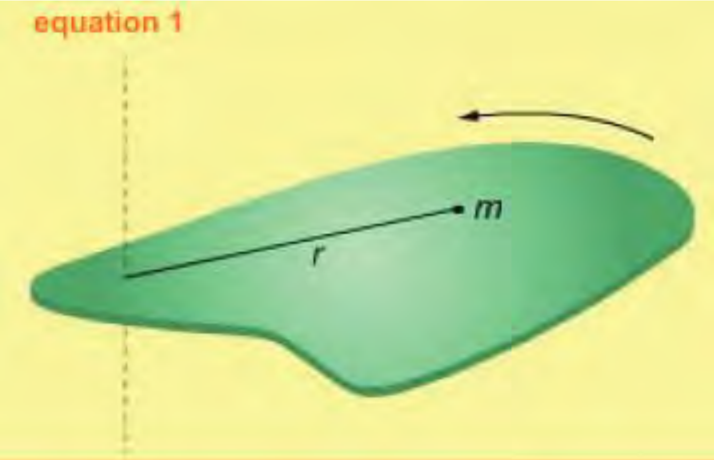
Consider the case of a boy sitting on a seesaw. When he sits close to the axis of rotation, it takes a certain amount of torque to cause him to have a given rate of angular acceleration. When he sits farther away, it takes more torque to create the same rate of acceleration. Even though the boy's (and the seesaw's) mass stays constant, he can increase the system's moment of inertia by sitting farther away from the axis. When a rigid object or system of particles rotates about a fixed axis, each particle in the object contributes to its moment of inertia. The formula in Equation 1 to the right shows how to calculate the moment of inertia. The moment equals the sum of each particle's mass times the square of its distance from the axis of rotation.

A single object often has a different moment of inertia when its axis of rotation changes. For instance, if you rotate a baton around its center, it has a smaller moment of inertia than if you rotate it around one of its ends. The baton is harder to accelerate when rotated around an end. Why is this the case? When the baton rotates around an end, more of its mass on average is farther away from the axis of rotation than when it rotates around its center. If the mass of a system is concentrated at a few points, we can calculate its moment of inertia using multiplication and addition. You see this in Example 1, where the mass of the object is concentrated in two balls at the ends of the rod. The moment of inertia of the rod is very small compared to that of the balls, and we do not include it in our calculations. We also consider each ball to be concentrated at its own center of mass when measuring its distance from the axis of rotation (marked by the ×). This is a reasonable approximation when the size of an object is small relative to its distance from the axis.

Not all situations lend themselves to such simplifications. For instance, let's assume we want to calculate the moment of inertia of a CD spinning about its center. In this case the mass is uniformly distributed across the entire CD. In such a case, we need to use calculus to sum up the contribution that each particle of mass makes to the moment, or we must take advantage of a table that tells us the moment of inertia for a disk rotating



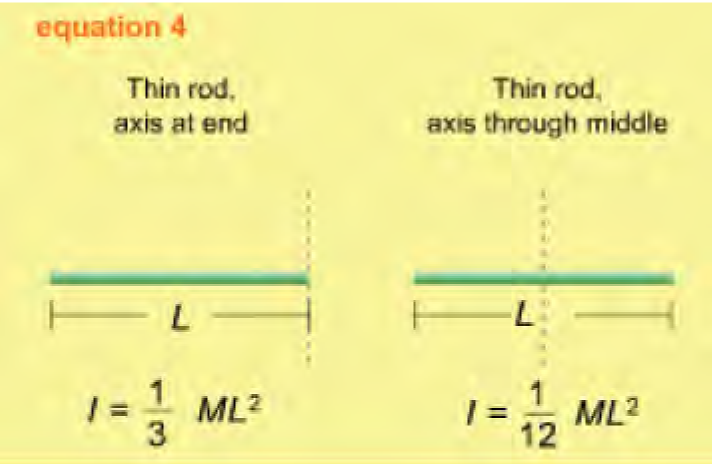
Moment of inertia
Sum of each particle's
· Mass times its
· Distance squared from the axis



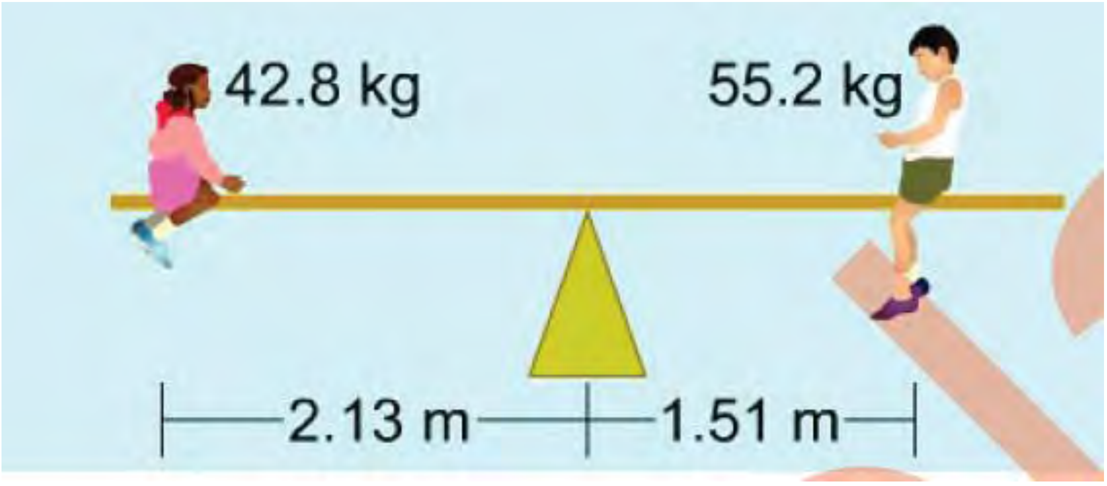
Moment of inertia

$$I = \Sigma mr^2$$

I = moment of inertia



Thin rods



The seesaw plank is horizontal. Its mass is 36.5 kg, and it is 4.40 m long. What is the initial angular acceleration of this system?

The axis of rotation is the point where the fulcrum touches the midpoint of the plank. The plank itself creates no net torque since it is balanced at its middle. For every particle at a given distance from the axis that creates a clockwise torque, there is a matching particle at the same distance creating a counterclockwise torque. However, the plank does factor into the moment of inertia.

Draw a diagram

Variables

mass of seesaw plank	$m_S = 36.5$ kg
seesaw plank's moment of inertia	I_S

	girl	boy
mass	$m_G = 42.8$ kg	$m_B = 55.2$ kg
distance from axis	$r_G = 2.13$ m	$r_B = 1.51$ m
moment of inertia	I_G	I_B

What is the strategy?

1. Calculate the moment of inertia of the system: the sum of the moments for the children, and the moment of the plank.

2. Calculate the net torque by summing the torques created by each child. The torques of the left and right sides of the plank cancel, so you do not have to consider them.

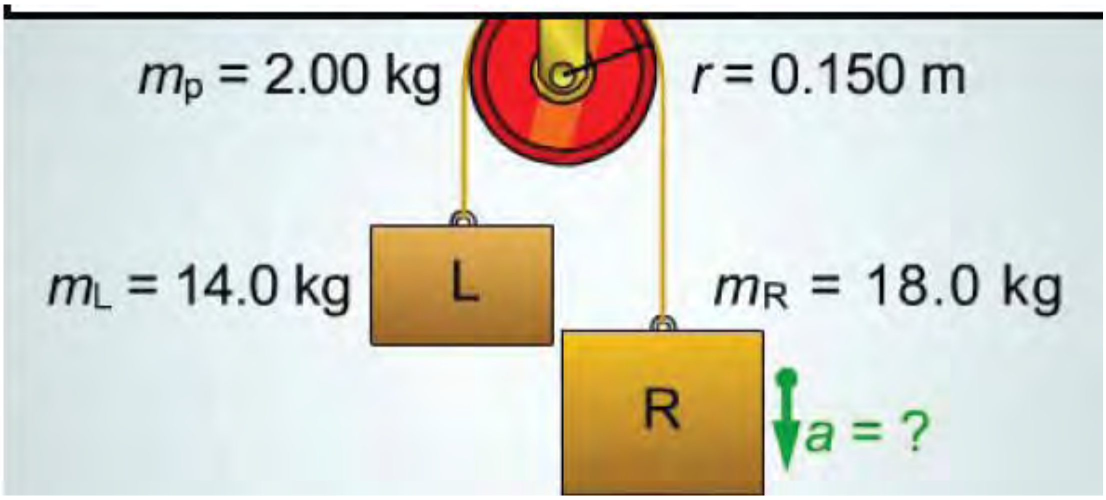
3. Divide the net torque by the moment of inertia to determine the initial angular acceleration.

Physics principles and equations

We will use the definitions of torque and moment of inertia.

□ = *rF* sin □, *I* = □*mr*2

To calculate the moments of inertia of the children, we consider the mass of each to be concentrated at one point. The plank can be considered as a slab rotating on an axis parallel to an edge through the center, with moment of inertia



What is the magnitude of the acceleration of the block on the right?

Here we consider an Atwood machine, factoring in the moment of inertia of the pulley. We will model the pulley as a uniform solid disk. We still assume that the rope is massless and does not stretch and that the pulley is frictionless. The blocks' accelerations are equal and opposite, but the tension exerted on each block by the rope is different because of the pulley's moment of inertia. We will use the convention that downward acceleration and force are negative, and upward acceleration and force are positive. We know the block on the right will accelerate downward because it is more massive than the block on the left.

Variables

	Block L	Block R
tension	T_L	T_R
mass	$m_L = 14.0$ kg	$m_R = 18.0$ kg
weight	$-m_L g$	$-m_R g$
acceleration	a	$-a$

	Pulley
torque from left block	τ_L
torque from right block	τ_R
mass	$m_P = 2.00$ kg
radius	$r = 0.150$ m
moment of inertia	I
angular acceleration	α

What is the strategy?

1. Calculate the net force on each block. The forces are strictly vertical. Use Newton's second law for each block to find expressions for the tension force in the rope on each side of the pulley.
2. Calculate the net torque acting on the pulley. The forces that create torques on the pulley are the tensions of the rope. The tension forces are perpendicular to the radius of the pulley at the points where they act. Use Newton's second law for rotation to set $I\alpha$ equal to this net torque.
3. Use the definition of angular acceleration to introduce the linear acceleration of the masses into the previously derived equation, and solve for the linear acceleration.
4. Use the expression for the moment of inertia of a solid disk as the moment of inertia of the pulley, and use the values given in the problem to compute the acceleration.

Physics principles and equations

Newton's second law

$$\Sigma F = ma$$

Equations for torque

$$\tau = rF \sin \theta$$

$$\Sigma\tau = I\alpha$$

Moment of inertia of a solid disk

$$I = \tfrac{1}{2}mr^2$$

Angular acceleration

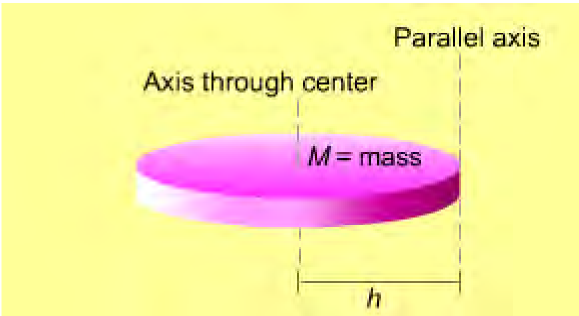
$$\alpha = a/r$$

The acceleration a is the positive magnitude of the acceleration of the blocks. Since the right block falls, this acceleration results in a clockwise

Parallel axis theorem

The parallel axis theorem is a tool for calculating the moment of inertia of an object. You use it when you know the moment of inertia for an object rotating about an axis that passes through its center of mass and want to know the moment when it rotates around a different but parallel axis of rotation.

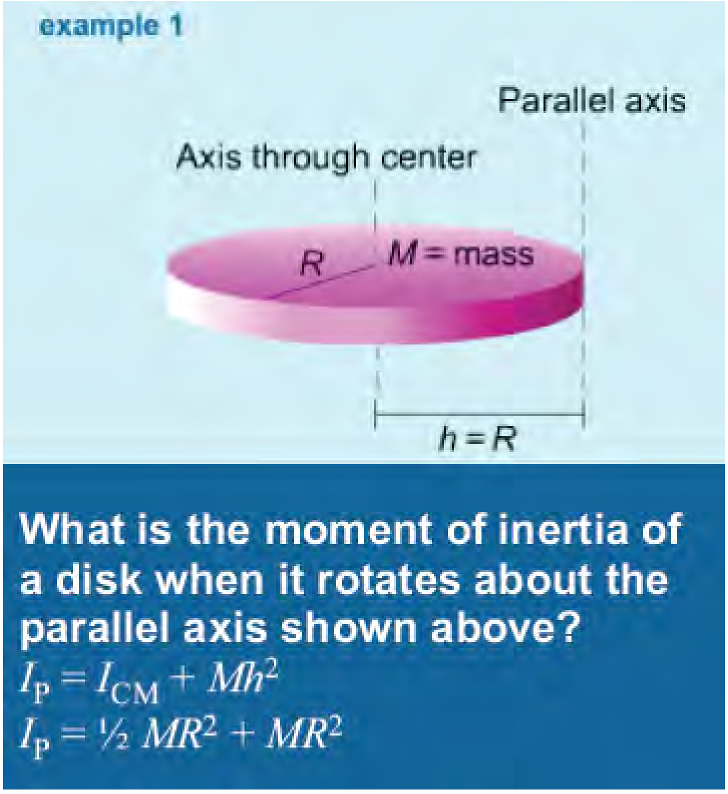
The illustration for Equation 1 shows two such parallel axes. The axis on the left passes through the center of mass of a cylindrical disk, the other is at the edge of the disk. The theorem states that the moment of inertia when the disk rotates about the axis on its edge will be the sum of two values: the moment of inertia when the disk rotates about its center of mass, and the product of the disk's mass and the square of the distance between the two axes (shown as h in our diagram). This is stated as an equation to the right. The usefulness of the parallel axis theorem lies in this fact: It is usually much easier to calculate the moment of inertia of an object around an axis through its center of mass than around an off-center axis. For example, if we must use an integral to calculate the moment of inertia, doing so around the center of mass lets us more readily take advantage of any symmetry of the object. The parallel axis theorem can then be used to find the moment of inertia around another parallel axis. In sum, the parallel axis theorem lets us use an easier integral and some algebra to calculate the moment for the parallel axis. The disk to the right has a moment of inertia of ½*MR*2 when it rotates about its center. We can use this formula as the starting point in our calculation of the disk's moment of inertia when it is rotated about an axis at its edge. You see this computation worked out



Parallel axis theorem

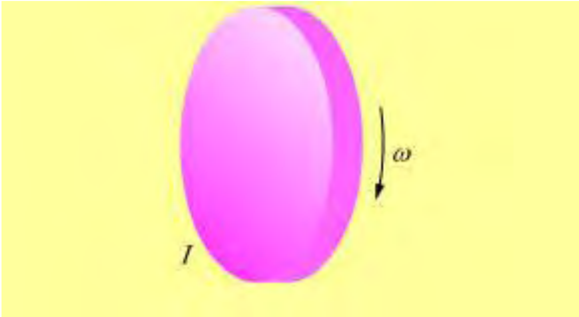
$$I_P = I_{CM} + Mh^2$$

I_P = moment of inertia, parallel axis
I_{CM} = moment, center of mass axis
M = mass
h = distance between axes



Rotational kinetic energy

The equation on the right enables you to calculate the rotational kinetic energy (*KE*) of a rigid, rotating object, the kinetic energy of an object due to its rotational motion. It is analogous to the equation for linear kinetic energy. The rotational *KE* equals one-half the moment of inertia times the square of the angular velocity. This equation can be derived from the definition of linear kinetic energy. The rotating object consists of a large number of individual particles, each moving at a different linear (tangential) velocity. The kinetic energies of all the particles can be added to determine the kinetic energy of the entire object. To derive the equation to the right, the key insight is to see that the distance from the axis of rotation figures both in a particle's tangential velocity and in calculating its contribution to the disk's moment of inertia. In the derivation, we start with a particle of mass *m* situated somewhere in a rigid object, as shown in the second illustration to the right. We derive the equation by first calculating the kinetic energy of the single particle. The total *KE* is the sum of the kinetic energies of all the particles.



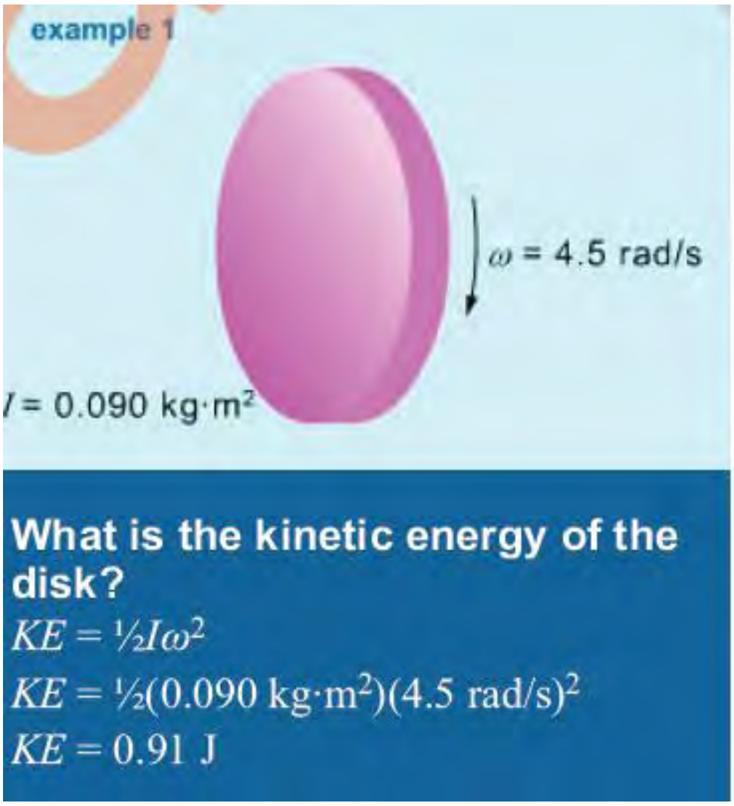
Rotational kinetic energy

$$KE = \tfrac{1}{2}I\omega^2$$

KE = rotational kinetic energy
I = moment of inertia
ω = angular velocity

Variables

mass of a particle on rotating object	m
linear speed of particle	v
distance of particle from axis of rotation	r
angular velocity of rotating object	ω
kinetic energy of particle	KE_p
kinetic energy of object	$KE = \Sigma KE_p$
moment of inertia of particle	I_p
moment of inertia of object	$I = \Sigma I_p$



Strategy

1. State the equation for the linear KE of a particle in terms of its speed. Restate the equation in rotational terms and simplify.
2. Rewrite the expression for the KE of a particle in terms of its moment of inertia.
3. Sum the individual kinetic energies of all the particles to derive the desired equation.

Physics principles and equations

The equation for the kinetic energy of a particle

$$KE = \tfrac{1}{2}mv^2$$

The relationship between linear speed and angular velocity is

$$v = \omega r$$

The moment of inertia of a single particle of mass m at distance r from the axis of rotation

$$I = mr^2$$

Step-by-step derivation

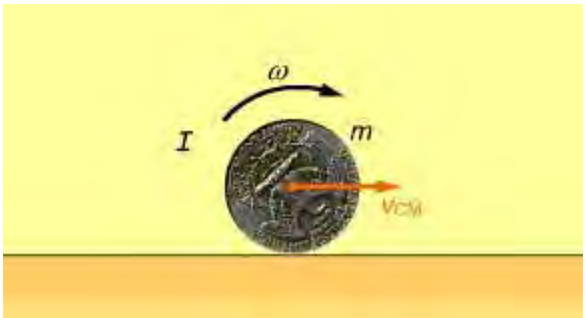
For a particle on a rotating object, its linear speed as it moves along a circular path is its tangential speed. We use the definition of kinetic energy and the relation between linear and angular speed to write the kinetic energy of a particle in rotational terms.

Step	Reason
1. $KE_p = ½mv^2$	definition of kinetic energy
2. $v = \omega r$	linear speed and angular velocity
3. $KE_p = ½m(\omega^2 r^2)$	substitute equation 2 into equation 1
4. $KE_p = ½(mr^2)\omega^2$	rearrange

Rolling objects and kinetic energy

The coin shown in Equation 1 rolls without slipping. That is, it rotates and moves linearly as it travels to the right. Its total kinetic energy is the sum of its linear and

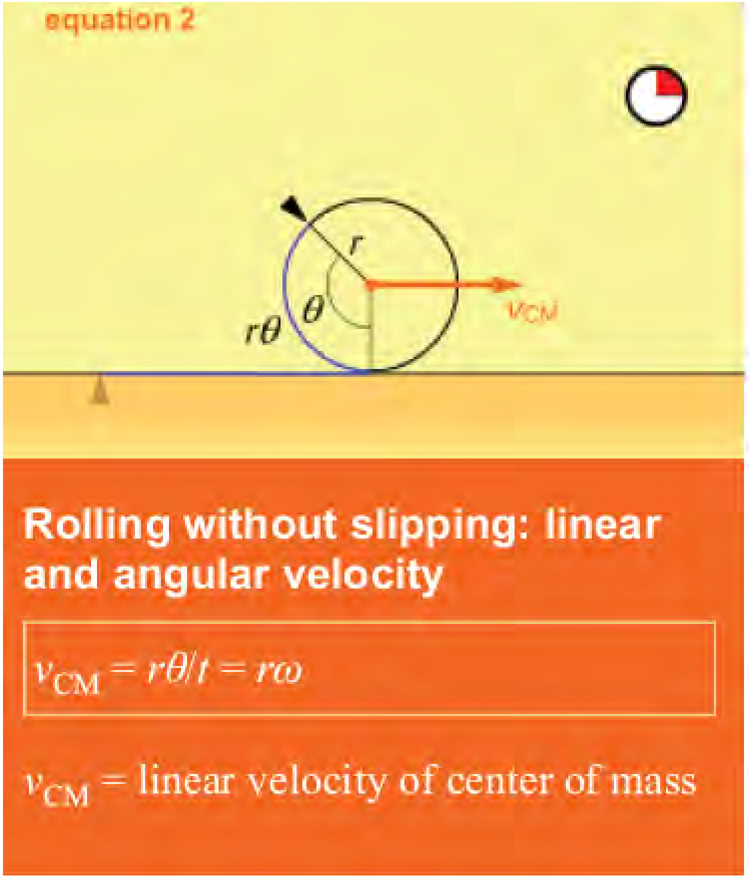
rotational kinetic energies. We can use two equations discussed earlier to determine the coin's total kinetic energy. The coin's rotational kinetic energy equals ½ *I*□2. We measure □ with respect to the coin’s axis of rotation, perpendicular to the center of the coin. Its linear kinetic energy equals ½ *mv*CM
2. The “CM” subscript indicates that the point used in calculating the linear speed is the coin's center of mass. As Equation 1 shows, the sum of these two types of kinetic energy equals the total kinetic energy. When an object rolls without slipping, it is often useful know the relationship between its linear and angular velocities. Equation 2 shows this relationship. As the rolling object with radius *r* rotates through an angle □, an arc of length *r*□ makes contact with the ground. This means the object moves linearly the same distance *r*□. Its linear speed *v*CM is that distance divided by *t*, or *r*□/*t*. Since □/*t* equals □, we can also say that *v*CM equals *r*□. The kinetic energy equation and the relationship in Equation 2 are both used to solve the example problem.



Kinetic energy of rolling object

$$KE = \tfrac{1}{2} m v_{CM}^2 + \tfrac{1}{2} I \omega^2$$

m = mass
v_{CM} = velocity of center of mass
I = moment of inertia
ω = angular velocity



Variables

The radius of the ball is not stated in the problem. It will cancel out and not be a factor in the answer.

initial (potential) energy	$PE = 0.75$ J
mass	$m = 0.10$ kg
radius	r
angular velocity	ω
speed of center of mass	v_{CM}
moment of inertia	I

What is the strategy?

1. Use the equation for the moment of inertia of a hollow sphere to write an expression for the rotational kinetic energy at any instant in terms of mass, radius and angular velocity.

2. Write an expression for the linear kinetic energy at any instant, also in terms of mass, radius and angular velocity.

3. Find the ratio of the two kinetic energies. This does not change as the ball moves.

4. The total energy at the top of the ramp is all potential energy. At the bottom of the ramp, all the energy is kinetic energy. Distribute the total energy between linear and rotational kinetic energy, according to the constant ratio you just calculated.

Physics principles and equations

We use the conservation of energy. In this case, all the energy at the top of the ramp is potential, and all the energy at the bottom is kinetic.

PE (top) = *KE*L + *KE*R (bottom)

The equations for linear and rotational kinetic energy

*KE*L = ½ *mv*CM 2, *KE*R = ½ *I*□2

The relationship between the linear velocity of the ball's center of mass and its angular velocity

*v*CM = *r*□

The moment of inertia of a hollow sphere

$$I = \frac{2}{3}mr^2$$

Step-by-step solution

We use the moment of inertia of a hollow sphere to find an expression for the rotational kinetic energy.

Step	Reason
1. $KE_R = ½ I\omega^2$	rotational kinetic energy
2. $I = 2mr^2/3$	moment of inertia
3. $KE_R = ½(2mr^2/3)\omega^2$	substitute equation 2 into equation 1
4. $KE_R = mr^2\omega^2/3$	simplify

Now we write an expression for the linear kinetic energy.

Step	Reason
5. $KE_L = ½\, mv_{CM}^2$	linear kinetic energy
6. $v_{CM} = r\omega$	rolling without slipping
7. $KE_L = ½\, m(r\omega)^2$	substitute equation 6 into equation 5
8. $KE_L = mr^2\omega^2/2$	simplify

The expressions for kinetic energy in steps 4 and 8 are the same except for a constant factor. We can write the linear kinetic energy as a constant times the rotational kinetic energy.

Step	Reason
9. $KE_L/KE_R = (mr^2\omega^2/2)/(mr^2\omega^2/3)$	divide equation 8 by equation 4
10. $KE_L = 1.5KE_R$	simplify

$$KE = ½mv^2 + ½I\omega^2$$

Moments of inertia

$I = mr^2$ (hollow cylinder)

$I = ½mr^2$ (solid cylinder)

The relationship of the speed and angular velocity for a rolling object

$$v_{CM} = r\omega$$

We apply the principle of the conservation of energy.

Step-by-step solution

We start by finding a general equation for the speed of an object that has rolled down the ramp, starting from rest at height h. We define the potential energy to be zero at the bottom of the ramp.

Step	Reason
1. $E_i = E_f$	conservation of energy
2. $E_i = PE = mgh$	initial energy is all potential
3. $E_f = KE = ½mv_{CM}^2 + ½I\omega^2$	energy at bottom of ramp is all kinetic
4. $mgh = ½mv_{CM}^2 + ½I\omega^2$	substitute equations 2 and 3 into equation 1
5. $\omega = v_{CM}/r$	relation of angular velocity and speed of center of mass
6. $mgh = ½mv_{CM}^2 + ½I(v_{CM}/r)^2$	substitute equation 5 into equation 4
7. $v_{CM} = \sqrt{\dfrac{2mgh}{\left(m+\dfrac{I}{r^2}\right)}}$	solve for v_{CM}

Solid cylinder. Now that we have a general equation for the speed of an object at the bottom of the ramp, we can apply the moment of inertia formulas to find equations for the speeds of the cylinders. We start with the solid cylinder.

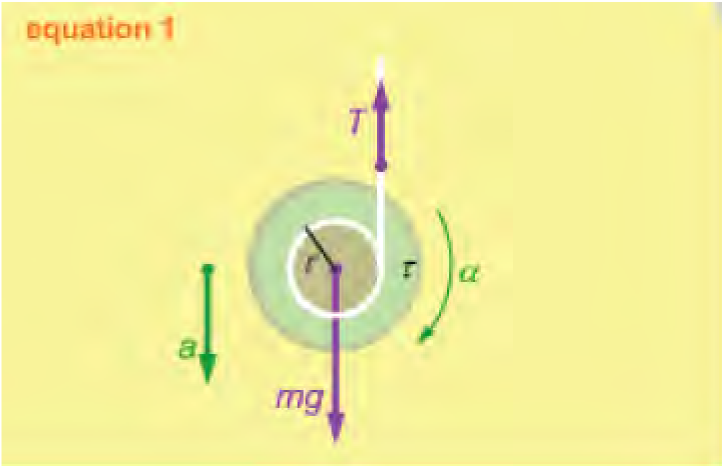
Step	Reason
8. $I_s = ½mr^2$	moment of inertia for a solid cylinder
9. $v_{CM} = \sqrt{\dfrac{2mgh}{\left(m+\dfrac{\frac{1}{2}mr^2}{r^2}\right)}}$	substitute equation 8 into equation 7
10. $v_{CM} = \sqrt{\frac{4}{3}gh}$	simplify

Hollow cylinder. Notice that the expression for the speed of the solid cylinder is independent of its mass and radius. This means that any solid cylinder will have the same speed as it rolls down a ramp of the same height. Next, we consider the speed of the hollow cylinder.

Step	Reason
11. $I_h = mr^2$	moment of inertia for a hollow cylinder
12. $v_{CM} = \sqrt{\dfrac{2mgh}{\left(m + \dfrac{mr^2}{r^2}\right)}}$	substitute equation 11 into equation 7
13. $v_{CM} = \sqrt{gh}$	simplify

For this reason we abandon our usual convention and take downward as the positive linear direction.

mass of yo-yo	m
acceleration of yo-yo	a
acceleration of gravity	$g = 9.80\ \text{m/s}^2$
upward force exerted by string	T
torque of string on yo-yo spindle	τ
radius of spindle	r
moment of inertia of yo-yo	I
angular acceleration of yo-yo	α



Acceleration of descending yo-yo

$$a = \frac{g}{1 + (I/mr^2)}$$

a = acceleration of yo-yo
g = acceleration of gravity
I = moment of inertia of yo-yo
m = mass of yo-yo
r = radius of spindle

Strategy

1. Analyze the linear motion of the yo-yo using Newton's second law. The equation will contain the tension T in the yo-yo string.
2. Analyze the angular motion of the yo-yo using the rotational form of Newton's second law. This equation, too, will contain T.
3. Combine the equations for linear and rotational motion, using the common variable T, and solve for the downward acceleration of the descending yo-yo.

Physics principles and equations

We will use two versions of Newton's second law to calculate linear and angular acceleration.

$$\Sigma F = ma, \quad \Sigma\tau = I\alpha$$

For a force perpendicular to the line from the axis of rotation to the point where the force is applied, the torque is

$$\tau = rF$$

The resulting tangential acceleration is

$$a = r\alpha$$

Step-by-step derivation

In the first steps we analyze the linear motion of the yo-yo. Note that the equation in the third step contains the tension force exerted by the string.

Step	Reason
1. $\Sigma F = ma$	Newton's second law
2. $\Sigma F = mg + (-T)$	inspection
3. $mg - T = ma$	substitute equation 2 into equation 1

We now analyze the angular acceleration of the yo-yo using Newton's second law for rotation. We solve the resulting equation for the tension force exerted by the string.

Step	Reason
4. $\Sigma\tau = I\alpha$	Newton's second law for rotation
5. $\Sigma\tau = rT$	torque and tangential force
6. $\alpha = a/r$	relationship of angular and tangential acceleration
7. $rT = I(a/r)$	substitute equations 5 and 6 into equation 4
8. $T = Ia/r^2$	solve for T

Now we combine the linear and rotational analyses and solve for the linear acceleration.

Step	Reason
9. $mg - Ia/r^2 = ma$	substitute equation 8 into equation 3
10. $a = \dfrac{g}{1+(I/mr^2)}$	solve for a

Steps

First, calculate the moment of inertia of each of the cylinders making up the yo-yo. We start with the identical disks A, and then calculate the moment of the spindle B.

Step	Reason
1. $I_A = ½\, m_A r_A{}^2$	moment of inertia of solid cylinder
2. $I_A = ½(0.030\ \text{kg})(0.075\ \text{m})^2$ $I_A = 8.44 \times 10^{-5}\ \text{kg}\cdot\text{m}^2$	enter values and multiply
3. $I_B = ½ m_B r_B^2$ $I_B = ½(0.0050\ \text{kg})(0.0065\ \text{m})^2$ $I_B = 1.06 \times 10^{-7}\ \text{kg}\cdot\text{m}^2$	moment of inertia of spindle

Now we calculate the moment of inertia of the yo-yo.

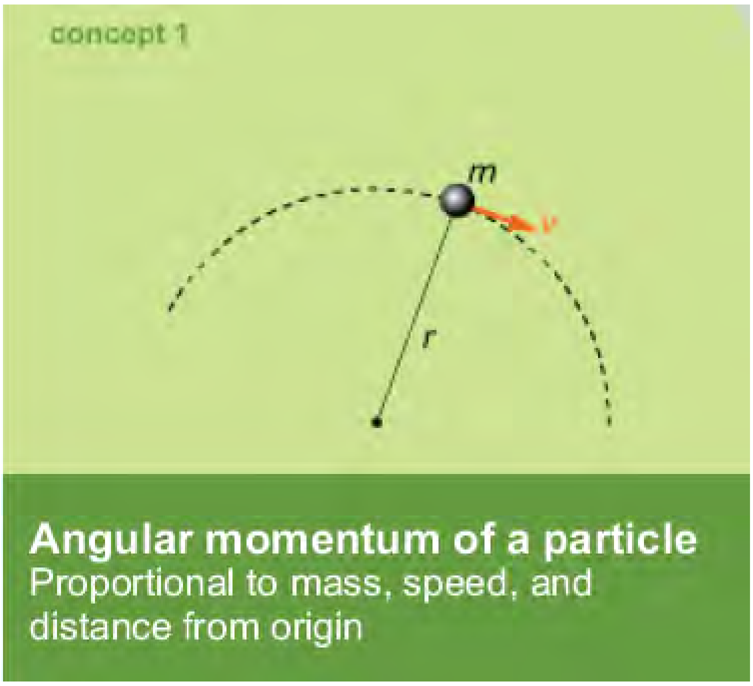
Step	Reason
4. $I = 2I_A + I_B$	yo-yo has two disks and spindle
5. $I = 2\,(8.44 \times 10^{-5}\ \text{kg}\cdot\text{m}^2) + 1.06 \times 10^{-7}\ \text{kg}\cdot\text{m}^2$ $I = 1.69 \times 10^{-4}\ \text{kg}\cdot\text{m}^2$	sum of moments

Now we use the equation stated above for the acceleration of a yo-yo.

Step	Reason
6. $a = \dfrac{g}{1+(I/mr_B^2)}$	acceleration of a yo-yo
7. $m = 2m_A + m_B$ $m = 2(0.030 \text{ kg}) + 0.0050 \text{ kg}$ $m = 0.065 \text{ kg}$	total mass
8. $a = \dfrac{9.80 \text{ m/s}^2}{1+\dfrac{1.69\times10^{-4} \text{ kg}\cdot\text{m}^2}{(0.065 \text{ kg})(0.0065 \text{ m})^2}}$	enter values into equation 6
9. $a = 0.16 \text{ m/s}^2$	evaluate

Angular momentum of particle in circle motion:

The concepts of linear momentum and conservation of linear momentum prove very useful in understanding phenomena such as collisions. *Angular momentum* is the rotational analog of linear momentum, and it too proves quite useful in certain settings. For instance, we can use the concept of angular momentum to analyze an ice skater's graceful spins.

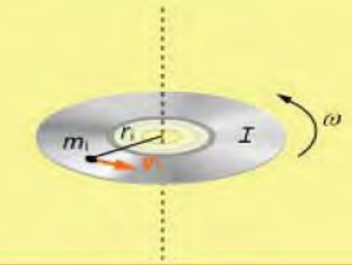


In this section, we focus on the angular momentum of a single particle revolving in a circle. Angular momentum is always calculated using a point called the origin. With circular motion, the simple and intuitive choice for the origin is the center of the circle, and that is the point we will use here. The letter **L** represents angular momentum. As with linear momentum, angular momentum is proportional to mass and velocity. However, with rotational motion, the distance of the particle from the origin must be taken into account, as well. With circular motion, the amount of angular momentum equals the product of mass, speed and the radius of the circle: *mvr*. Another way to state the same thing is to say that the amount of angular momentum equals the linear momentum *mv* times the radius *r*. Like linear momentum, angular momentum is a vector. When the motion is counterclockwise, by convention, the vector is positive. The angular momentum of clockwise motion is negative. The units for angular momentum are kilogram-meter2 per second (kg·m2/s).

Variables

mass of a particle	m_i
tangential (linear) speed of a particle	v_i
radius of a particle	r_i
angular momentum of particle	L_i
angular momentum of CD	L
angular velocity of CD	ω
moment of inertia of CD	I

equation 1



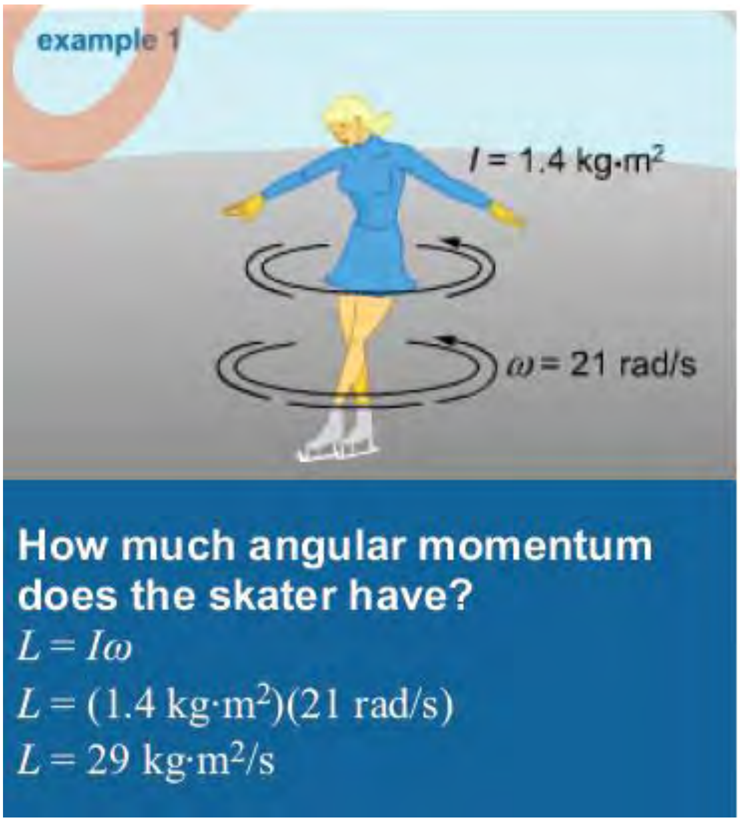
Angular momentum of a rigid body

$$L = I\omega$$

L = angular momentum
I = moment of inertia
ω = angular velocity

Strategy

1. Express the angular momentum of the CD as the sum of the angular momenta of all the particles of mass that compose it.
2. Replace the speed of each particle with the angular velocity of the CD times the radial distance of the particle from the axis of rotation.
3. Express the sum in concise form using the moment of inertia of the CD.



Physics principles and equations

The angular momentum of a particle in circular motion

$$L = mvr$$

We will use the equation that relates tangential speed and angular velocity.

$$v = r\omega$$

The formula for the moment of inertia of a rotating body

$$I = \Sigma m_i r_i^2$$

Step-by-step derivation

First, we express the angular momentum of the CD as the sum of the angular momenta of the particles that make it up.

Step	Reason
1. $L_i = m_i v_i r_i$	definition of angular momentum
2. $L = \Sigma m_i v_i r_i$	angular momentum of object is sum of particles

We now express the speed of the *i*th particle as its radius times the constant angular velocity ω, which we then factor out of the sum. The angular velocity is the same for all particles in a rigid body.

Step	Reason
3. $v_i = r_i \omega$	tangential speed and angular velocity
4. $L = \Sigma m_i (r_i \omega) r_i$	substitute equation 3 into equation 2
5. $L = (\Sigma m_i r_i^2)\omega$	factor out ω

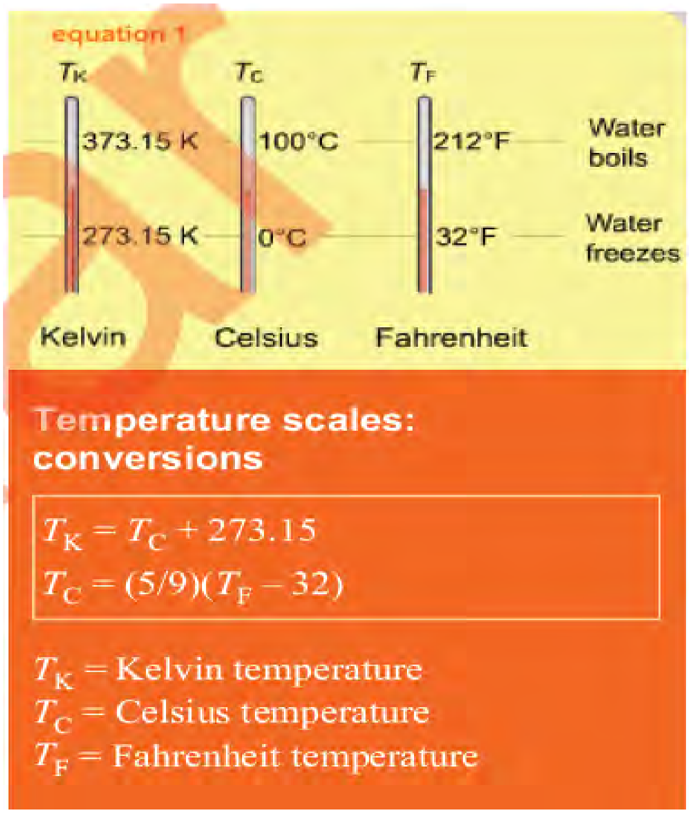
In the final steps we express the above result concisely, replacing the sum in the last equation by the single quantity I.

Step	Reason
6. $I = \Sigma m_i r_i^2$	moment of inertia
7. $L = I\omega$	substitute equation 6 into equation 5

Another important concept is shown in the illustration to the right: absolute zero. At this temperature, molecules (in essence) cease moving. Reaching this temperature is not theoretically possible, but temperatures quite close to this are being achieved. Absolute zero is 0 K, or □273.15°C. To standardize temperatures, scientists have agreed on a common reference point called the *triple point*. The triple point is the sole combination of pressure and temperature at which solid water (ice), liquid water, and gaseous water (water vapor) can coexist. It equals 273.16 K at a pressure of 611.73 Pa. The triple point is used to define the kelvin as an SI unit. One kelvin equals 1/273.16 of the difference between absolute zero and the triple point. If you are a sharp-eyed reader, you may have noticed the references to both 273.16 and 273.15 in this section. The freezing point of water is typically stated as 273.15 K (0°C) because this is its value at standard atmospheric pressure, but at the triple point pressure, water freezes at 273.16 K (0.01°C).

Temperature scale conversions

Since the Celsius and Kelvin scales have the same number of units between the freezing and boiling points of water, it takes just one step to convert between the two systems, as you see in the first conversion formula in Equation 1. To convert from degrees Celsius to kelvins, add 273.15. To convert from kelvins to degrees Celsius, subtract 273.15. Since water freezes at 32° and boils at 212° in the Fahrenheit system, there are 180 degrees Fahrenheit between these points, compared to the 100 units in the Celsius and Kelvin systems. To convert from degrees Fahrenheit to degrees Celsius, first subtract 32 degrees (to establish how far the temperature is from the freezing point of water) and then multiply by 100/180, or 5/9, the ratio of the number of degrees between freezing and boiling on the two systems. That conversion is shown as the second equation in Equation 1. If you further needed to convert to kelvins, you would add 273.15. To switch from Celsius to Fahrenheit, you first multiply the number of degrees Celsius by 9/5 (the reciprocal of the ratio mentioned above) and then add 32. In Example 1, you see the conventionally normal human body temperature, 98.6°F, converted to degrees Celsius and kelvins



example 1

Convert 98.6°F to Celsius and Kelvin.

$T_C = (5/9)(T_F - 32.0)$

$T_C = (5/9)(98.6°F - 32.0)$

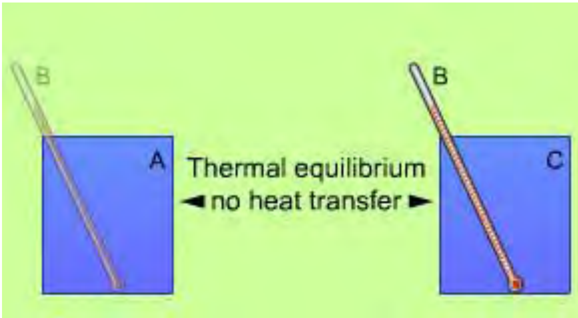
$T_C = (5/9)(66.6) = 37.0°C$

$T_K = T_C + 273.15$

$T_K = 37.0 + 273.15 = 310 \text{ K}$

Zeroth law of thermodynamics: If objects A and B are in thermal equilibrium, and objects B and C are in thermal equilibrium, then A and C will be in equilibrium as well.

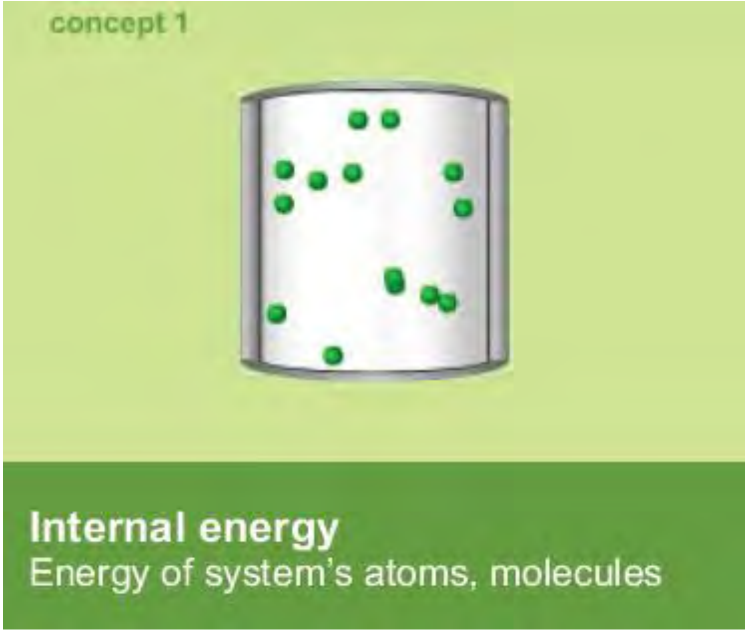
When you place two objects with different temperatures next to each other, the warmer object will cool off and the cooler object will warm up. Heat will flow until the objects reach *thermal equilibrium*, meaning they have the same temperature. For instance, place a pint of ice cream in a warm car, and the result will be warmer ice cream and a cooler car. Thermometers rely on heat flowing until they reach thermal equilibrium with the substance whose temperature they are measuring. Their practical use also relies on another principle, called the zeroth law of thermodynamics. This principle states that if object A is in thermal equilibrium with object B, and object B is in equilibrium with object C, then A and C will be in equilibrium when they are placed in direct contact, and no heat will flow between them. We illustrate this law on the right. Let's say you put thermometer B in a container of water A. When the thermometer's reading stabilizes at a constant value, say 20°C, it has reached thermal equilibrium with the water. If you then place the thermometer in a second container C and its reading remains 20°C there, you can conclude that A and C would be in thermal equilibrium when placed in direct contact with each other. They have the same temperature and heat would not flow between them. This may seem commonsense, but it is an important assumption in thermodynamics. Its importance was realized after the first and second laws of thermodynamics (which you will study later) were already codified hence it became the zeroth law, since it is an underlying assumption for the other laws.



Zeroth law of thermodynamics
If A, B in thermal equilibrium,
and B, C in thermal equilibrium,
then A, C in thermal equilibrium
(no heat transfer)

Internal energy: The energy associated with the molecules and atoms that make up a system.

In the study of mechanics, energy is an overall property of an object or system. The energy is a function of factors like how fast a car is moving, how high an object is off the ground, how fast a wheel is rotating, and so forth. In thermodynamics, the properties of the molecules and/or atoms that make up the object or system are now the focus. They also have energy, a form of energy called internal energy. The internal energy includes the rotational, translational and vibrational energy of individual molecules and atoms. It also includes the potential energy within and between molecules. To contrast the two forms of energy: If you lift a pot up from a stovetop, you will increase its gravitational potential energy. But in terms of internal energy, nothing has changed. The potential energy of the pot's molecules based on their relationship to each other has not changed. However, if you turn on the burner under the cooking pot, the flow of heat will increase the kinetic energy of its molecules. The molecules will move faster as heat flows to the pot, which means the internal energy of the molecules of the pot increases.



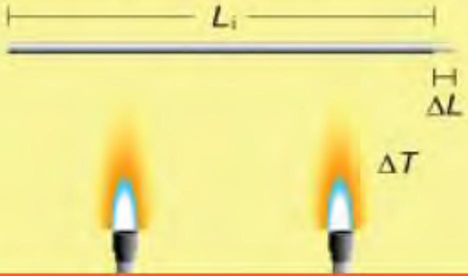
Thermal expansion: The increase in the length or volume of a material due to a change in its temperature.

You buy a jar of jelly at the grocery store and store it on a pantry shelf. When it comes time to open the jar, the lid refuses to budge. Fortunately, you know that placing the jar under hot water will increase your odds of being able to twist open the lid.

Expansion joints allow bridge sections to expand without breaking.

equation 1

α = coefficient of expansion



Linear expansion

$$\Delta L = L_i \alpha \Delta T$$

L = length
α = coefficient of linear expansion
ΔT = change in temperature
Coefficient calibrated for K or °C

example 1

$\alpha = 1.65\text{x}10^{-5}$ 1/C°

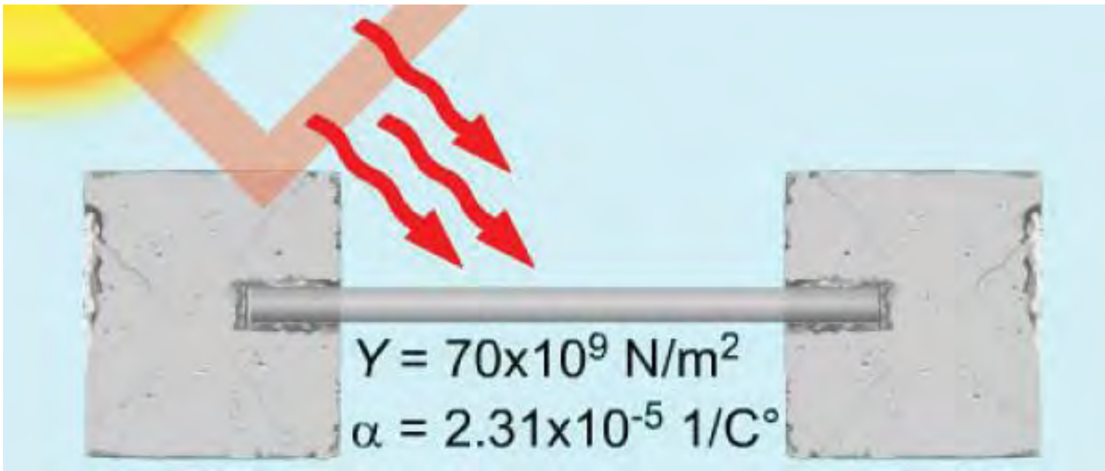
The copper rod is heated from 15°C to 95°C. What will its increase in length be?

$\Delta L = L_i \alpha \Delta T$

$\Delta T = 95°\text{C} - 15°\text{C} = 80\ \text{C}°$

$\Delta L = (0.5\ \text{m})(1.65\times10^{-5}\ 1/\text{C}°)(80\ \text{C}°)$

$\Delta L = 6.6\times10^{-4}\ \text{m}$



What stress does the aluminum rod exert when its temperature rises 20 K?

Above, you see an aluminum rod heated by the Sun and held in place with concrete blocks. Since the rod increases in temperature, its length also increases. This exerts a force on the concrete blocks. Stress is force per unit area, and an equation for tensile stress was presented in another chapter. Young's modulus for aluminum is given; it relates the fractional increase in length (the strain) to stress. You are asked to find the stress that results from the increase in temperature.

until 4°C
From 4°C to 0°C, water expands and stays on top
Then ice forms on top and floats

of ice that insulates the water below. Water is also atypical in that its solid form, ice, is less dense than its liquid form and floats on top of it. Fish and other aquatic life can live in the relatively warm (and liquid) water below, protected by a shield of ice. If water always expanded with increasing temperature for all temperatures above 0°C, and contracted with decreasing temperature, the coldest water would sink to the bottom where it might never warm up. Water's negative coefficient of expansion in the temperature range from 0°C to 4°C is crucial to life on Earth. If ice did not float, oceans and lakes would freeze from the bottom to the top. This would increase the likelihood that they would freeze entirely, since they would not have a top layer of ice to insulate the liquid water below and their frozen depths would not be exposed to warm air during the spring and summer.

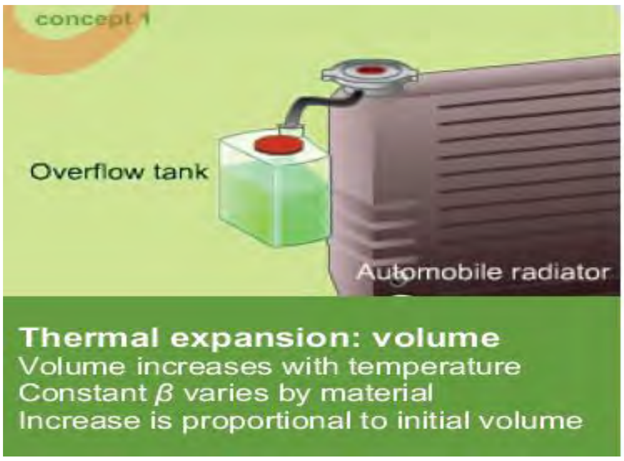
Coefficient of volume expansion (1/C°)

Liquids		*Solids*	
Mercury	19.6×10^{-5}	Glass*	2.14×10^{-5}
Water	20.7×10^{-5}	Copper*	5.00×10^{-5}
Glycerin	50.4×10^{-5}	Silver*	5.64×10^{-5}
Olive Oil	72.0×10^{-5}	Lead*	8.37×10^{-5}
Methyl Alcohol	120×10^{-5}	Ice (-26°C)	11.3×10^{-5}
Acetone	149×10^{-5}		

* between 0 -100°C

Thermal volume expansion: Change in volume due to a change in temperature.

The equation for thermal linear expansion is used to calculate the thermally induced change in the size of an object in just one dimension. Thermal expansion or contraction also changes the volume of a material, and for liquids (and many solids) it is more useful to determine the change in volume rather than expansion along one dimension. The expansion in volume can be significant. Automobile cooling systems have tanks that capture excess coolant when the heated fluid expands so much it exceeds the radiator's capacity. A radiator and its overflow tank are shown in Concept 1 on the right. The formula in Equation 1 resembles that for linear expansion: The increase is proportional to the initial volume, a constant, and the change in temperature. The constant β is called the *coefficient of volume expansion*. Above, you see a table of coefficients of volume expansion for some liquids and solids. The coefficients for liquids are valid for temperatures at which these substances remain liquid.

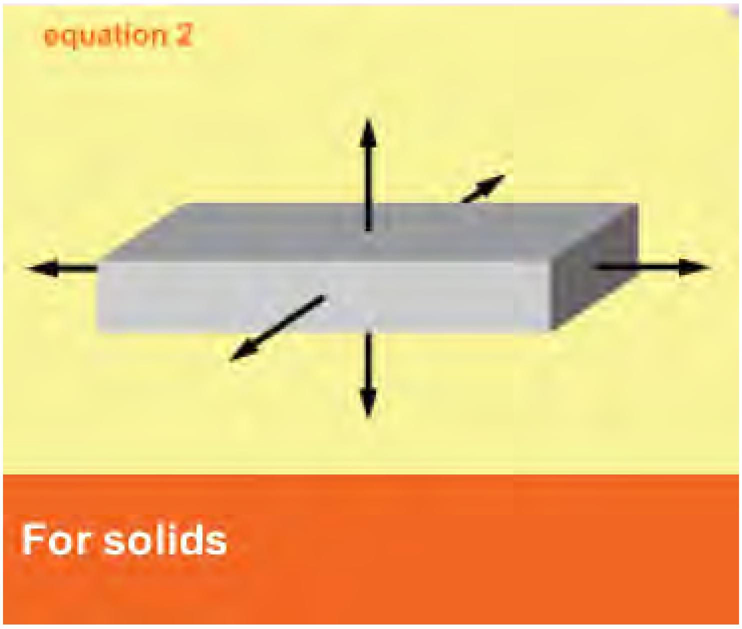


equation 1

Thermal expansion: volume

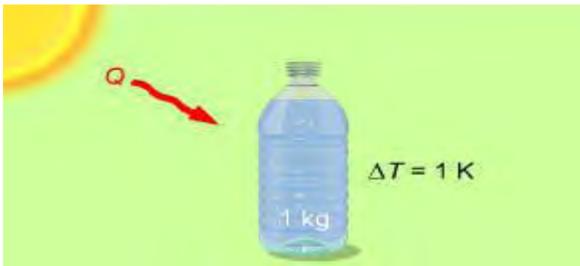
$$\Delta V = V_i \beta \Delta T$$

V = volume
β = coefficient of volume expansion
ΔT = change in temperature
Coefficient calibrated for K or °C

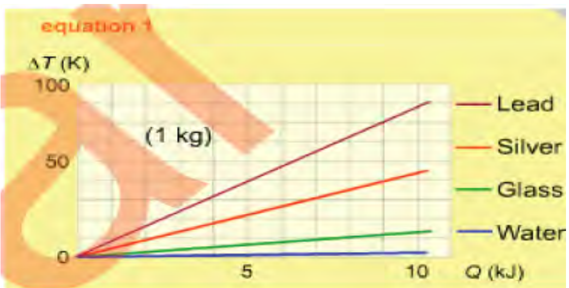


A material with a greater specific heat requires more heat per kilogram to increase its temperature a given amount than one with a lesser specific heat. In spite of its name, specific heat is not an amount of heat, but a constant relating heat, mass, and temperature change. The specific heat of a material is often used in the equation shown in Equation 1. The heat flow equals the product of a material's specific heat *c*, the mass of an object

consisting of that material, and its change in temperature. The illustration in Equation 1 shows how specific heat relates heat flow to change in temperature. As you can see from the graph, lead increases in temperature quite readily when heat flows into it, because of its low specific heat. In contrast, water, with a high specific heat, can absorb a lot of energy without changing much in temperature. Temperatures in locations at the seaside, or having humid atmospheres, tend to change very slowly because it takes a lot of heat flow into or out of the water to accomplish a small change in temperature. Summer in the desert southwest of the United States is famous for its blazing hot days and chilly nights, while on the east coast of the country the sweltering heat of the day persists long into the night. Materials with large specific heats are sometimes informally called “heat sinks” because of their ability to store large amounts of internal energy without much temperature change. Above, you see a table of some specific heats, measured in joules per kilogram· kelvin. The specific heat of a material varies as its temperature and pressure change. The table lists specific heats for materials at 25°C to 30°C (except for ice) and 105 Pa pressure, about one atmosphere. Specific heats vary somewhat with temperature, but you can use these values over a range of temperatures you might encounter in a physics lab (or a kitchen).



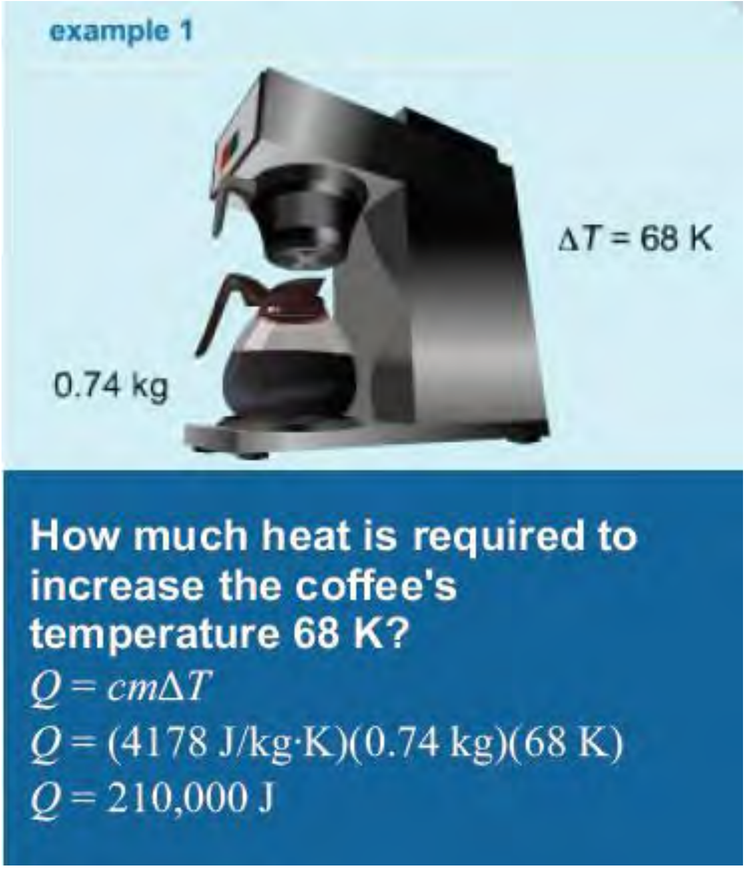
Specific heat of a material
Relates heat and temperature change, per kilogram



Specific heat of a material

$$Q = cm\Delta T$$

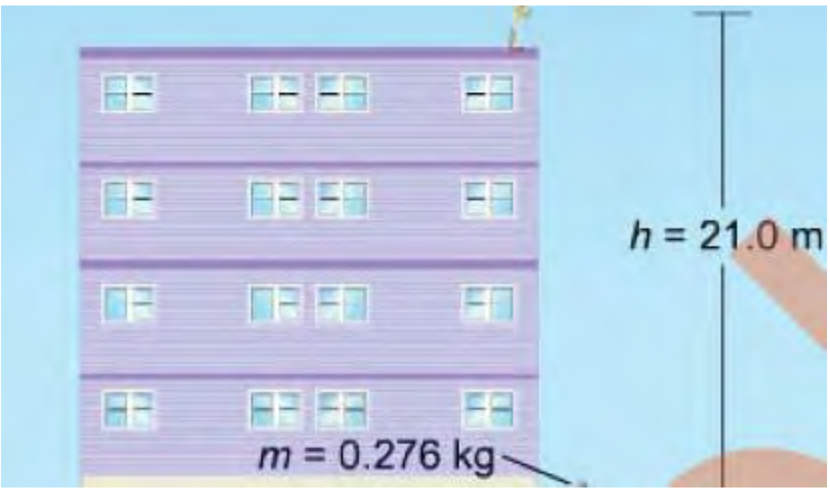
Q = heat
c = specific heat (J/kg·K)
m = mass
ΔT = temperature change in C° or K



Now we solve the equation for the unknown specific heat of the object and evaluate.

Step	Reason
6. $c_o = -\dfrac{c_w m_w (T_f - T_w)}{m_o (T_f - T_o)}$	solve for specific heat of object
7. $c_o = -\dfrac{(4178\ \text{J/kg}\cdot\text{K})(0.744\ \text{kg})(25.6\,^\circ\text{C} - 23.2\,^\circ\text{C})}{(0.197\ \text{kg})(25.6\,^\circ\text{C} - 67.8\,^\circ\text{C})}$	substitute
8. $c_o = 897\ \text{J/kg}\cdot\text{K}$	evaluate

Based on the values in the table of specific heats, it appears that the material may consist of aluminum.

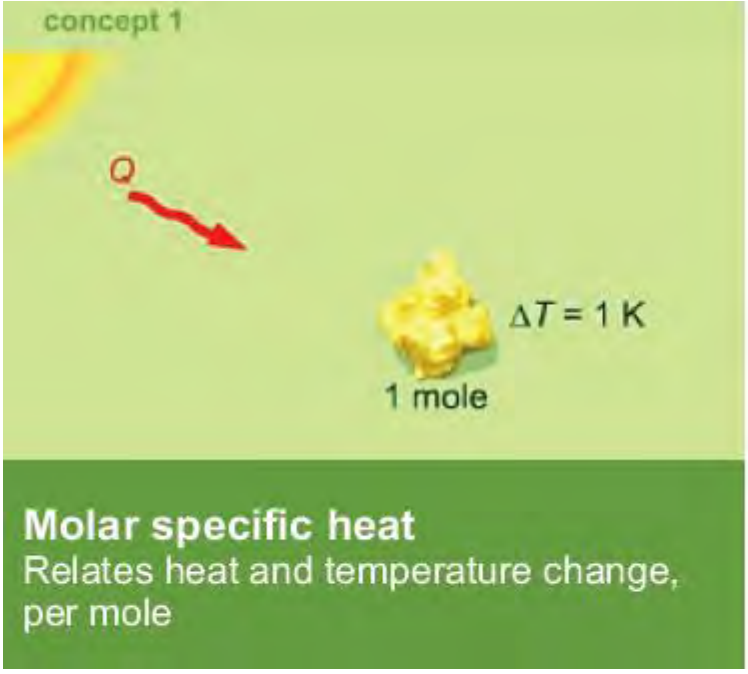


A 0.276 kg lead ball falls from a height of 21.0 m and lands on the ground without bouncing. Assume that half the energy generated by the impact of the ball with the ground becomes internal energy in the ball. The specific heat of lead is 129 J/kg·K. What is the temperature change of the ball?

Molar specific heat: A proportionality constant that relates the amount of heat flow per mole to a material's change in temperature.

	Molar specific heat (J/mol·K)
Aluminum	24.20
Copper	24.44
Iron	25.10
Silver	25.35
Lead	26.65
	at 10^5 Pa, 25°C

Scientists find it convenient at times to measure substances in terms of moles. If you have studied chemistry, you probably studied moles. Briefly, one mole of a substance contains 6.022×10^{23} particles (typically molecules; moles are explained in more depth later). Measuring in moles focuses particularly on the number of molecules in an object instead of its mass.



Specific heat and molar specific heat are both proportionality constants, relating the heat transfer per an amount of a material to the resulting change in temperature. Specific heat is stated in terms of joules per kilogram, and molar specific heat in terms of joules per mole.

A material's molar specific heat is determined by how many joules are required to heat one mole of the substance one kelvin. A material with a greater molar specific heat requires more heat per mole to produce a given change in temperature than a material with a lesser molar specific heat. This is quantified in Equation 1. The table above lists the molar specific heats of some metals at room temperature. Measuring specific heat in terms of moles reveals an interesting fact: The values do not vary much. In fact, the molar specific heats of all solids approach a value of about 25 J/mol·K as their temperatures increase. When they turn into liquids or gases, their molar specific heats change. This consistency means that the differences in specific heat values for solids (when measuring by kilograms) are due mainly to the number of molecules contained in a kilogram, rather than differences in the properties of the solids.

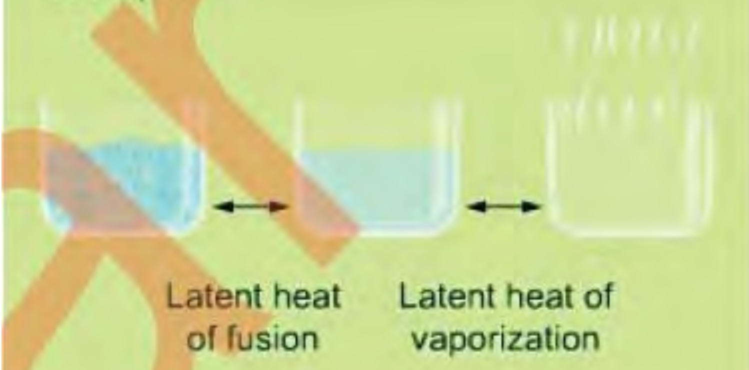
Latent heat: Energy required per kilogram to cause a phase change in a given material.

Heat flow can cause a substance to change phases by converting it between a solid and a liquid, or a liquid and a gas. Latent heat describes how much energy per kilogram is required for a given substance to change phase. It is a proportionality constant, expressing the relationship between heat and mass as shown in Equation 1. The constant depends on the material and on the phase change. Different amounts of energy are required to transform a material between its liquid and solid states than between its liquid and gaseous states. The *latent heat of vaporization* is the amount of heat per kilogram consumed when a given substance transforms from a liquid into a gas, or released when the substance transforms from a gas back to a liquid. The *latent heat of fusion* is the heat flow per kilogram during a change in phase between a solid and a liquid. The table above shows the latent heats of fusion and vaporization for various substances. For instance, you need 3.34×105 J of energy to convert a kilogram of ice (at 0°C) to liquid water. Continued flow of heat into the water will raise its temperature until it reaches 100°C. At this temperature, it will take 2.26×106 joules of heat to turn it into a gas, about seven times as much as it took to convert it to a liquid. Salt causes ice to melt, a phenomenon called "freezing point depression." When you add rock salt to the crushed ice in a hand-cranked ice cream freezer, you force the ice to melt. Heat flows from the resulting saltwater solution into the ice as it changes phase from solid to liquid, resulting in a slurry having a temperature far colder than 0°C. Heat then flows from the ice cream solution into this mixture, and the ice cream freezes

	Melting point (°C)	Latent heat of fusion (J/kg)	Boiling point (°C)	Latent heat of vaporization (J/kg)
Aluminum	660	3.97×10^5	2519	1.09×10^7
Carbon	4489	9.74×10^6		
Copper	1085	2.09×10^5		
Iron	1538	2.47×10^5		
Lead	327	2.30×10^4	1749	8.66×10^5
Mercury	-39	1.14×10^4	357	2.95×10^5
Nitrogen	-210	2.53×10^4	-196	1.99×10^5
Table salt	801	3.78×10^5		
Water	0	3.34×10^5	100	2.26×10^6

at standard pressure

concept 1



Latent heat

Energy required per kg to change state
Latent heat of fusion: solid to liquid
Latent heat of vaporization: liquid to gas
Amount same in either "direction"

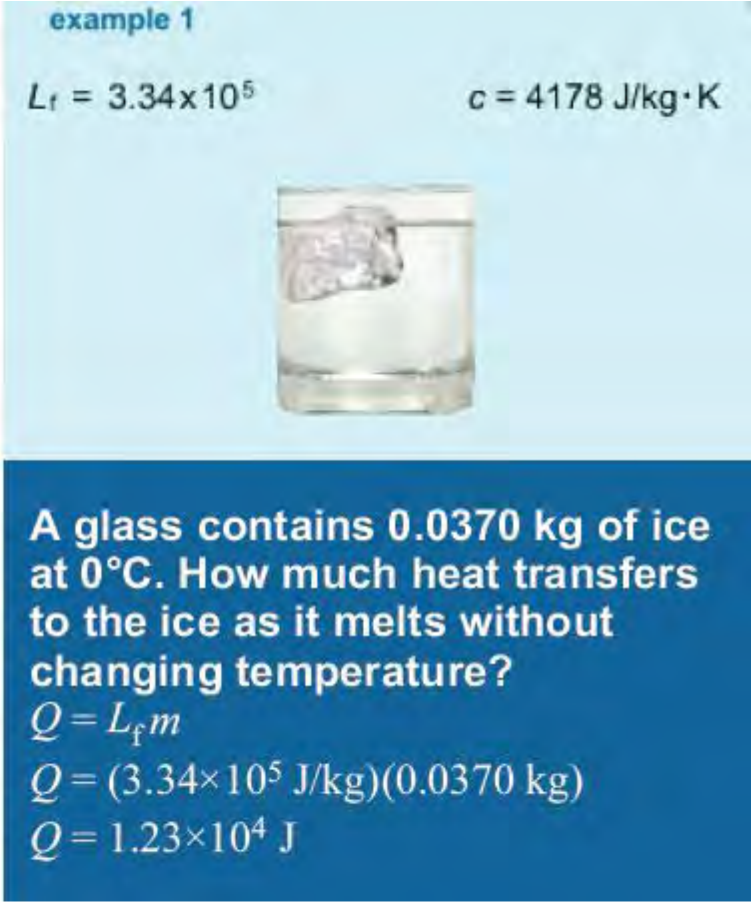
equation 1

Heat required for phase changes

$$Q = L_f m$$

$$Q = L_v m$$

Q = heat
m = mass
L_f = latent heat of fusion (J/kg)
L_v = latent heat of vaporization (J/kg)



Step-by-step solution

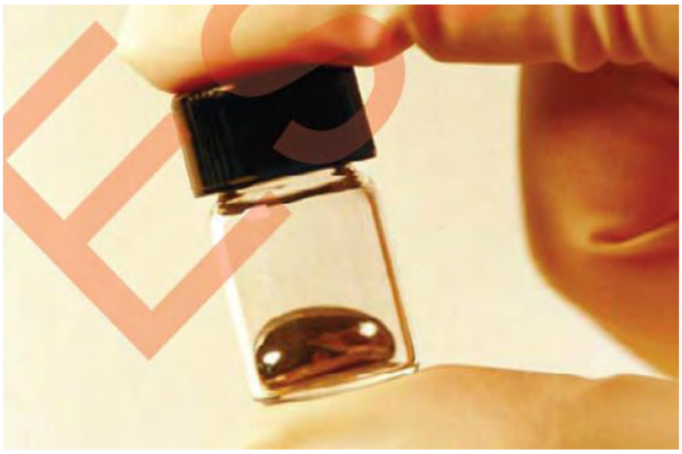
First we calculate the temperature of the liquid water after it gives up heat to melt the ice. We use the specific heat of water, 4178 J/kg·K.

Step	Reason
1. $Q = cm\Delta T$	specific heat equation
2. -1.23×10^4 J = (4178 J/kg·K)(0.160 kg)ΔT	substitute values
3. $\Delta T = -18.4$ K $= -18.4$ C°	solve
4. $T_{Lf} = T_{Li} + \Delta T$ $T_{Lf} = 30.0$°C + (−18.4°C) $T_{Lf} = 21.6$°C	calculate water temperature

Now we use the fact that the heat transfers sum to zero as the two masses of water reach thermal equilibrium to calculate the final temperature of the total mass of water.

Step	Reason
5. $Q_L + Q_S = 0$	equation above
6. $cm_L(T - T_{Lf}) + cm_S(T - T_{Sf}) = 0$	specific heat equation
7. $T = \dfrac{m_L T_{Lf} + m_S T_{Sf}}{m_L + m_S}$	solve for T
8. $T = \dfrac{(0.160\text{ kg})\,(21.6\,^\circ\text{C}) + (0.0370\text{ kg})\,(0.00\,^\circ\text{C})}{0.160\text{ kg} + 0.0370\text{ kg}}$	substitute values
9. $T = 17.5°C$	evaluate

Interactive checkpoint: vaporizing mercury



A vial of 0.0500 kg of mercury is at room temperature (20.0°C). What amount of heat must be transferred to the mercury in order to vaporize it? Mercury boils at 357°C, its specific heat is 140 J/kg·K (assume this is constant over the temperature range of interest) and its latent heat of vaporization is 2.95×10^5 J/kg.

Conduction: The flow of thermal energy directly through a material without motion of the material itself.

When a frying pan is placed on a burner, heat flows from the burner to the pan. The heat then spreads through the pan, soon reaching the handle even though the handle is not in direct contact with the burner. This process illustrates the flow of thermal energy via conduction. effective insulators and can be combined with other reasonably good insulators such as wood for even greater energy efficiency. Third, materials can be combined. Double-paned windows trap a quantity of an inert gas like argon between two layers of glass. Argon has a high *R* value and considerably reduces the rate of heat transfer through the window.

$$P_c = \frac{kA\Delta T}{L}$$

k = thermal conductivity
A = area
ΔT = temperature difference
L = thickness
k units: J/s·m·K = W/m·K

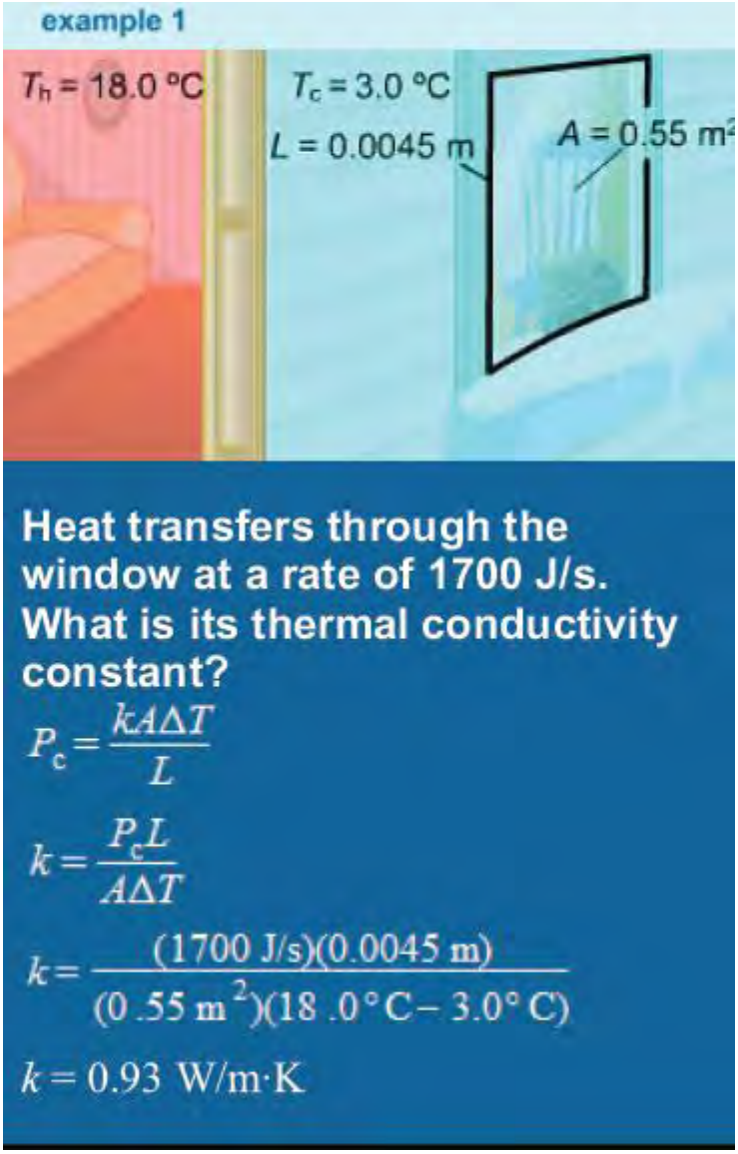
equation 3

Thermal resistance

$$R = L/k$$

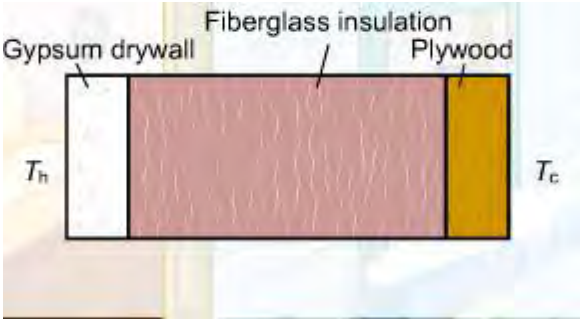
$$P_c = \frac{A\Delta T}{R}$$

R = thermal resistance
R units (SI): $m^2 \cdot K/W$
R units (British): $ft^2 \cdot F^\circ \cdot h/Btu$



Conduction through composite objects

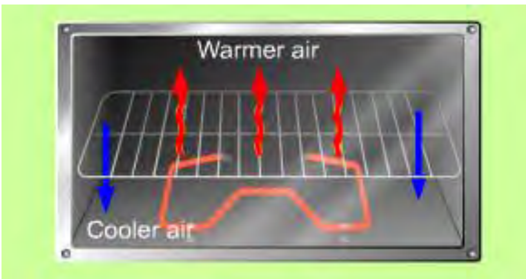
Real-world objects such as the walls of a house are often a composite of different materials. For example, a house wall may consist of gypsum drywall, fiberglass insulation and plywood. At the right, you see a schematic of a wall made of materials of varying thicknesses. To calculate the rate of heat flow through this composite object, the overall thermal resistance is calculated by summing the resistance of each object. This value can be used as the R-value of a single object in other equations. You see this in Equation 1 on the right. When designing buildings, the rate at which heat will flow through the walls is an important consideration. Example 1 shows a calculation of the rate of heat flow using R values for three common building materials



To calculate rate of heat transfer:

Convection: Heat transfer through a gas or liquid caused by movement of the fluid.

Gases and liquids usually decrease in density when they are heated (liquid water near 0°C is a notable exception). When part of a body of liquid or gas is heated, the warmed component rises because of its decreased density, while the cooler part sinks. This occurs in homes, where heat sources near the floor heat the nearby air, which rises and moves throughout the room. The warmer air displaces cooler air near the ceiling, causing it to move near the heat source, where it is heated in turn. This transfer of heat by the movement of a gas or liquid is called *convection*. All kitchen ovens, like the one shown in Concept 1, rely largely on convection for baking. The heating element at the bottom of the oven warms the air next to it, causing it to rise. The heated air then reaches the food in the oven to warm it, while the cooler air sinks to the bottom of the oven. So-called "convection ovens" speed this process with fans that cause the air to circulate more quickly. Convection occurs in liquids as well as in gases. If you stir spaghetti sauce as it heats, you are accelerating the process of convection. Again, your goal is to uniformly distribute the thermal energy. If you see a hawk soaring upward without flapping its wings, it may be riding what is called a "thermal." As the Sun warms the ground, the nearby air also becomes warmer. In the process, it becomes less dense, and is forced upward by air that is cooler and denser. A bird can ride this upward draft.

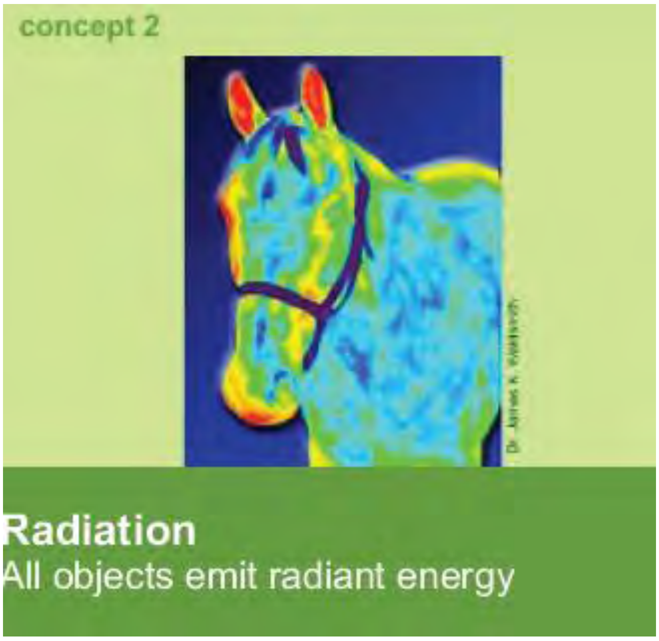


Radiation: Heat transfer by electromagnetic waves.

If you place your hand near a red-hot heating element and feel your hand warm up, you are experiencing thermal radiation: the transfer of energy by electromagnetic waves. You correctly think of objects like the heating element as radiating heat; in fact, every object with a temperature above absolute zero radiates energy. Radiation consists of electromagnetic waves, which are made up of electric and magnetic fields. Radiation needs no medium in which to travel; it can move through a vacuum. The wavelength of radiation varies. For instance, red light has a wavelength of about 700 nm, and blue light a wavelength of about 500 nm. Infrared and ultraviolet radiation are two forms of radiation whose wavelengths are, respectively, longer and shorter than those of visible light. All objects radiate electromagnetic radiation of different wavelengths. For instance, you see the red-hot stove coil because it emits some visible light. The coil also emits infrared radiation that you cannot see but do feel as heat flowing to your hand, and it emits a minimal amount of ultraviolet radiation too.

Although any particular object radiates a range of wavelengths, there is a peak in that range, a wavelength at which the power output is the greatest. This peak moves to shorter wavelengths as the temperature of the object increases. Understanding the exact form of the spectrum of thermal radiation wavelengths requires concepts from quantum physics, and its derivation was one of the early triumphs and verifications of quantum theory. Bodies with temperatures near the temperature of the surface of the Earth emit mostly infrared radiation. In the photograph in Concept 2, called an *infrared thermograph*, yousee the radiation emitted by a horse. Since areas of inflammation in the body are unusually warm, and emit extra thermal radiation, veterinarians can use photographs like this to diagnose an animal's ailments. They are created by a digital or film camera that assigns different (visible) colors to different intensities of (invisible) infrared radiation in a process called false color reproduction.

Sunlight is a form of radiation and is crucial to life on Earth. The Sun emits massive amounts of energy in the form of radiation: 3.9×1026 joules every second. Some of that strikes the Earth, where it warms the planet and supplies the energy that plants use in photosynthesis. The amount of power radiated by a body is proportional to the fourth power of its absolute temperature, its surface area, and a factor called its emissivity. The Sun emits tremendous amounts of radiation energy because it is quite hot (about 6000 K) and vast (with a surface area of about 6×1018 m2).Only a small portion of the total power emitted by the Sun reaches the Earth. Even this fraction is an enormous amount: 1.8×1017 watts, about 100 times what human civilization consumes. The average solar power striking the Earth's atmosphere in regions directly facing the Sun is about 1370 watts per square meter. This value is called the *solar constant*.



This example serves to illustrate the role of the greenhouse effect. The Earth's atmosphere "traps" a substantial amount of the radiation emitted by the Earth's surface. Without this effect, the temperature at the surface of the Earth would be cooler.

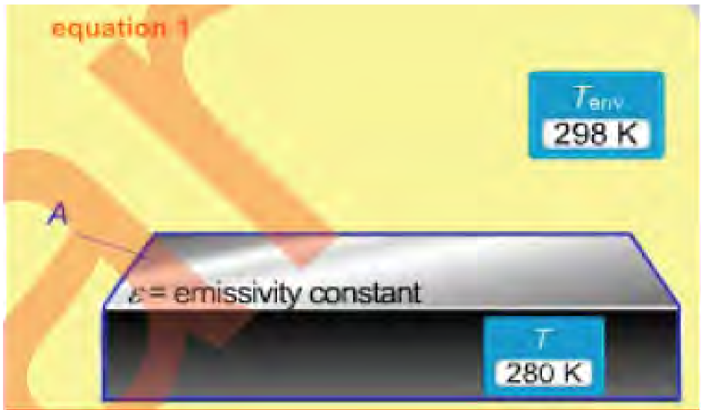
concept 2

	Emissivity
Polished silver, gold, aluminum	0.02-0.04
Mercury (the element)	0.09-0.12
Venus	0.24
Earth average	0.67
White enamel paint	0.87-0.91
Mercury (the planet)	0.9
Flat black lacquer	0.92-0.96
Candle soot	0.95

Emissivity

ε = ratio absorbed/incident radiation

equation 1



Radiated and absorbed power

$$P_{rad} = \sigma \varepsilon A T^4$$

$$P_{abs} = \sigma \varepsilon A T_{env}^4$$

P_{rad} = power radiated

P_{abs} = power absorbed

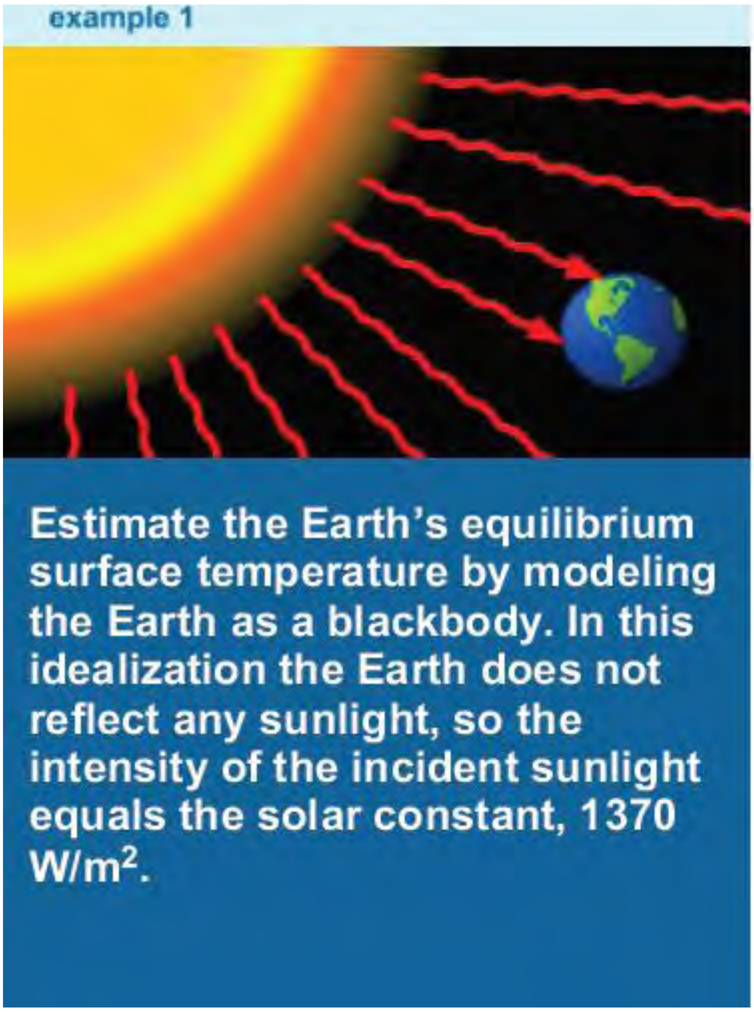
ε emissivity, A = surface area

T = temperature of object (K)

T_{env} = temperature of environment (K)

$\sigma = 5.67\times10^{-8}$ W/m^2·K^4

(Stefan-Boltzmann constant)



Interactive summary problem: pop the cork

You just bought a bottle of Pierrot, the water from ancient limestone caves deep in the French Alps, filtered by pure quartz crystals. But you did not realize the bottle came with a cork, and you have no corkscrew. Fortunately, your knowledge of thermal physics comes in handy. You remember that the density of ice is 9% less than the density of water. This means that water expands quite a bit when it freezes into ice. If you let the water in the bottle freeze, the expansion of the ice will push the cork out. If 89.0% of the water freezes, the expanding ice will just push the cork out. But if more than 89.0% of the water freezes, the ice will expand too much and the bottle will break. You want to remove just enough heat from the water so that exactly 89.0% of it turns to ice. In the interactive simulation on the right, you control the amount of heat removed from the water. The bottle contains 0.750 kg of water and its temperature is now 15.0°C. You need to remove enough heat to reduce the temperature of all the water to 0°C, and then remove enough additional heat to freeze 89.0% of it. To do these calculations, you will need to use the specific heat of water, 4178 J/kg·K, and the latent heat of fusion of water, 3.34×105 J/kg. We ignore the glass bottle itself in these calculations. Heat is removed from the bottle, but much more heat (about 50 times more) is removed from the water. Also, while the volume of the glass bottle decreases slightly as it cools, the expansion of the ice is much greater. Similarly, the small air space at the top of the bottle has little effect. Set the amount of heat to be removed from the water, then press GO. If you are right, the ice will push the cork out. Press RESET to try again. If you have trouble calculating the correct amount of heat transfer, review the sections in this chapter on specific heat and latent heat, and the sample problem that combines the two concepts.

Gotchas

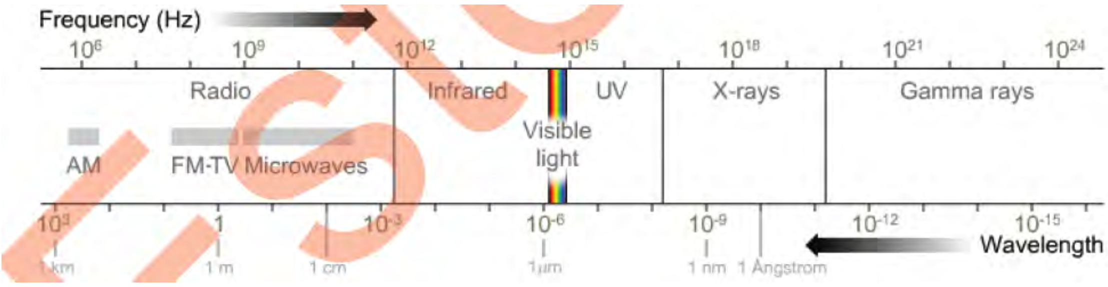
Heat is the same as temperature. No, heat is a flow of energy. Temperature is a property of an object. The flow of heat will change the temperature of an object, and a thermometer measures the object's temperature.*The Fahrenheit temperature system is the wave of the future.* If you think so, can I interest you in buying a record player? *Two rods of the same material experience the same increase in temperature, which means they must have expanded by the same amount of length.* Only if they were the same initial length. Their percentage increase would be the same in any case. *You throw a football upward. You have not increased the internal energy of the air within the football.* Correct: You have not increased the internal energy of the molecules inside the football. (You have increased its translational kinetic energy and its rotational kinetic energy and its

gravitational PE by throwing it upward, but not its internal energy.)

x-rays, microwaves

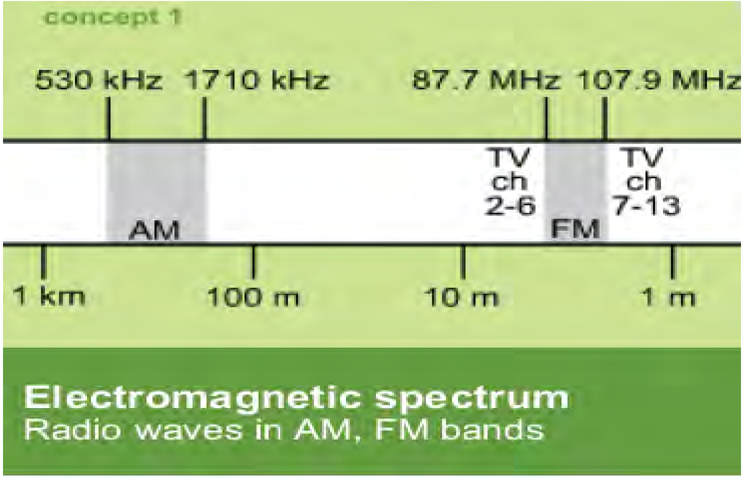
Radio and television signals, x-rays, microwaves: Each is a form of electromagnetic radiation. If steam and internal combustion engines symbolize the Industrial Revolution, and microprocessors and memory chips now power the Information Revolution, it almost seems that we have neglected to recognize the "Electromagnetic Revolution." Think about it: Can you imagine life without television sets or cell phones? You may long for such a life, or wonder how people ever survived without these devices! These examples are from the world of engineered electromagnetic radiation. Even if you think we might all prosper without such technologies to entertain us, do our cooking, carry our messages, and diagnose our illnesses, you would be hard-pressed to survive without light. This form of electromagnetic radiation brings the Sun's energy to the Earth, warming the planet and supplying energy to plants, and in turn to creatures like us that depend on them. There are primitive forms of life that do not depend on the Sun's energy, but without light there would be no seeing, no room with a view, no sunsets, and no Rembrandts. Some of the electromagnetic radiation that reaches your eyes was created mere nanoseconds earlier, like the light from a lamp. Other electromagnetic radiation is still propagating at its original speed through the cosmos, ten billion years or more after its birth. An example of this is the microwave background radiation, a pervasive remnant of the creation of the universe that is widely studied by astrophysicists. Back here on Earth, this chapter covers the fundamental physical theory of electromagnetic radiation. Much of it builds on other topics, particularly the studies of waves, electric fields and magnetic fields. **Electromagnetic radiation: Rainbows and radios. Sundazzled reflections. Shadowlamps and lampshadows. Red, white, and blue.**

The electromagnetic spectrum



Electromagnetic spectrum: Electromagnetic radiation ordered by frequency or wavelength.

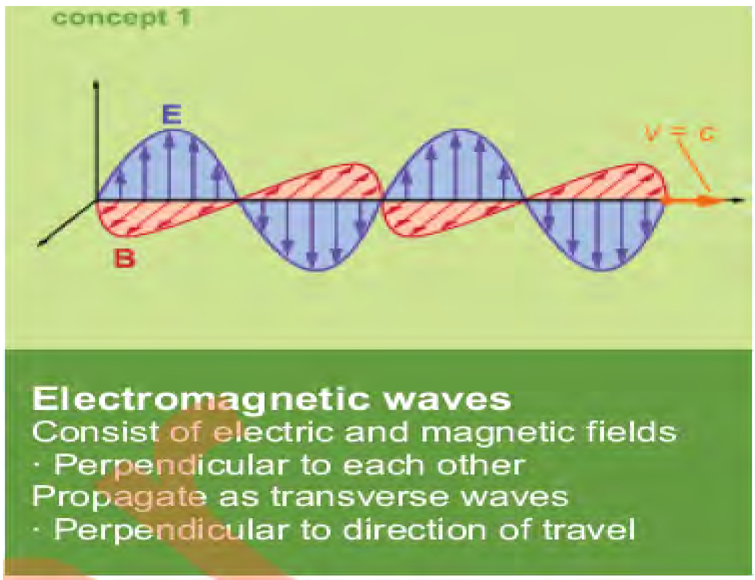
Electromagnetic radiation is a traveling wave that consists of electric and magnetic fields. Before delving into the details of such waves, we will discuss the electromagnetic spectrum, a system by which the types of electromagnetic radiation are classified. The illustration of the electromagnetic spectrum above orders electromagnetic waves by frequency and by wavelength. In the diagram, frequency **increases** and wavelength **decreases** as you move from the left to the right. The chart's scale is based on powers of 10. Wavelengths range from more than 100 meters for AM radio signals to as small as 10□16 meters for gamma rays. All electromagnetic waves travel at the same speed in a vacuum. This speed is designated by the letter *c* and is called the speed of light. (The letter *c* comes from *celeritas,* the Latin word for speed. It might be more accurate to refer to it as the speed of electromagnetic radiation.) The speed of light in a vacuum is exactly 299,792,458 m/s, and it is only slightly less in air. The unvarying nature of this speed has an important implication: The wavelength of electromagnetic radiation is inversely proportional to its frequency. As you may recall, the speed of a wave equals the product of its frequency and wavelength. This means that if you know the wavelength of the wave, you can determine its frequency (and vice versa). For instance, an electromagnetic wave with a wavelength of 300 meters, in the middle of the AM radio band, has a frequency of 1×106 Hz. This equals 3×108 m/s, the speed of light, divided by 300 m. The frequencies of electromagnetic waves range from less than one megahertz, or 106 Hz, for long radio waves to over 1024 Hz for gamma rays. We will now review some of the bands of electromagnetic radiation and their manifestations. The lowest frequencies are often utilized for radio

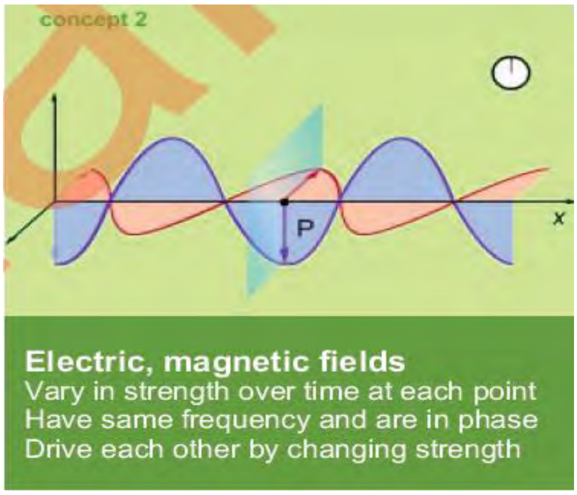


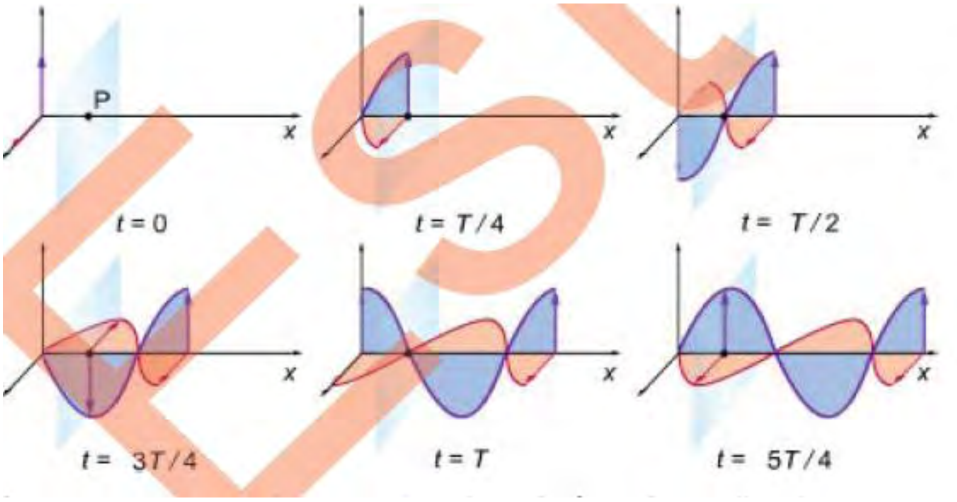
Electromagnetic wave: A wave consisting of electric and magnetic fields oscillating transversely to the direction of propagation.

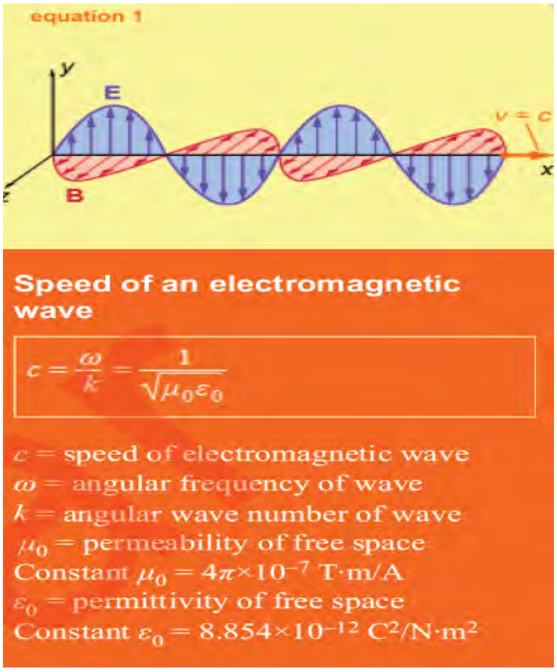
Physicist James Clerk Maxwell's brilliant studies pioneered research into the nature of electromagnetic waves. He correctly concluded that oscillating electric and magnetic fields can constitute a self-propagating wave that he called electromagnetic radiation. His law of induction (a changing electric field causes a magnetic field) combined with Faraday's law (a changing magnetic field causes an electric field) supplies the basis for understanding this kind of wave. As the diagrams to the right show, the electric and magnetic fields in an electromagnetic wave are perpendicular to each other and to the direction of propagation of the wave. These illustrations also show the amplitudes of the fields varying sinusoidally as functions of position and time. Electromagnetic waves are an example of *transverse waves*. The fields can propagate outward from a source in all directions at the speed of light; for the sake of visual clarity, we have chosen to show them moving only along the x axis. The animated diagram in Concept 2 and the illustrations below are used to emphasize three points. First, the depicted wave moves away from the source. For example, if you push the "transmit" button on a walkie-talkie, a wave is initiated that travels away from the walkie-talkie. Second, at any fixed location in the path of the wave, both fields change over time. The wave below is drawn at intervals that are fractions $T/4$ of the period T. Look at the point P below, on the light blue vertical plane. The vectors from point P represent the direction and strength of the electric and magnetic fields at this point. As you can see, the vectors, and the fields they represent, change over time at P. Concept 2 shows them varying continuously with time at the point P.

Third, the diagrams reflect an important fact: The electric and magnetic fields have the same frequency and phase. That is, they reach their peaks and troughs simultaneously. A wave on a string provides a good starting point for understanding electromagnetic waves. Both electromagnetic radiation and a wave on a string are transverse waves. The strengths of the two fields constituting the radiation can be described using sinusoidal functions, just as we can use a sinusoidal function to calculate the transverse displacement of a particle in a string through which a wave is moving. There is a crucial difference, though: Electromagnetic radiation consists of electric and magnetic fields, and does not require a medium like a string for its propagation. Electromagnetic waves can travel in a vacuum. If this is troubling to you, you are in good company. It took some brilliant physicists a great deal of hard work to convince the world that light and other electromagnetic waves do not require a medium of transmission. Furthermore, when electromagnetic waves radiate in all directions from a compact source like an antenna or a lamp, the radiation emitted at a particular instant travels outward on the surface of an expanding sphere, and its strength diminishes with distance from the source. The waves cannot be truly sinusoidal, since the amplitude of a sinusoidal function never diminishes. In the sections that follow we will analyze *plane waves*, which propagate through space, say in the positive *x* direction, in parallel planar wave fronts rather than expanding spherical ones. They are good approximations to physical waves over small regions that are distant from the source of the waves. Plane waves never diminish in strength; they can be accurately modeled using sinusoidal functions, and we will do so.

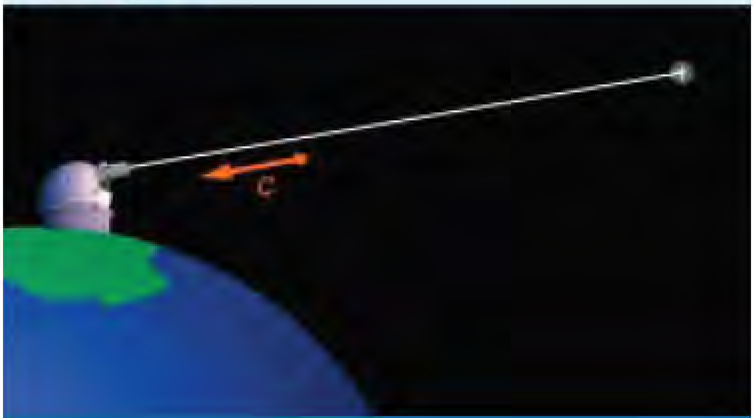



concept 2
P
x
Electric, magnetic fields
Vary in strength over time at each point
Have same frequency and are in phase
Drive each other by changing strength


P
x
$t = 0$
$t = T/4$
$t = T/2$
$t = 3T/4$
$t = T$
$t = 5T/4$


equation 1
y
E
B
$v = c$
x
Speed of an electromagnetic wave
$c = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$
c = speed of electromagnetic wave
ω = angular frequency of wave
k = angular wave number of wave
μ_0 = permeability of free space
Constant $\mu_0 = 4\pi \times 10^{-7}$ T·m/A
ε_0 = permittivity of free space
Constant $\varepsilon_0 = 8.854 \times 10^{-12}$ C²/N·m²

example 1



What is the speed of an electromagnetic wave in a vacuum?

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\mu_0 \varepsilon_0 = (4\pi \times 10^{-7})(8.854 \times 10^{-12})$$

$$\mu_0 \varepsilon_0 = 1.113 \times 10^{-17}\ \mathrm{s^2/m^2}$$

$$\sqrt{\mu_0 \varepsilon_0} = 3.336 \times 10^{-9}\ \mathrm{s/m}$$

$$c = 2.998 \times 10^{8}\ \mathrm{m/s}$$

Creating electromagnetic waves: antennas Radio antennas create electromagnetic waves. A radio antenna is part of an overall system called a radio transmitter that converts the information contained in sound waves into electromagnetic waves. A radio receiver then reverses the process, converting the signals from electromagnetic waves back to sound waves. The system depicted to the right shows the fundamentals of a radio transmitter. In the illustrations, the terminals of an AC generator are connected to two rods of conducting material: an antenna. The AC generator produces an emf □ that varies sinusoidally over time. The emf drives positive

through the surface by the wave, per unit area, is called the *area power density* of the wave. The area power density is equal to the magnitude *S* of the Poynting vector. The surface area through which the instantaneous power density is measured is perpendicular to the direction of the wave's propagation. When radiation reaches a physical surface obliquely, the cosine of its angle with the area vector can be used to calculate the power conveyed to the surface. This is analogous to the calculation of electric or magnetic flux. As Equation 1 shows, the Poynting vector equals the cross product of the vectors representing the electric and magnetic fields of the electromagnetic radiation, divided by the permeability constant. Since these fields are always perpendicular to one another, the sine of the angle between them, used to evaluate the magnitude of the cross product, always equals one, and can be effectively ignored when calculating the instantaneous area power density *S*. The units of the Poynting vector are watts per square meter. The direction of **S** is determined by the right-hand rule. If you apply the rule, wrapping your fingers from **E** to **B** and noting the direction of your thumb, you can correctly determine that it is parallel to the direction of propagation of the wave. When **E** reverses its direction, so does **B**, and the direction of **S** remains the same, "pointing" (heh, heh) in the direction of the wave's motion. As an electromagnetic wave passes through a surface, the strengths of its electric and magnetic fields there change sinusoidally with time. Since the Poynting vector is the product of these fields, it changes sinusoidally over time, as well. In fact, it varies with values between zero and $E_{max}B_{max}/\mu_0$, with a frequency twice that of the fields. If you are curious why it has this frequency, recall from the field equations that *E* and *B* are both cosine functions of time at a fixed point. Then use the trigonometric identity $\cos^2 t = [1 + \cos 2t]/2$.

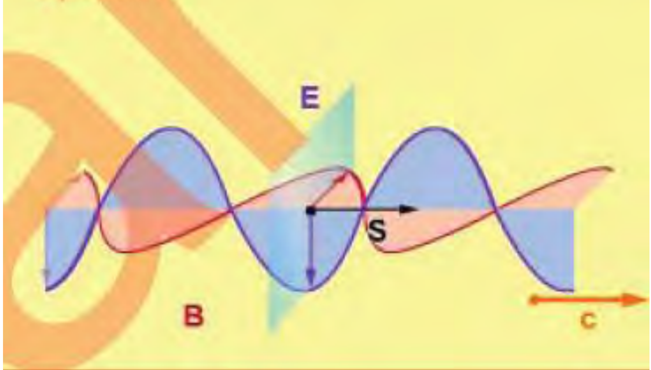
concept 1

Poynting vector

Power per unit surface area

Surface perpendicular to wave direction

equation 1



Poynting vector

$$\mathbf{S} = \frac{1}{\mu_0}\,\mathbf{E} \times \mathbf{B}$$

$\mathbf{S}$ = Poynting vector

S = instantaneous area power density

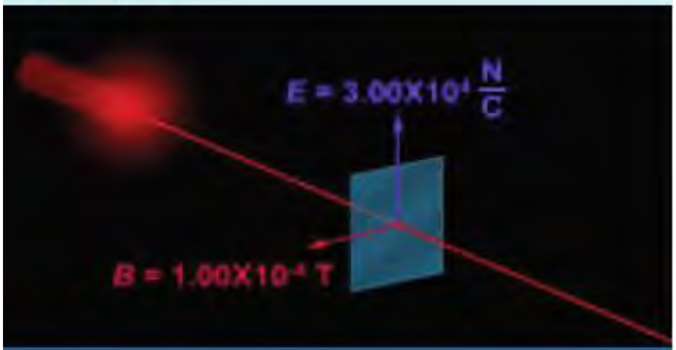
μ_0 = permeability of free space

$\mathbf{E}$ = electric field

$\mathbf{B}$ = magnetic field

Units: watts per square meter (W/m^2)

example 1



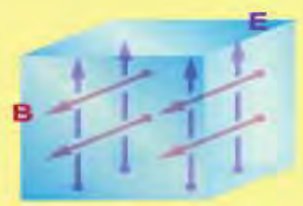
At this instant, what is the area power density of the ruby laser light?

$$S = EB / \mu_0$$

$$S = \frac{(3.00 \times 10^{4} \frac{\text{N}}{\text{C}})(1.00 \times 10^{-4}\ \text{T})}{4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}}$$

$$S = 2.39 \times 10^{6}\ \text{W/m}^2$$

equation 2



Electric, magnetic energy densities

$$u_E = \frac{\varepsilon_0 E^2}{2} \qquad u_B = \frac{B^2}{2\mu_0}$$

Since $E^2/B^2 = c^2 = 1/\mu_0\varepsilon_0$, then

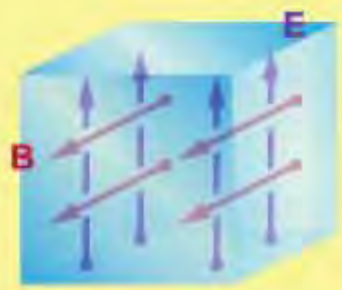
$$u_E = u_B$$

The total energy density is

$$u = u_E + u_B = 2u_E = 2u_B$$

u_E = electric field energy density
u_B = magnetic field energy density
u = total energy density

equation 3



Average energy per unit volume

Average value of u over time

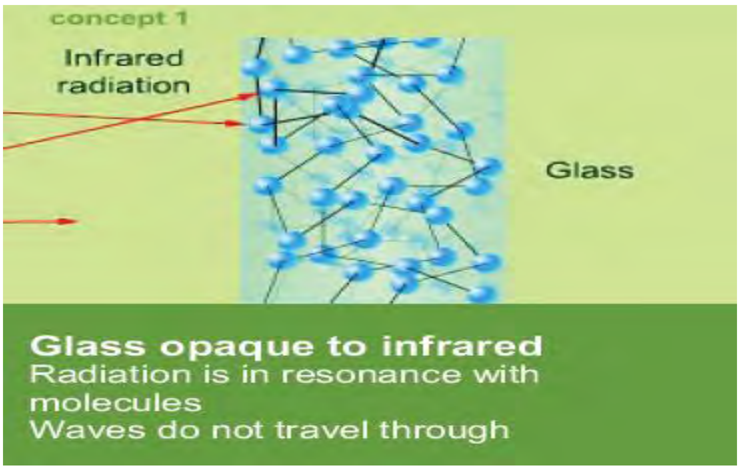
$$u_{avg} = \frac{\varepsilon_0 E_{max}^2}{2}$$

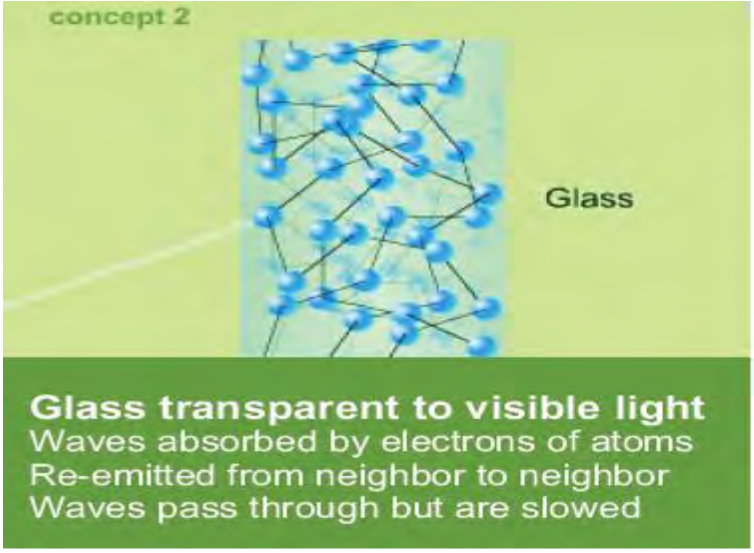
u_{avg} = average total energy density

Units: joules per cubic meter (J/m^3)

How electromagnetic waves travel through matter

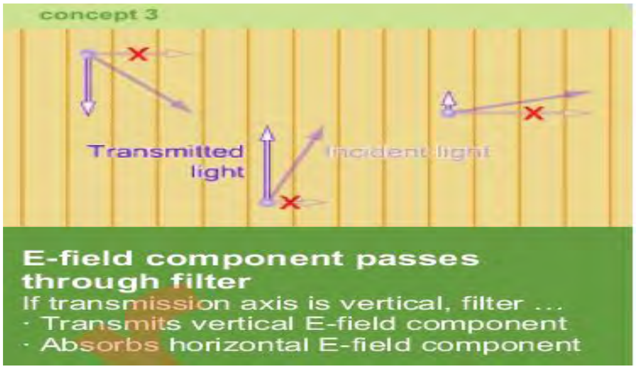
Light and other forms of electromagnetic radiation can travel through a vacuum, and it is often simplest to study them in that setting. However, radiation can also pass through matter: If you look through a glass window, you are viewing light that has passed through the Earth’s atmosphere and the glass. Other forms of radiation such as radio waves pass through matter, as well. This section focuses on how such transmission occurs. It relies on a classical model of electrons and atoms that predates quantum theory. In this model, electrons orbit an atom. They have a resonant frequency that depends on the kind of atom. On a larger scale, atoms themselves and the molecules composed of them also have resonant thermal frequencies at which they can vibrate or rotate. We will use the example of light striking the glass in a window to discuss how substances transmit (or do not transmit) electromagnetic radiation. When an electromagnetic wave encounters a window, it collides with the molecules that make up the glass. If the frequency of the wave is near the resonant thermal frequency of the glass molecules, which is true for infrared radiation, the amplitude of the molecules' vibrations increases. They absorb the energy transported by the wave, and dissipate it throughout the glass by colliding with other molecules and heating up the window. Because it absorbs so much infrared energy, the glass is opaque to radiation of this frequency, preventing its transmission. Scientists in the 19th century noted a phenomenon in greenhouses caused by the opaqueness of glass to infrared radiation, which they called the *greenhouse effect*. The glass in a greenhouse admits visible light from the Sun, which is then absorbed by the soil and plants inside. They reradiate the solar energy as longer infrared waves, which cannot pass back out through the glass and so help warm up the greenhouse. The same phenomenon occurs on a vaster scale in the atmosphere as gases like methane and carbon dioxide trap solar energy near the Earth's surface. In contrast to infrared radiation, higher frequency radiation such as visible light does not resonate thermally with atoms or molecules, but may resonate with the electrons of the atoms of a substance. In glass, visible light experiences much less reduction in the amplitude of its waves than infrared radiation does, and most of its energy passes through the glass quite easily. Atoms with resonant electrons that do absorb energy from a light wave quickly pass on that energy by re-emitting it as radiation of the same frequency to other atoms, which in turn pass it on to their neighbors. This chain of absorptions and re-emissions, called *forward scattering*, follows a path close to the light's original direction of travel. A beam of light that strikes a pane of glass will reach the "last atom" on the far side of the pane in an extremely short time. We see the light after it emerges, and think of glass as transparent.





Radiation also can be *partially polarized*, having a few waves oscillating in all planes, but with most of its waves concentrated in a single plane. This is true of sunlight scattered by the atmosphere. As the photo above shows, the sky in certain directions is partially polarized in a vertical plane so that most of its light can pass through a pair of sunglasses whose transmission axis is vertical. Less light (but still some) passes through the rotated sunglasses. (Polarizing sunglasses are specifically intended to reduce horizontally polarized glare reflected from roadways and water, not skylight.)

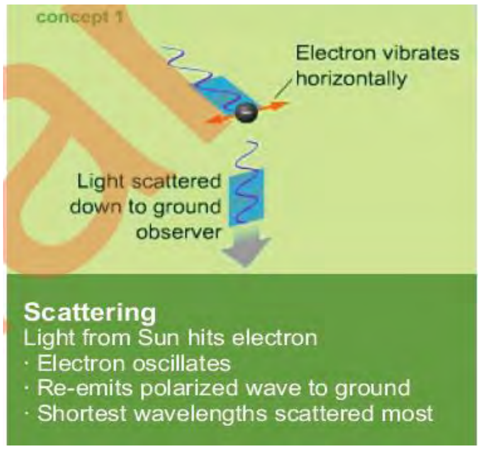
Many forms of artificial electromagnetic radiation are polarized. A radio transmitter emits polarized radiation. If the rods of its antenna are vertical, then so is the electric field of every radio wave it creates. In this case, the most efficient receiving antenna is also vertically oriented; a horizontal receiving antenna would absorb radio waves much less efficiently. You may be familiar with this fact if you have ever tried to maneuver a radio antenna wire or a set of television “rabbit ears” to get the best reception. (If you do not know what “rabbit ears” are for television, well, before there was cable television, there was….)



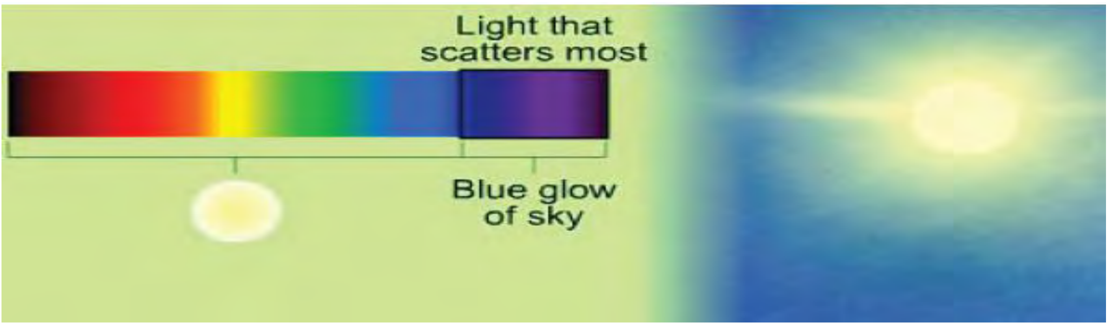
Scattering: Absorption and re-emission of light by electrons, resulting in dispersion and some polarization.

The answer to a classic question Why is the sky blue? rests in a phenomenon called scattering. In this section, we give a classical (as opposed to quantum mechanical) explanation of how scattering occurs. When light from the Sun strikes the electrons of various atoms in the Earth's atmosphere, the electrons can absorb the light's energy, oscillating and increasing their own energy. The electrons in turn re-emit this energy as light of the same wavelength. In effect, the oscillating electrons act like tiny antennas, emitting electromagnetic radiation in the frequency range of light.

An electron oscillates in a direction parallel to the electric field of the wave that energizes it, as shown in Concept 1. The electron then emits light polarized in a plane parallel to its vibration. We show a particular polarized wave that is re-emitted downward toward the ground, since we are concerned with what an observer on the surface of the Earth sees. Other light is scattered in other directions, including light scattered upward and light scattered forward in its original direction of travel. Scattering explains why we see the sky: Light passing through the atmosphere is redirected due to scattering toward the surface of the Earth. In contrast, for an astronaut observer in the vacuum of space, sunlight is not scattered at all so there is no sky glow: Except for the stars, the sky appears black. To the astronaut, the disk of the Sun, a combination of all colors, looks white. We illustrate this below: The full spectrum combines to form white light. The question remains, why is our sky blue rather than some other color? Light at the blue end of the visible spectrum, which has the shortest wavelength, is 10 times more resonant with the electrons of atmospheric atoms than red light. This means blue light is scattered more than red, so that more of it is redirected toward the ground. Scattering also explains why we see the Sun as yellow rather than white. When you look up at the disk of the Sun from the Earth's surface, the bluest portion of its light has been scattered away to the sides. The remaining part of the Sun's direct light appears somewhat yellowish. You may also have noted how the Sun appears to change color when it sets. As the Sun's disk descends toward the horizon, its light must pass through a greater and greater thickness of atmosphere in order to reach you. Since a certain amount of sunlight is scattered aside for each kilometer of atmosphere it passes through, its position at sunset causes it to lose large amounts of light at the blue end and even toward



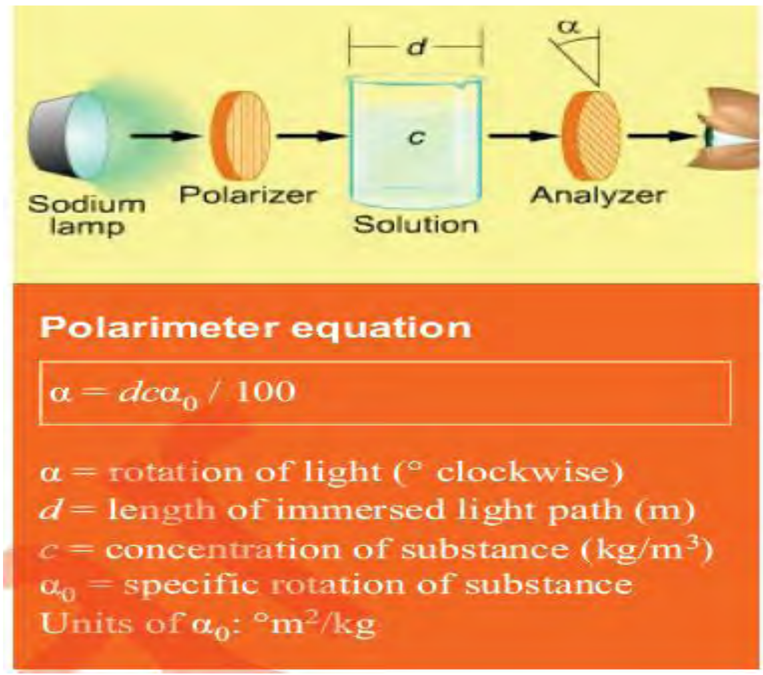
View from space
No scattering: sky is black
Sun appears white



Why the sky is blue (and the Sun is yellow)
Shortest waves are scattered: sky is blue
With blue scattered, Sun appears yellowish white

the concentration of the dissolved substance. The rotation is also proportional to a constant □0 called the *specific rotation* of the substance, which reflects the rotating power of its molecules. These relationships are summarized to the right in the *polarimeter equation*. Note that (and this is unusual for a physics equation) the rotation angle □ is measured in degrees rather than radians, and the clockwise direction is considered positive. The *polarimeter* is a device that can be used to measure the net rotation of polarized light passing through an optically active solution. An experimenter directs polarized light through a container of the solution to be analyzed. The analyzer, which starts out parallel to the polarizer, does not transmit all the light from the polarizer because the light's plane of polarization has been rotated by the solution. The experimenter turns the analyzer to one side or the other until the transmitted light has maximum brightness. Then she knows that the analyzer's transmission axis matches the rotated polarized light, and she can measure the angle □ through which the analyzer has turned. The polarimeter equation gives an expression for the angle □ of the analyzer at which the transmitted light will be the brightest. If the polarized light encounters more molecules of the optically active substance, either because the solution is more concentrated or because the immersed light path is longer, the rotation will be greater. Since the amount of rotation also depends on the wavelength of the light, the specific rotations □0 given in tables for particular dissolved substances are based on a polarimeter employing the 589 nm light that is emitted by a sodium vapor lamp. Dextrose and fructose molecules are chemically identical (they have the same atoms arranged in the same pattern) but they are mirror images of each other. Because of this they rotate polarized light by the same amount in opposite directions. Organic molecules such as *carvone* may exist in two mirror image forms; you smell carvone as caraway or spearmint, depending on which way the molecule twists. The scents are different because the smell receptors in the nose react differently to the mirror image forms. Using a polarimeter is one way to distinguish between the two forms of mirror image compounds. Also, if the specific rotation of a particular substance is known, the device can be used together with the polarimeter equation to determine the concentration of the substance in a solution. You are asked to perform such an analysis in the example problem to the right.

The diagrams below show the mirror image molecular forms of the citrus oil *limonene*, which is the essence of either orange or lemon, depending on the orientation of its molecules! (The gray spheres represent carbon atoms, and the blue spheres are hydrogen atoms.)



Carvone's specific rotation is +62.5 (caraway) or –62.5 (spearmint). What is the concentration of the carvone in this beaker?

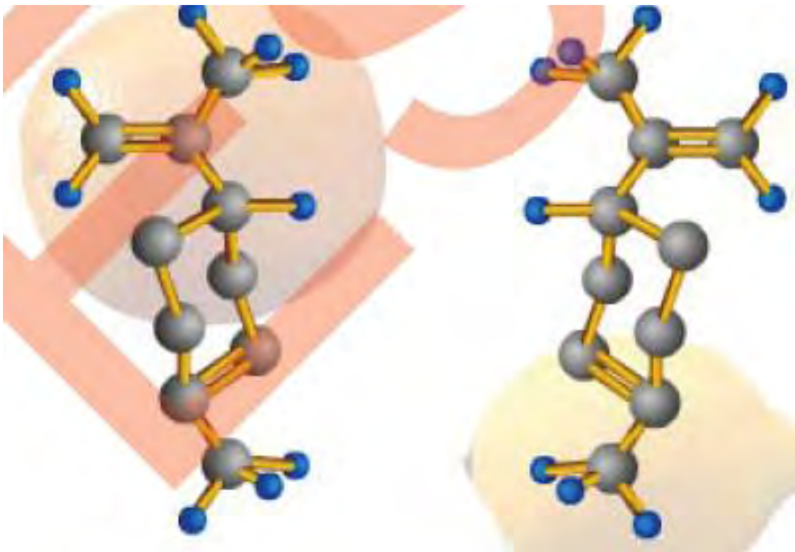
$\alpha = dc\alpha_0 / 100$

$$c = \frac{100\alpha}{d\alpha_0}$$

Counterclockwise rotation means spearmint, so we use $\alpha_0 = -62.5$:

$$c = \frac{(100)(-35°)}{(0.12\ \text{m})(-62.5\ °\text{m}^2/\text{kg})}$$

$c = 470\ \text{kg/m}^3$



Equations

Proportionality of fields

$$\frac{E}{B} = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Poynting vector

$$\mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B}$$

Intensity of electromagnetic radiation

$$I = S_{avg} = \frac{E_{max}^2}{2\mu_0 c} = \frac{E_{rms}^2}{\mu_0 c}$$

$$I = \frac{P}{4\pi r^2}$$

Energy density

$$u_E = \frac{\varepsilon_0 E^2}{2}, \quad u_B = \frac{B^2}{2\mu_0}$$

$$u_E = u_B$$

$$u = u_E + u_B = 2u_E = 2u_B$$

$$u_{avg} = \frac{\varepsilon_0 E_{max}^2}{2}$$

Momentum transferred by radiation absorption

$$\Delta p = \frac{\Delta U}{c} \quad \text{for a blackbody}$$

Reflection: Light "bouncing back" from a surface.

When you look at yourself in a mirror, you are seeing a reflection of yourself. When you look at the Moon at night, you are seeing sunlight reflecting off that distant body. Not all the light that reaches a surface reflects. In fact, you see an object like a tree as having different colors because its varied parts reflect some wavelengths of light and absorb others. Light can pass through a material, as it does with a glass window. It can also be absorbed by a material, as evidenced by how a black rock warms up during a sunny day. All this can happen simultaneously: Light will reflect off the surface of a lake (which is why you see the lake), penetrate the water (otherwise, it would be completely dark below the surface), and be absorbed by the water, warming it. To understand reflection, it is often useful to treat light as a stream of particles that move in a straight line and change direction only when they encounter a surface. Each light "particle" acts like a ball bouncing off of a surface, and like a ball, it reflects off the surface at a rebound angle equal to its incoming angle. You see yourself in a mirror because the light bounces back to your eyes from the mirror. The term "reflection" likely conjures up images of light and perhaps mirrors. Studying mirrors is a good way to learn about reflection because they are designed to reflect light in a way that creates a clear visual image. However, it is worth noting that reflection does not apply only to light. Some creatures use the reflection of sound (echoes) to help them perceive their surroundings and stalk their prey. For example, bats, seals and dolphins emit high frequency sound and then listen for the reflected waves. By analyzing these reflections, they can "see" with great precision. *Radar*, used to track airplanes, is based on the reflection of radio waves. A sophisticated understanding of reflection can be used to design "stealth" aircraft that are difficult to detect with radar. Stealth aircraft register on radar screens as being about as large as a BB, in part because of their ability to reflect incoming waves in "random" directions.

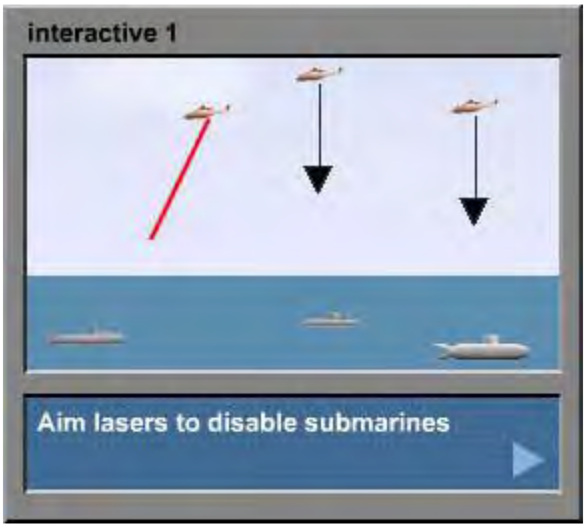
Light can refract

□ change direction □ as it moves from one medium to another. For instance, if you stand at the edge of a pool and try to poke something underwater with a stick, you may misjudge the object's location. This is because the light from the object changes direction as it passes from the water to the air. You perceive the object to be closer to the surface than it actually is because you subconsciously assume that light travels in a straight line. Although refraction can cause errors like this, it can also serve many useful purposes. Optical microscopes, eyeglass lenses, and indeed the lenses in your eyes all use refraction to bend and focus light, forming images and causing objects to appear a different size or crisper than they otherwise would. Where a lens focuses light, and whether it magnifies an object, is determined by both the curvature of the lens and the material of which it is made. Scientists have developed quantitative tools to determine the nature of the images created by a lens. We will explore these tools thoroughly later, "focusing" first, so to speak, on the principle of refraction underlying them. To begin your study of refraction, try the simulation to the right. Each of your helicopters can fire a laser □ a sharp beam of light □ at any of three submarines lurking under the sea. The submarines have lasers, too, and will shoot back at your craft. Your mission is to disable the submarines before they disarm your helicopters. When you make a hit, you can shoot again. Otherwise, the submarines get their turn to shoot until they miss. You play by dragging the aiming arrow underneath any one of your helicopters. Press FIRE and the laser beam will follow the direction of this arrow until it reaches the water, where refraction will cause the beam to change direction. In addition to hitting the submarines before they get you, you can conduct some basic experiments concerning the nature of refraction. As with reflection, the angle of incidence is measured from a line normal (perpendicular) to a surface. In this case, the surface is the horizontal boundary between the water and the air. Observe how the light bends at the boundary when you shoot straight down, at a zero angle of incidence, or grazing the water, at a large angle of incidence. You can create a large angle of incidence by having the far right helicopter, for example, aim at the submarine on the far left. You can also observe how refraction differs when a laser beam passes from air to water (your lasers) and from water to air (the submarines' lasers). Observe the dashed normal line at each crossover point and answer the following question: Does the laser beam bend toward or away from that line as it changes media? You should notice that the laser beams of the submarines behave differently than those of the helicopters when they change media. As a final aside:

You may see that some of the laser beams of the submarines never leave the water, but reflect back from the surface between the water and the air. This is called total internal reflection.

Refraction: The change in the direction of light as it passes from one medium to another.

A material through which light travels is called a *medium* (plural: *media*). When light traveling in one medium encounters another medium, its direction can change. It can reflect back, as it would with a mirror. It can also pass into the second medium and change direction. This phenomenon, called refraction, is shown to the right. In the photo, a beam of light from a laser refracts (bends) as it passes from the air into the water. Light refracts when its speeds in the two media are different. Light travels faster through air than in water, and it changes direction as it moves from air into water, or from water into air. Although we are primarily interested in the refraction of light, all waves, including water waves, refract. Above, you see a photograph of surf wave fronts advancing parallel to a beach. Deep-ocean swells may approach a coastline from any angle, but they slow down as they encounter the shallows near the shore. The parts of a wave that encounter the shallow water earliest slow down first, and this causes the wave to refract. Sound waves can also refract. During a medical ultrasound scan, an acoustic lens can be used to focus the sound waves. The lens is made of a material in which sound travels faster than in water or body tissues. The surface between two media, such as air and water, is called an *interface*. As with mirrors, light rays are often used to depict how light refracts when it meets an interface. Lasers are often used to demonstrate refraction because they can create thin beams of light that do not

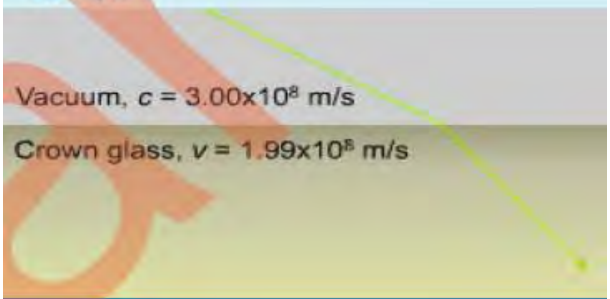


equation 2

	Index of refraction
Air	1.0003
Water	1.33
Vegetable oil	1.47
Crown glass	1.51
Salt	1.54
Flint glass	1.61
Corundum (ruby, sapphire)	1.77*
Diamond	2.42
At 20° C, λ = 589 nm	*Approximate value

Indices of refraction

example 1

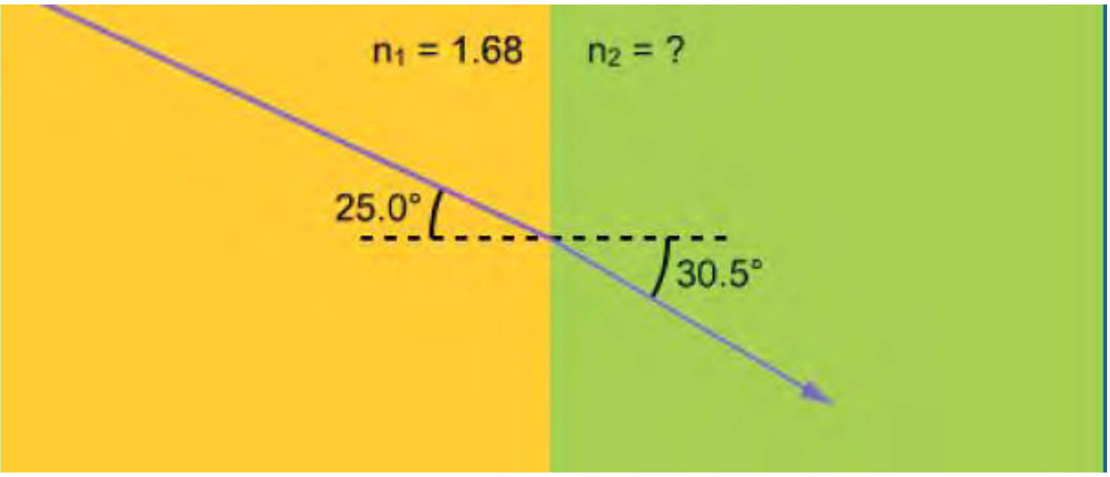


Green light travels at 1.99×10^8 m/s in crown glass. What is the index of refraction of the glass for this light?

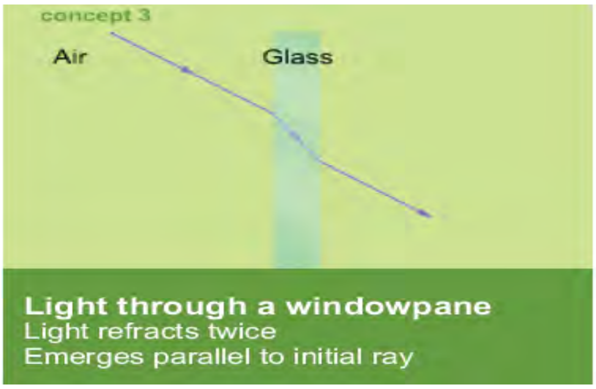
$$n = \frac{c}{v}$$

$$n = \frac{3.00\times10^8 \text{ m/s}}{1.99\times10^8 \text{ m/s}}$$

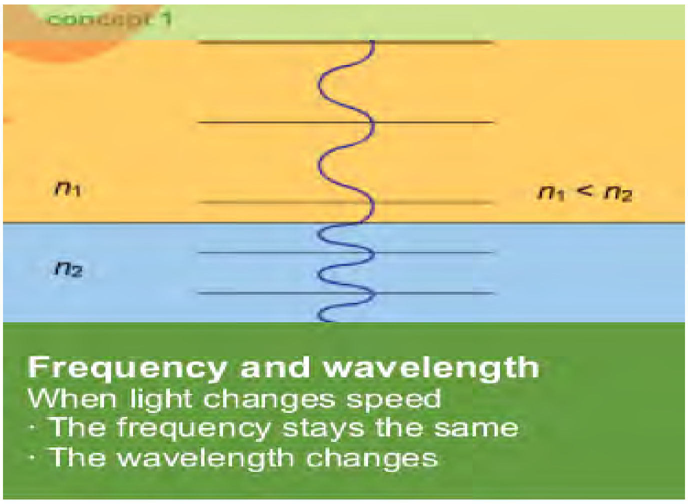
$$n = 1.51$$

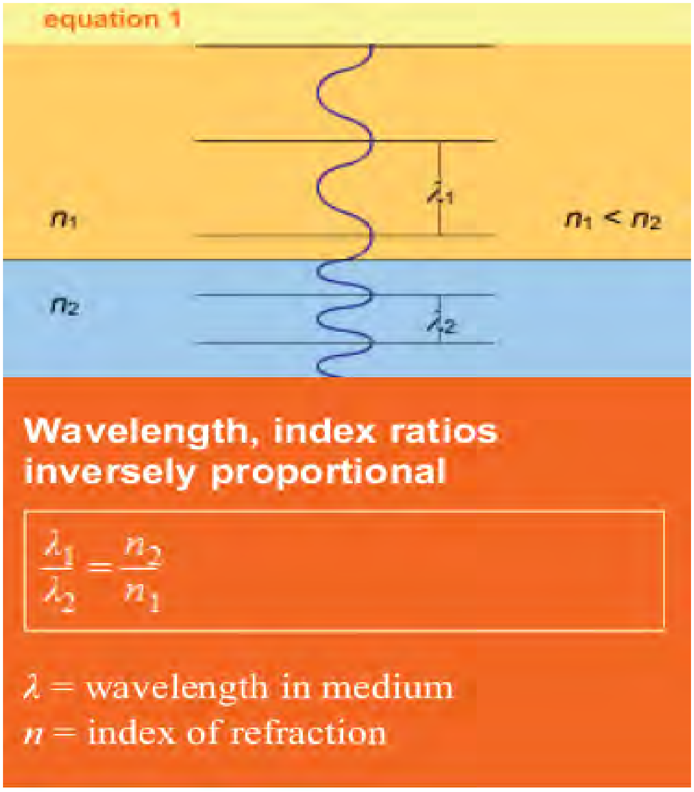


What is the index of refraction of the second material?



Wavelength of light in different media When light changes speed as it moves from one medium to another, its frequency stays the same but its wavelength changes. The ratio of its wavelengths in the two media is the inverse of the ratio of the indices of refraction. We show this as an equation to the right and derive it below. Before deriving the equation, let's consider why the frequency stays the same, since this is an essential part of the derivation. The frequencies in the media must be the same, because if they were not, waves would either pile up at the interface or be destroyed. Neither occurs. You can witness this at the beach, where wave speed and wavelength may change as waves approach the beach, but the frequency of the waves does not change.





Variables

In this derivation, c represents the speed of light in a vacuum. For the other two media we define the variables in the following table:

	medium 1	medium 2
light speed	v_1	v_2
frequency	f_1	f_2
wavelength	λ_1	λ_2
index of refraction	n_1	n_2

Strategy

1. Use the equality of frequencies in the two media together with the wave speed equation to obtain a proportionality of the light speeds and wavelengths in the media.
2. Use the definition of the index of refraction to convert the previous proportion to one involving wavelengths and indices of refraction.

Physics principles and equations

The wave speed equation states that for any wave, the speed is the product of the wavelength and the frequency:

$$v = \lambda f$$

As a wave passes from one medium to another, its speed and wavelength may change, but its frequency must remain the same. The definition of the index of refraction of a medium is

$$n = \frac{c}{v}$$

Step-by-step derivation

we explain the diagram you see above. The purple line is a light ray refracting at an interface. In the diagram, light travels more slowly in the lower medium than the upper. This could represent, for example, light passing from air into water. The gray lines perpendicular to the ray represent wave fronts. You see the wavelength labeled as □ (□i in the upper medium, □r in the lower medium). There are two right triangles in the diagram that share the hypotenuse labeled *x*. The bright yellow triangle shows elements of a wave front that has not yet entered the lower medium. The dark orange triangle shows elements of a wave front that is now traveling in the lower medium. The angles of incidence and refraction □i and □r are also shown in the diagram.

Because the wave fronts are perpendicular to the light rays, we can identify angles in each of the triangles that are equal to □i and □r. These base angles are shown in the diagram.

Variables

In this derivation, *x* represents the common hypotenuse of the two triangles in the diagram. For the incident and refractive media we define the variables in the following table.

	incident medium	refractive medium
angle	θ_i	θ_r
wavelength	λ_i	λ_r
index of refraction	n_i	n_r

Strategy

1. Consider the two triangles in the diagram. State the sines of their base angles □i and □r as trigonometric ratios of the triangles' sides.
2. Construct the ratio sin □i / sin □r. The common hypotenuse *x* will cancel out, leaving a ratio of wavelengths.
3. Restate the ratio of wavelengths as a ratio of indices of refraction to obtain Snell's law.

Physics principles and equations

The ratio of the wavelengths is inversely proportional to the ratio of the indices of refraction.

$$\frac{\lambda_i}{\lambda_r} = \frac{n_r}{n_i}$$

Step-by-step derivation

We construct the fraction sin □i / sin □r, and calculate the sines as the ratios of the sides of triangles. This leads to a ratio of wavelengths that can be replaced by a ratio of indices of refraction, yielding Snell's law.

Step	Reason
1. $\sin\theta_i = \lambda_i / x$, $\sin\theta_r = \lambda_r / x$	definition of sine
2. $\dfrac{\sin\theta_i}{\sin\theta_r} = \dfrac{\lambda_i/x}{\lambda_r/x}$	ratio using definition of sine
3. $\dfrac{\sin\theta_i}{\sin\theta_r} = \dfrac{\lambda_i}{\lambda_r}$	simplify
4. $\dfrac{\lambda_i}{\lambda_r} = \dfrac{n_r}{n_i}$	change of wavelength
5. $\dfrac{\sin\theta_i}{\sin\theta_r} = \dfrac{n_r}{n_i}$	substitute equation 4 into equation 3

Light is a particle.

Many of the great scientists of the 17th and 18th centuries who made fundamental contributions to the study of optics, including Isaac Newton, thought that light consisted of a stream of "corpuscles," or particles. In the 20th century, Albert Einstein explained the photoelectric effect. His explanation, for which he was awarded the 1921 Nobel Prize, depended on the fact that light acts like a particle. This property of light led to the coining of the term "photon" for a single particle of light by the chemist Gilbert Lewis.

Light is a wave.

Between the 18th and 20th centuries, physicists discovered many wave-like properties of light. They found that a number of phenomena they routinely observed with water waves they could also observe with light. For instance, the English scientist Thomas Young (1773-1829) showed that light could produce the same kinds of interference patterns that water waves produce. At the right, you see examples of interference patterns formed by light and by water waves. The similarities are striking. In this chapter, you will apply to light some of what you have studied about the interference of sound waves and traveling waves in strings.

Let there be light.

Is light a particle, a wave, or both? Perhaps an Early Authority had it right. Light is light. It is a combination of electric and magnetic fields. Trying to classify light as a particle or as a wave may be a fruitless effort □ better to revel in its unique properties.

In this chapter, we will revel in its wave-like properties, and discuss the topic of interference. Your prior study of electromagnetic radiation modeled as a wave phenomenon will prove useful.

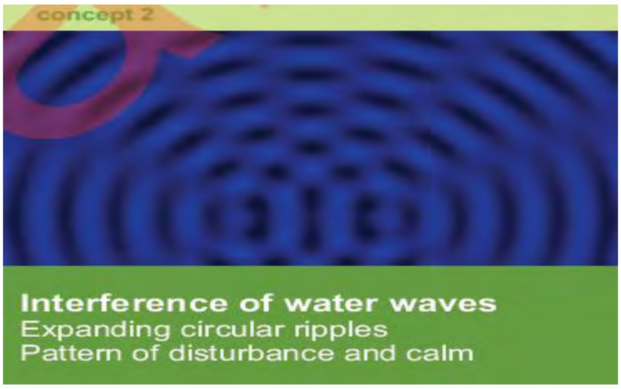
Interference

In Concept 1, you see an *interference pattern* created by causing a beam of light to pass through two parallel slits to illuminate a viewing screen. Constructive interference of light waves accounts for the bright regions (called bright *fringes*) while destructive interference causes the dark fringes. In this section, we review some of the fundamentals of interference, and discuss the conditions necessary for light to make the pattern you see to the right. You may have already studied the interference of mechanical waves; for instance, what occurs when two waves on a string interact. In this chapter, you will study what happens when electromagnetic waves meet. Some of the same principles and terminology are used in discussing both kinds of interference.

When two light waves meet, the result can be constructive or destructive interference. In the following discussion, we assume that the waves have equal amplitude. Constructive interference creates a wave of greater amplitude and more intensity than either source wave; destructive interference results in a wave of smaller amplitude and less intensity than either source wave. At any point in a two-slit interference pattern such as that to the right, light waves from the two sources meet and interfere constructively, destructively, or partially (exhibiting a degree of interference somewhere between complete constructive and destructive interference). To create an interference pattern, a physicist needs light that is:

1. *Monochromatic*. This means light with a specific wavelength. For instance, experimenters can produce the pattern you see in Concept 1 by using pure red light.

2. *Coherent*. This means the phase difference between the light waves arriving at



The pattern of bright and dark fringes extends to both the left and the right on the screen. The light is interfering constructively at the bright fringes, and destructively at the dark fringes, because of different path lengths to these regions and the resulting phase differences. There are a few limitations to showing Young's apparatus in a compact diagram. First, the diagram is far from being drawn to scale. The screen should be much farther from the double-slit barrier than we show here, and the slits should be narrower and closer together. In actual interference experiments, the interfering rays from the two slits are practically parallel. Second, we vastly exaggerate the wavelength of the light. You may have a question about what you would see if you conducted this experiment yourself. What if, at some instant, two waves meet at the screen and are in phase, but their electric and magnetic fields both happen to be zero at that point? Would you see "flickering" as the two reinforcing waves moved from peak to trough and back again? The answer is no: The frequency of light is so great that you only perceive the average brightness of a region; the human eye does not perceive changes in intensity due to the oscillation of a light wave. You do not even perceive flicker in systems oscillating at far lower frequencies, much less than the frequency of visible light, which is on the order of 1014 Hz. For example, a computer monitor refreshes its display 60 times a second, but you do not ordinarily perceive any flicker when you look at it.