

Lecture No.3: Fluid motion in the presence of solid particles; Fixed & Fluidized Beds

1. Relative motion between a fluid and a single particle

The relative motion is considered the following cases are covered:

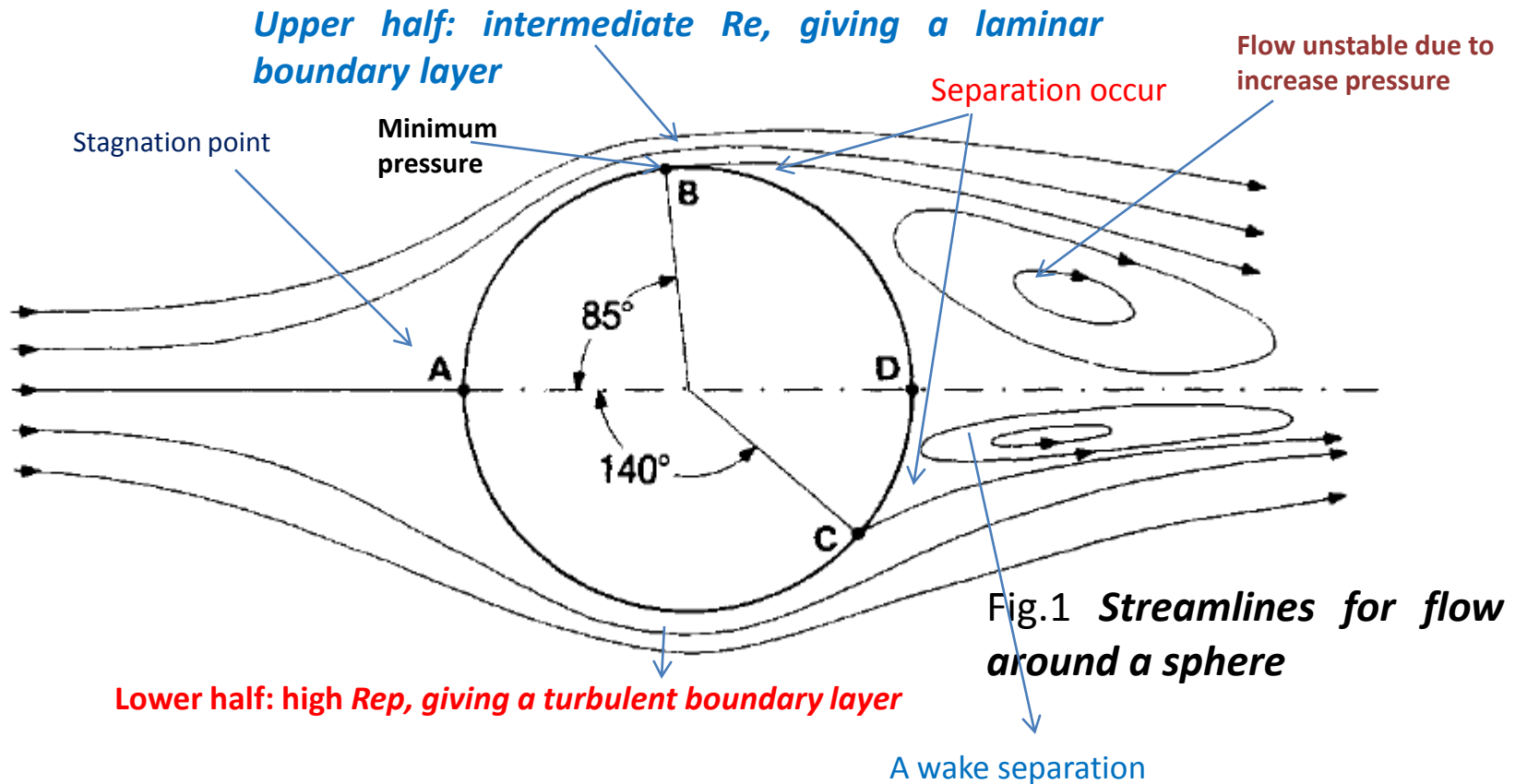
- a stationary particle in a moving fluid;
- a moving particle in a stationary fluid;
- a particle and a fluid moving in opposite directions;
- a particle and a fluid both moving in the same direction but at different velocities.

Assume flow around a spherical particle of diameter d_p , as shown in Fig.1 the appropriate definition of the Reynolds No. is :

$$Re_p = \frac{\rho u_p d_p}{\mu} \quad \text{—————} \quad (1)$$

where u_p is the speed of the particle relative to the fluid.

ρ density of fluid, and μ viscosity of fluid



Note: The whole of the wake is a region of relatively low pressure, very close to that at the point of separation, and much lower than the pressure near point A. Therefore the force arising from this pressure difference is known as form drag because it is due to the (bluff) shape of the particle. Then the total drag force is a combination of:

- Skin friction and
- Form drag

2. Terminal settling or Falling velocity of the Particle u_p u_t

Consider a spherical particle as shown in Fig.2 of diameter d_p and density ρ_p falling with a velocity u_p under the influence of gravity in a fluid of density ρ . The net gravitational force F_1 on the particle (gravity force F_g -buoyancy F_b Archimedes' principle= F_1) is given by the equation:

$$F_1 = \frac{\pi d_p^3}{6} (\rho_p - \rho) g \quad (2)$$

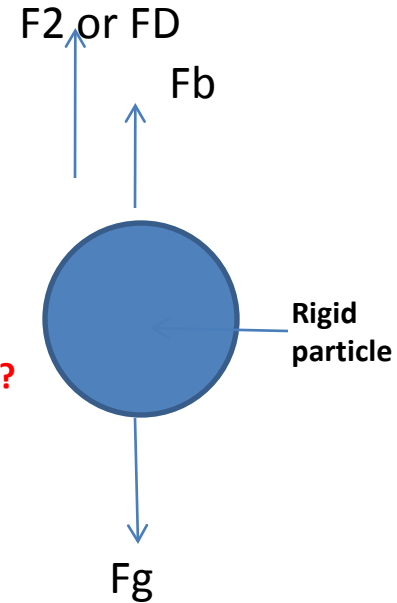
↑
volume of spherical particle

The retarding force or drag force F_2 on the particle from the fluid is given by:

$$F_2 = C_d S_p \frac{\rho u_p^2}{2} \quad (3) \text{ Is derived by ?}$$

↙
dimensionless drag coefficient is similar to the friction factor in pipe

↘
projected area $S_p = \pi d_p^2 / 4$



For steady flow the $F_1 = F_2$ and F_2 and opposite and the particle reaches a constant speed *ut*. Eqs. 2&3 can be combined and written as:

$$\frac{\pi d_p^3}{6} (\rho_p - \rho) g = C_d S_p \frac{\rho u_t^2}{2} \quad (4) \quad \rightarrow \quad u_t = \sqrt{\frac{4 d_p (\rho_p - \rho) g}{3 C_d \rho}} \quad (5)$$

↑
terminal velocity or falling velocity

3. Calculation Drag Coefficient Cd:

A-For $Re_p < 0.2$ (laminar flow or creeping flow, Stokes' region)

the drag coefficient C_d is a function of the Reynolds number. For the streamline flow range of Reynolds numbers, $Re_p < 0.2$, the drag force F_2 is given by:

$$F_2 = 3\pi d_p \mu u_t \quad (6)$$

Therefore Eq.3=Eq.6 to calculate C_d

$$C_d S_p \frac{\rho u_p^2}{2} = 3\pi d_p \mu u_t \quad (7)$$

Therefore sub. Eq.7 in Eq.5 to calculate u_t

$$u_t = \frac{d_p^2 (\rho_p - \rho) g}{18\mu} \quad (8)$$

Eq.8 is called Stokes' Equation, apply for Laminar or creeping flow only, it can be used for $Re_p < 2$

B-For $0.2 < Re_p < 500$ or $2 < Re_p < 500$ (Intermediate region or Schiller region)

$$C_d = \frac{24}{Re_p} (1 + 0.15 Re_p^{0.687}) \quad (9) \quad 0.2 < Re_p < 500,$$

$$C_d = \frac{18.5}{Re_p^{0.6}} \quad (10) \quad 2 < Re_p < 500$$

C- For Newton s' region, $500 < Re_p < 200\,000$, C_d constant=0.44

$$C_d = 0.44 \quad (11)$$

Note :1-When the Reynolds number *Rep* reaches a value of about 300000, transition from a laminar to a turbulent boundary layer occurs and the point of separation moves towards the rear of the sphere as discussed above. As a result, the drag coefficient suddenly falls to a value of **0.10** and remains constant at this value at higher values of *Rep*.

2-For the most part, solid particles in fluid streams have Reynolds numbers which are much lower than 500.

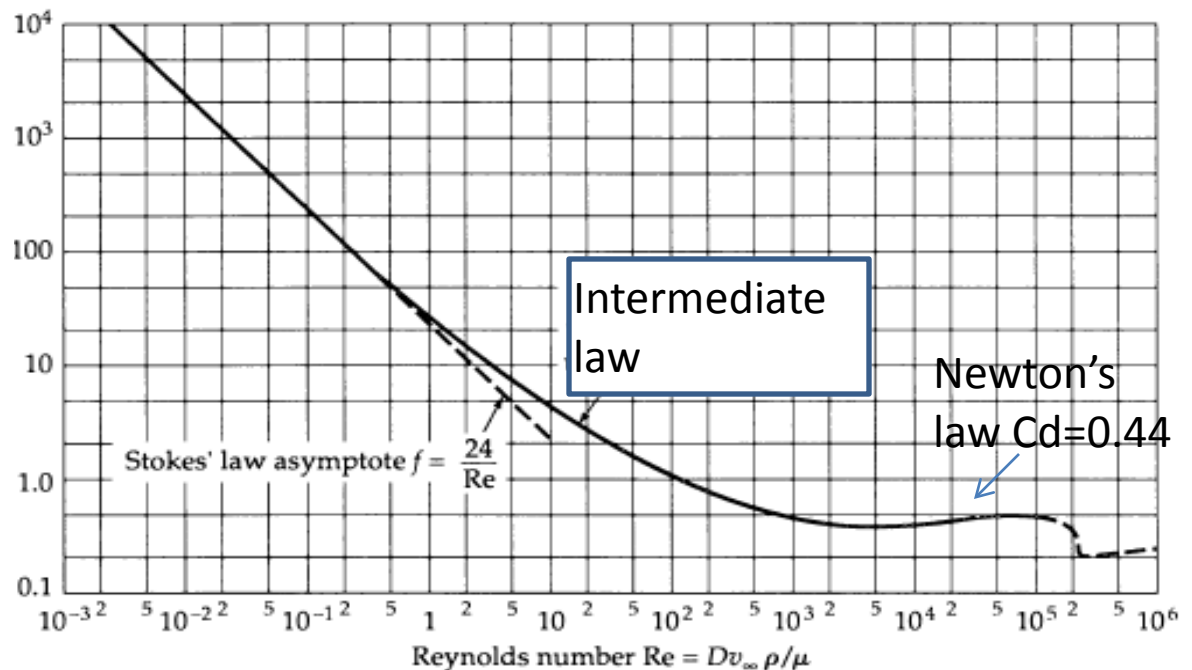
From above in A, B, & C the relationship between Cd & Rep can be drawn in the following Fig. (note: similar with friction factor with Re in pipe flow:-

NOTE :Eq. 5 is derived for spherical particle but if non spherical shape then Eq5 can be written as following Cd form:

$$u_t = \sqrt{\frac{4d_p \psi (\rho_p - \rho) g}{3C_d \rho}} \quad (12)$$

ψ =shape factor

ψ =1 for spherical shape



Example:1 Air at 37.8 C and 101.3 Kpa absolute pressure at a velocity of 23m/s past a sphere having a $d_p=24\text{mm}$. What are the drag coefficient and the force on the sphere? $\mu_{\text{air}}=1.9 \times 10^{-5}$, $\rho=1.137 \text{ Kg/m}^3$.


Sol: Calculate $Re_p = \rho u_p d_p / \mu = 57810$  Newton's region from Fig.

$C_d = 0.44$,

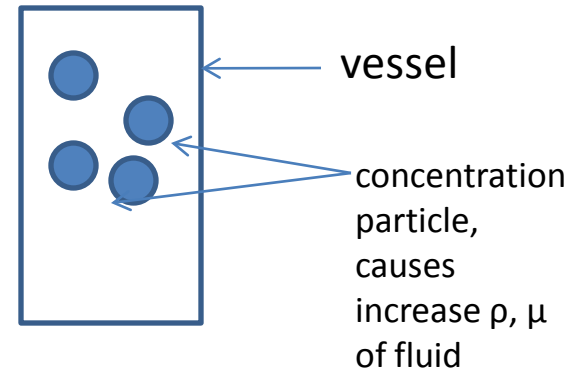
The drag force $F_2 = C_d S_p \frac{\rho u_p^2}{2}$

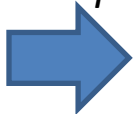
$S_p = \pi d_p^2 / 4$ $F_2 = ? \text{ N}$



4. Relative motion between a fluid and a concentration of particles

So far the relative motion between a fluid and a single particle has been considered. This process is called  **free settling.**

When a fluid contains a concentration of particles in a vessel, as shown in Fig the settling of an individual particle may be hindered by the other particles and by the walls. When this is the case, the process is called **hindered settling.**



Richardson and Zaki showed that in the Reynolds number range $Re_p < 0.2$, the velocity u_c of a suspension of coarse spherical particles in water relative to a fixed horizontal plane is given by the equation:  $\frac{u_c}{u_t} = \epsilon^{4.6}$ (13)

u_t  terminal settling velocity of single particle
 ϵ  voidage fraction of the suspension

Relation Between Two Particles in Fluid:

i. **If the same diameter $d_{p1}=d_{p2}$**

but of different densities settling freely in a fluid of density ρ in the streamline Reynolds number range $Re_p < 0.2$. The ratio of the terminal settling velocities u_{t1}/u_{t2} is given by Eq. 8 rewritten in the form:

$$\frac{u_{t1}}{u_{t2}} = \frac{\rho_{p1} - \rho}{\rho_{p2} - \rho} \quad \text{_____} \quad (14)$$

ii. **If the same density $\rho_{p1}=\rho_{p2}$**


But of different diameters settling freely in a fluid of density ρ in the streamline Reynolds number range $Re_p < 0.2$, the ratio of the terminal settling velocities u_{t1}/u_{t2} is given by Eq.8 rewritten in the form:

$$\frac{u_{t1}}{u_{t2}} = \left(\frac{d_{p1}}{d_{p2}} \right)^2 \quad \text{_____} \quad (15)$$

iii. **If $u_{t1}=u_{t2}$** in the same fluid in the streamline flow regime if their densities and diameters are related by the following Eq.

$$\frac{d_{p1}}{d_{p2}} = \sqrt{\left(\frac{\rho_{p2} - \rho}{\rho_{p1} - \rho} \right)} \quad \text{_____} \quad (16)$$

5. Settling velocity of particle in Centrifugal separator

A particle of mass =m rotating at a **radius =r** with an angular velocity = ω is subject to a centripetal force  = $m r \omega^2$:h can be made very much > than the vertically directed gravity force $F_g = mg$.

The terminal settling velocity **ut** for a **single spherical particle** in a centrifugal separator can be calculated from **Eq.5**




$$u_t = \sqrt{\frac{4d_p(\rho_p - \rho)r\omega^2}{3C_d\rho}} \quad \text{_____} \quad (17)$$

A very small particle may still be in laminar flow in a centrifugal separator, therefore, ut is given by Eq.5:

$$u_t = \frac{d_p^2(\rho_p - \rho)r\omega^2}{18\mu} \quad \text{_____} \quad (18)$$

Effect particles as slurry on viscosity of fluid; Einstein showed that the distortion of the streamlines around the particles caused the dynamic viscosity of the slurry to increase according to the following Eq.

$$\mu = \mu_L(1 + 2.5\alpha) \quad \text{_____} \quad (19)$$

viscosity of slurry viscosity of liquid Volume fraction of solid=(1- ϵ);
 $\alpha=0.02$ for low concentration

6. Fluid flow through packed beds, (Fixed Beds)

Application in Industry:-

- i. Fixed bed catalytic reactors, such as SO₂-SO₃ converters
- ii. Drying columns containing silica gel or molecular sieves, gases are passed through a bed particles
- iii. Gas absorption into liquid.
- iv. Filtration through a fixed bed.

Calculation of Pressure drop (Δp), & Velocity (u_b) through Beds.

Assume in packed bed with length (L), spherical particles with diameter d_p , & the surface area (S_o)= $6/d_p$, ϵ voidage, $1-\epsilon$ =solid particles

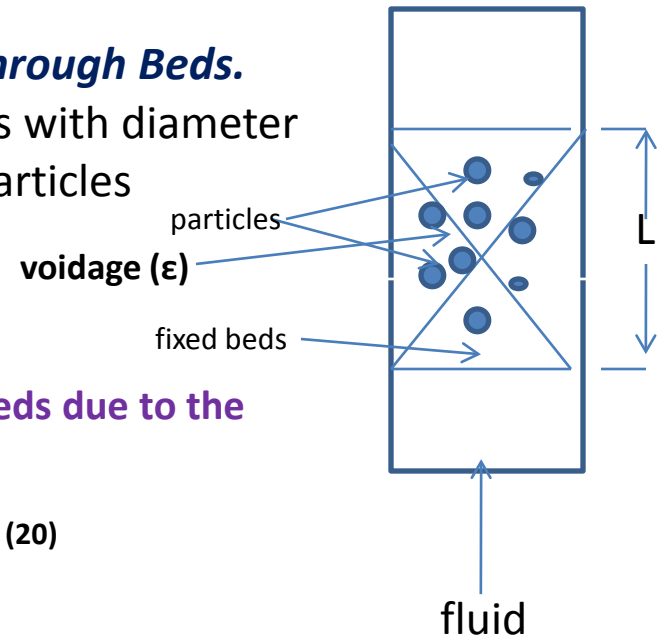
Thus total surface area $S_o = 6/d_p$.

For non-spherical particles $S_o = 6/(d_p \psi)$

When fluid flow through packed beds (L), there is Δp across beds due to the resistance caused by the presence of the particles.

equivalent diameter
through the bed

$$d_e = \frac{4\epsilon}{(1-\epsilon)S_o} \quad (20)$$



Superficial velocity (u) of fluid through the bed is $= Q$ (volumetric flow rate)/ A (whole cross-sectional flow area), therefore the mean velocity of fluid through beds is $\Rightarrow u_b = u/\epsilon$

$$Re_b = \frac{\rho u_b d_e}{\mu} \quad \text{-----} \quad (21)$$

A Reynolds number for flow through a packed bed

Eq.21 when combined with equation 20 can be written as

$$Re_b = \frac{4\rho u}{\mu(1-\epsilon)S_o} \quad \text{-----} \quad (22)$$

An alternative Reynolds number has been used to correlate data and is defined as

$$Re'_b = \frac{\rho u}{\mu(1-\epsilon)S_o} \quad \text{-----} \quad (23)$$

For a packed bed consisting of spherical particles, Eq. 23 can be written in the form

$$Re'_b = \frac{\rho u d_p}{6\mu(1-\epsilon)} \quad \text{-----} \quad (24)$$

The corresponding equation for non-spherical particles is

$$Re'_b = \frac{\rho u d_p \psi}{6\mu(1-\epsilon)} \quad \text{-----} \quad (25)$$

A pressure drop ΔP_f occurs in the bed because of frictional viscous and drag forces, is given by force balance across unit cross-sectional area gives:

$$\Delta P_f \epsilon = \tau_b L (1-\epsilon) S_o \quad \text{-----} \quad (26) \quad \div \rho u_b^2$$

resistance per unit area of surface be or (R)
similar to flow in pipe

$$\frac{f_b}{2} = \frac{\tau_b}{\rho u_b^2} = \left(\frac{\Delta P_f}{L} \right) \left[\frac{\varepsilon}{(1-\varepsilon)S_o \rho u_b^2} \right] \quad (27)$$

or since $u_b = u/\varepsilon$ as

$$\frac{f_b}{2} = \frac{\tau_b}{\rho u_b^2} = \left(\frac{\Delta P_f}{L} \right) \left[\frac{\varepsilon^3}{(1-\varepsilon)S_o \rho u^2} \right] \quad (28)$$

velocity through bed

where f_b is a dimensionless friction factor for flow through a packed bed.

It is calculate as following:

For laminar flow where $Re'_b \leq 2$

$$\frac{f_b}{2} = \frac{5}{Re'_b} \quad (29)$$

The transition to turbulent flow is gradual. Turbulence commences initially in the largest channels and eventually extends to the smaller channels. For the complete range of Reynolds number Carman gave the Eq.

$$\frac{f_b}{2} = \frac{5}{Re'_b} + \frac{0.4}{(Re'_b)^{0.1}} \quad (30)$$

Note: OR f can be calculate by drawn f vs. Re'_b as shown in following Fig. (Ref. VOL.2 p.197 5ed.)

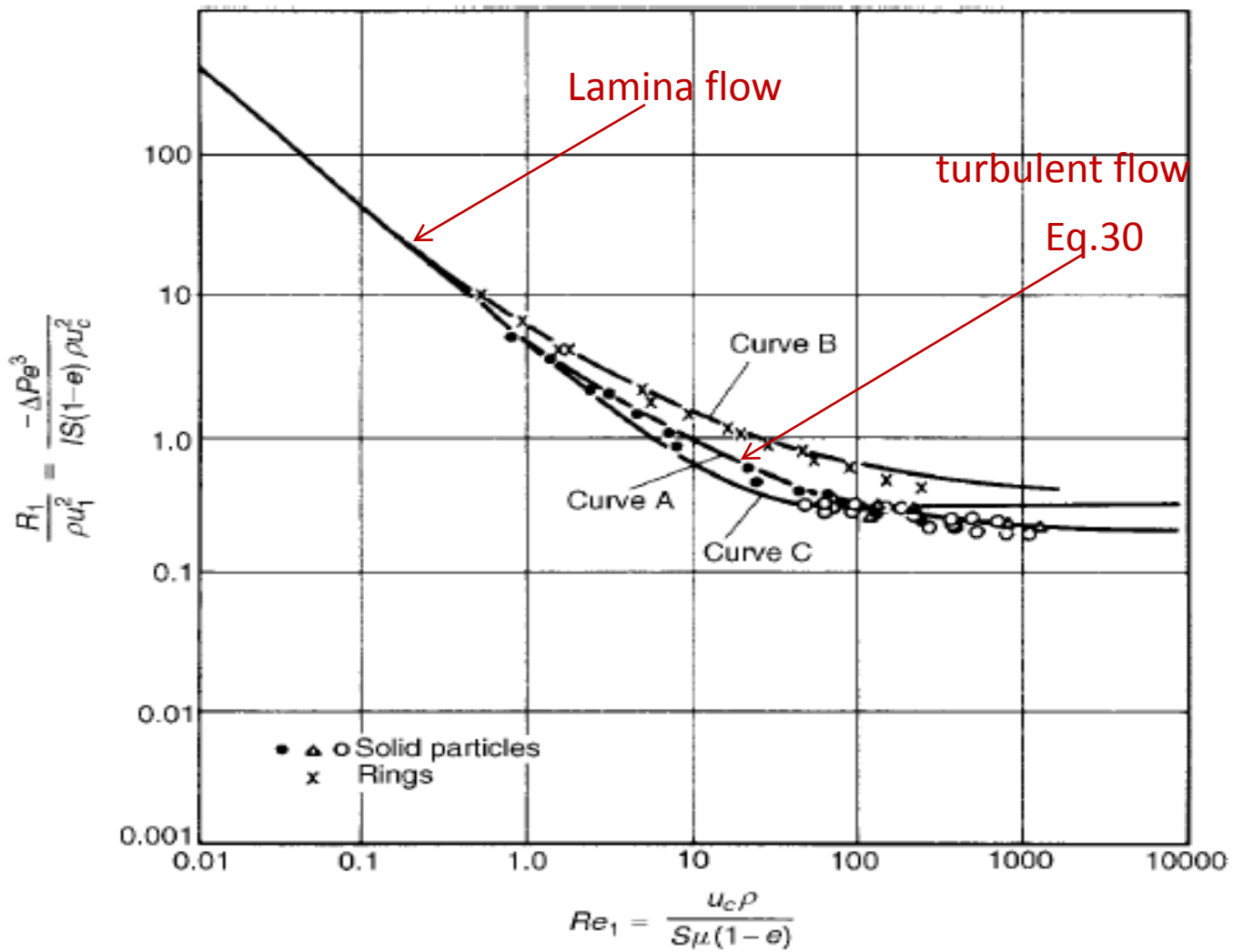


Figure 4.1. Carman's graph of $R_1/\rho u_1^2$ against Re_1

OR Δp & u calculated by Carman's-Kozeny Eqn. as following:

The Hagen–Poiseuille equation for steady laminar flow of Newtonian fluids in pipes and tubes can be written as

$$u = \left(\frac{\Delta P_f}{L} \right) \frac{d_i^2}{32\mu} \quad \text{—————} \quad * \text{ Eqn. * is rewritten for packed bed as following}$$

$$u_b = \left(\frac{\Delta P_f}{L} \right) \left(\frac{1}{32\mu} \right) \left[\frac{16\epsilon^2}{(1-\epsilon)^2 S_o^2} \right] \quad \text{—————} \quad (31) \text{ or, since } u_b = u/\epsilon$$

$$u = \left(\frac{\Delta P_f}{L} \right) \left(\frac{1}{2\mu} \right) \left[\frac{\epsilon^3}{(1-\epsilon)^2 S_o^2} \right] \quad \text{—————} \quad (32)$$

Eq.32 does not hold for flow through packed beds and should be replaced by Eqn:

$$u = \left(\frac{\Delta P_f}{L} \right) \left(\frac{1}{K_c \mu} \right) \left[\frac{\epsilon^3}{(1-\epsilon)^2 S_o^2} \right] \quad \text{—————} \quad (33)$$

Carman–Kozeny equation

Eq.33 can also be written in the form Δp

If $K_c=5$, $S=6/d_p$ Eq. 34 becomes

$$\Delta P_f = (180 \mu L) \left[\frac{(1-\epsilon)^2}{\epsilon^3 d_p^2} \right] u \quad \text{—————} \quad (35)$$

$$\Delta P_f = (K_c \mu L) \left[\frac{(1-\epsilon)^2 S_o^2}{\epsilon^3} \right] u \quad \text{—————} \quad (34)$$

Kc parameter 3.5-5.5
but most **value 5**

Example 2

A gas of density $\rho = 1.25 \text{ kg/m}^3$ and dynamic viscosity $\mu = 1.5 \times 10^{-5} \text{ Pa s}$ flows steadily through a bed of spherical particles of diameter $d_p = 0.005 \text{ m}$. The bed has a height of 3.00 m and a voidage of $\frac{1}{3}$. The superficial velocity $u = 0.03 \text{ m/s}$. Calculate the Reynolds number and the frictional pressure drop over the bed.

Calculations

$$\text{Reynolds number } Re'_b = \frac{\rho u d_p}{6\mu(1-\epsilon)}$$

Substituting the given values

$$\begin{aligned} Re'_b &= \frac{(1.25 \text{ kg/m}^3)(0.03 \text{ m/s})(0.005 \text{ m})(3)}{(6)(1.50 \times 10^{-5} \text{ Pa s})(2)} \\ &= 3.125 \end{aligned}$$

The frictional pressure drop is given by

$$\Delta P_f = (180\mu L) \left[\frac{(1-\epsilon)^2}{\epsilon^3 d_p^2} \right] u$$

Given that

$$(1-\epsilon)^2 = \frac{1}{9}$$

$$\frac{(1-\epsilon)^2}{\epsilon^3} = 12$$

$$d_p^2 = 2.5 \times 10^{-5} \text{ m}^2$$

$$u = 0.03 \text{ m/s}$$

$$\mu = 1.5 \times 10^{-5} \text{ Pa s}$$

$$L = 3.0 \text{ m}$$

$$\begin{aligned} \Delta P_f &= (180)(1.5 \times 10^{-5} \text{ Pa s}) \frac{(3.0 \text{ m})(12)(0.03 \text{ m/s})}{(2.5 \times 10^{-5} \text{ m}^2)} \\ &= \underline{\underline{116.6 \text{ Pa}}} \end{aligned}$$

Example 3

In a contact sulphuric acid plant the secondary converter is a tray type converter, 2.3 m in diameter with the catalyst arranged in three layers, each 0.45 m thick. The catalyst is in the form of cylindrical pellets 9.5 mm in diameter and 9.5 mm long. The void fraction is 0.35. The gas enters the converter at 675 K and leaves at 720 K. Its inlet composition is:

SO₃ 6.6, SO₂ 1.7, O₂ 10.0, N₂ 81.7 mole per cent

and its exit composition is:

SO₃ 8.2, SO₂ 0.2, O₂ 9.3, N₂ 82.3 mole per cent

The gas flowrate is 0.68 kg/m²s. Calculate the pressure drop through the converter. The viscosity of the gas is 0.032 mN s/m².

7. Fluidization

If a fluid is passed upwards in laminar flow through a packed bed of solid particles the superficial velocity u is related to the pressure drop Δp by Eq.33

$$u = \left(\frac{\Delta P_f}{L} \right) \left(\frac{1}{K_c \mu} \right) \left[\frac{\epsilon^3}{(1 - \epsilon)^2 S_o^2} \right]$$

As the fluid velocity is increased the drag on the particles increases and a point is reached when the viscous frictional and drag forces on the particles become= to the weight of the particles in the fluid stream. This start of fluidization, the bed become fluidized is known **incipient fluidization** or **minimum fluidization**, and the velocity in this case is called **minimum fluidization velocity u_{mf}** . **Then the force balance u_{mf} is given by the following Eqns.:**

$$(\Delta P)_{mf} = (1 - \epsilon_{mf})(\rho_p - \rho)L_{mf}g \quad \text{—————} \quad (36)$$

void fraction at
minimum fluidization

$$u_{mf} = \left[\frac{(\rho_p - \rho)g}{K_c \mu} \right] \left[\frac{\epsilon_{mf}^3}{(1 - \epsilon_{mf})S_o^2} \right] \quad \text{—————} \quad (37) \quad K_c=5 \ \& \ S=6/d_p$$

minimum fluidization
velocity u_{mf}

If the velocity is further increased, the bed expands , then the following cases occur:

- i. Particulate fluidization***, (at all velocities of liquid & low gas velocity).
- ii. Bubbling fluidization***, (at higher gas velocity), similar to gas bubbles in a boiling liquid. This case also called aggregative fluidization

Note: In this case velocity of fluid is given by Eq.35 but use u_f & ϵ instead of u_{mf} & ϵ_{mf}

The relationship between Δp vs. u is shown in Fig. (Ref.Vol2 p.232)

As the fluid velocity is increased, the bed expands and solid particles become entrained. Initially the smaller particles only are carried away. If the fluid is sufficiently increased, all particles will become entrained, then velocity particles is given by Eq.4.

$$u_t = \sqrt{\frac{4d_p(\rho_p - \rho)g}{3C_d\rho}}$$

Therefore in practice must be mean velocity of fluid is $u_{mf} < u < u_p$

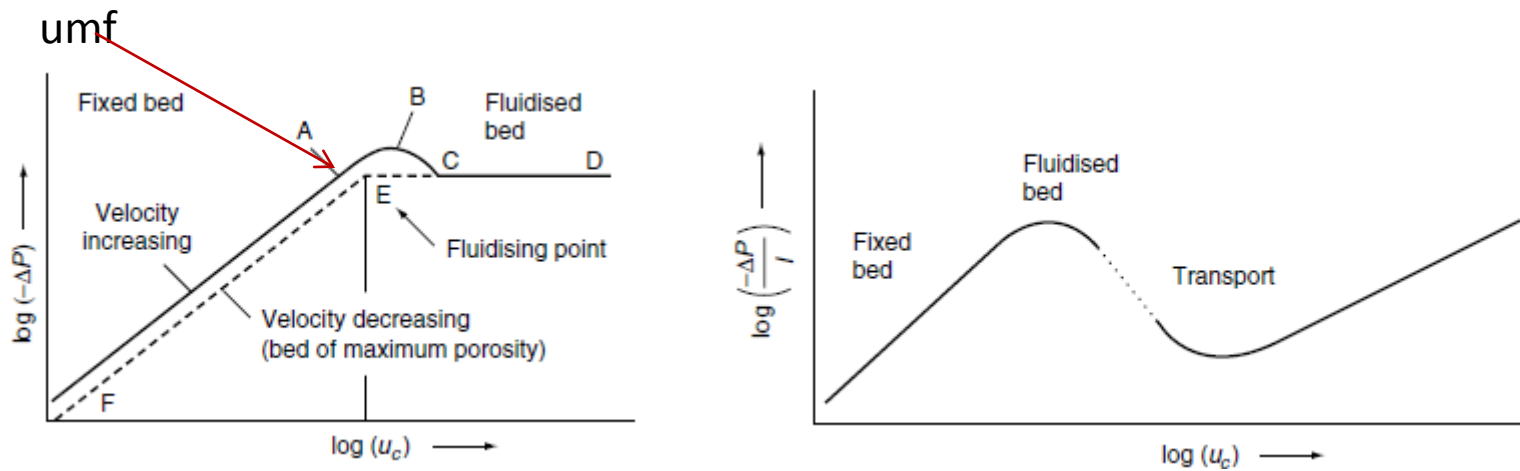


Fig. Pressure drop over fixed & fluidized beds

Note 1: If it is necessary to predict the minimum fluidization velocity the following correlation [Grace (1982)] may be used for gas-solid systems.

$$\frac{\rho u_{mf} d_p}{\mu} = (C^2 + 0.0408 Ar)^{1/2} - C \quad \text{----- (38)}$$

Where Ar Archimedes (or Galileo) number,
and C is the constant value between 27.2-33.7
but the average value C=30

$$Ar = \rho g d_p^3 (\rho_p - \rho) / \mu^2$$

Note 2: If it is possible to measure the height of the bed at incipient fluidization, *L_{mf}*, then *ε_{mf}* can be calculated from equation 36 or simply from the ratio of *L_{mf}* to the height of the packed bed if the void fraction in the latter is known.

Note 3: In the absence of pressure drop ΔP and void fraction measurements, *u_{mf}* is calculated from Eq. 38 and can be estimated from Eq. 37.

Ex. A bed consists of uniform spherical particles of diameter 3mm and density 4200 kg/m³. What will be the minimum fluidising velocity in a liquid of viscosity 3mNs/m² and density 1100 kg/m³?

Sol:

$$Ar\ No. = \frac{d_p^3 g \rho (\rho_p - \rho)}{\mu^2} = 1.003 \times 10^5$$

Hence from Eq. 38

$$\frac{\rho u_{mf} d_p}{\mu} = (C^2 + 0.0408 Ar)^{1/2} - C$$

Calculate $u_{mf} = 36.9 \text{ mm/s}$