7.2 TURBINES (EXPANDERS)

The expansion of a gas in a nozzle to produce a high-velocity stream is a process that converts internal energy into kinetic energy. This kinetic energy is in turn converted into shaft work when the stream impinges on blades attached to a rotating shaft. Thus a turbine (or expander) consists of alternate sets of nozzles and rotating blades through which vapor or gas flows in a steady-state expansion process whose overall effect is the efficient conversion of the internal energy of a high-pressure stream into shaft work.

When steam provides the motive force as in a power plant, the device is called a turbine; when a high-pressure gas, such as ammonia or ethylene in a chemical or petrochemical plant, is the working fluid, the device is often called an expander. The process for either case is shown in Fig. 7.3.

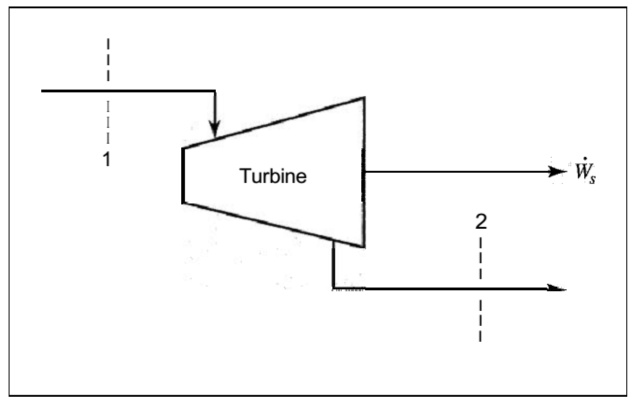
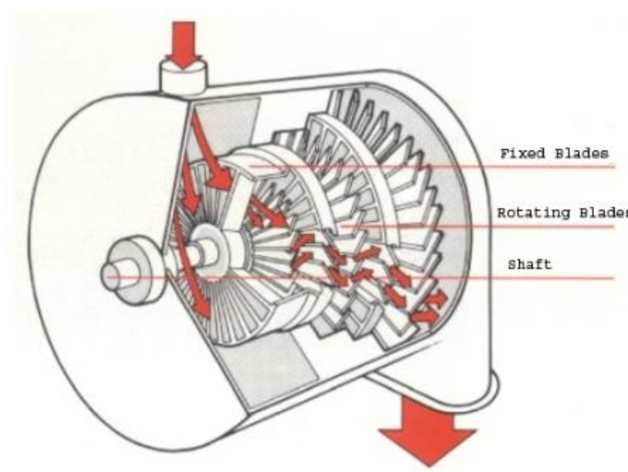


Figure 7.3 Steady-state flow through a turbine or expander



Equations (2.31) and (2.32) are appropriate energy relations. However, the potential-energy term can be omitted, because there is little change in elevation. Moreover, in any properly designed turbine, heat transfer is negligible and the inlet and exit pipes are sized to make fluid velocities roughly equal. Equations (2.3 1) and (2.32) therefore reduce to:

$$\dot{W}_s = \dot{m}\,\Delta H = \dot{m}(H_2 - H_1) \tag{7.13}$$

$$W_s = \Delta H = H_2 - H_1 \tag{7.14}$$

Normally, the inlet conditions T_1 and P_1 and the discharge pressure P_2 are known. Thus in Eq. (7.14) only H_1 is known, and both H_2 and W, remain as unknowns. The energy equation alone does not allow any calculations to be made. However, if the fluid in the turbine undergoes an expansion process that is reversible as well as adiabatic, then the process is isentropic, and $S_2 = S_1$. This second equation allows determination of the final state of the fluid and hence of H_2. For this special case, W_s is given by Eq. (7.14), written:

$$W_s(\text{isentropic}) = (\Delta H)_S \tag{7.15}$$

The shaft work $|W_s(isentropic)|$ is the maximum that can be obtained from an adiabatic turbine with given inlet conditions and given discharge pressure. Actual turbines produce less work, because the actual expansion process is irreversible. We therefore define turbine efficiency as:

$$\eta \equiv \frac{W_s}{W_s(\text{isentropic})}$$

where W_s is the actual shaft work. By Eqs. (7.14) and (7.15),

$$\eta = \frac{\Delta H}{(\Delta H)_S} \tag{7.16}$$

Values of η for properly designed turbines or expanders usually range from 0.7 to 0.8.

The reversible path is a vertical line of constant entropy from point 1 at the intake pressure P_1 to point 2' at the discharge pressure P_2. The line representing the actual irreversible process starts also from point 1, but is directed downward and to the right, in the direction of increasing entropy. Since the process is adiabatic, irreversibilities cause an increase in entropy of the fluid.

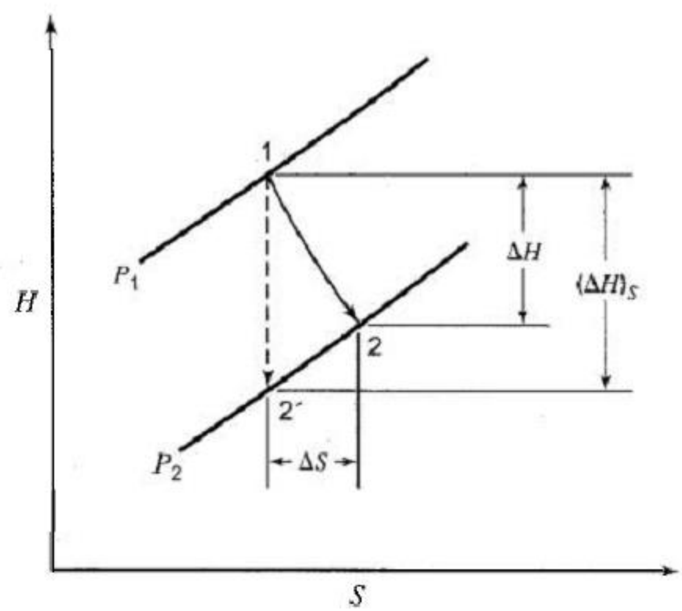


Figure 7.4 Adiabatic expansion process in a turbine or expander

Example 7.6

A steam turbine with rated capacity of 56,400 kW (56,400 kJ s^{-1}) operates with steam at inlet conditions of 8,600 kPa and 500°C, and discharges into a condenser at a pressure of 10 kPa. Assuming a turbine efficiency of 0.75, determine the state of the steam at discharge and the mass rate of flow of the steam.

Solution 7.6

At the inlet conditions of 8,600 kPa and 500°C, the steam tables provide:

$$H_1 = 3{,}391.6 \text{ kJ kg}^{-1} \qquad S_1 = 6.6858 \text{ kJ kg}^{-1}\text{ K}^{-1}$$

If the expansion to 10 kPa is isentropic, then, $S_2' = S_1 = 6.6858$. Steam with this entropy at 10 kPa is wet, and Eq. (6.82b), with $M = S$ and $x^v = x_2'$, yields:

$$S_2' = S_2^l + x_2'(S_2^v - S_2^l)$$

Then, $6.6858 = 0.6493 + x_2'(8.1511 - 0.6493) \qquad x_2' = 0.8047$

This is the quality (fraction vapor) of the discharge stream at point 2′. The enthalp H_2' is also given by Eq. (6.82b), written:

$$H_2' = H_2^l + x_2'(H_2^v - H_2^l)$$

Thus, $H_2' = 191.8 + (0.8047)(2{,}584.8 - 191.8) = 2{,}117.4 \text{ kJ kg}^{-1}$

$$(\Delta H)_S = H_2' - H_1 = 2{,}117.4 - 3{,}391.6 = -1{,}274.2 \text{ kJ kg}^{-1}$$

and by Eq. (7.16),

$$\Delta H = \eta(\Delta H)_S = (0.75)(-1{,}274.2) = -955.6 \text{ kJ kg}^{-1}$$

Whence, $H_2 = H_1 + \Delta H = 3{,}391.6 - 955.6 = 2{,}436.0 \text{ kJ kg}^{-1}$

Thus the steam in its actual final state is also wet, with its quality given by:

$$2{,}436.0 = 191.8 + x_2(2{,}584.8 - 191.8) \qquad x_2 = 0.9378$$

Then $S_2 = 0.6493 + (0.9378)(8.1511 - 0.6493) = 7.6846 \text{ kJ kg}^{-1}\text{ K}^{-1}$

This value may be compared with the initial value of $S_1 = 6.6858$.

The steam rate $\dot{m}$ is given by Eq. (7.13). For a work rate of 56,400 kJ s^{-1},

$$\dot{W}_s = -56{,}400 = \dot{m}(2{,}436.0 - 3{,}391.6) \qquad \dot{m} = 59.02 \text{ kg s}^{-1}$$

Example 7.6 is solved with data from the steam tables. When a comparable set of tables is not available for the working fluid, the generalized correlations of Sec. 6.7 may be used in conjunction with Eqs. (6.84) and (6.85), as illustrated in the following example.

Example 7.7

A stream of ethylene gas at 300°C and 45 bar is expanded adiabatically in a turbine to 2 bar. Calculate the isentropic work produced. Find the properties of ethylene by:

(*a*) Equations for an ideal gas. (*b*) Appropriate generalized correlations.

Solution 7.7

The enthalpy and entropy changes for the process are:

$$\Delta H = \langle C_P^{ig} \rangle_H (T_2 - T_1) + H_2^R - H_1^R \qquad (6.93)$$

$$\Delta S = \langle C_P^{ig} \rangle_S \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} + S_2^R - S_1^R \qquad (6.94)$$

Given values are $P_1 = 45$ bar, $P_2 = 2$ bar, and $T_1 = 300 + 273.15 = 573.15$ K.

(*a*) If ethylene is assumed an ideal gas, then all residual properties are zero, and the preceding equations reduce to:

$$\Delta H = \langle C_P^{ig} \rangle_H (T_2 - T_1) \qquad \Delta S = \langle C_P^{ig} \rangle_S \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

For an isentropic process, $\Delta S = 0$, and the second equation becomes:

$$\frac{\langle C_P^{ig} \rangle_S}{R} \ln \frac{T_2}{T_1} = \ln \frac{P_2}{P_1} = \ln \frac{2}{45} = -3.1135$$

or

$$\ln T_2 = \frac{-3.1135}{\langle C_P^{ig} \rangle_S / R} + \ln 573.15$$

Then,

$$T_2 = \exp\left(\frac{-3.1135}{\langle C_P^{ig} \rangle_S / R} + 6.3511 \right) \qquad (A)$$

Equation (5.17) provides an expression for $\langle C_P^{ig} \rangle_S / R$, which for computational purposes is represented by:

$$\frac{\langle C_P^{ig} \rangle_S}{R} = \text{MCPS(573.15,T2;1.424,14.394E-3,-4.392E-6,0.0)}$$

where the constants for ethylene come from Table C.1. Temperature T_2 is found by iteration. Assume an initial value for evaluation of $\langle C_P^{ig}\rangle_S/R$. Equation (A) then provides a new value of T_2 from which to recompute $\langle C_P^{ig}\rangle_S/R$, and the procedure continues to convergence on the final value: $T_2 = 370.8$ K. The value of $\langle C_P^{ig}\rangle_H/R$, given by Eq. (4.8), is for computational purposes represented by:

$$\frac{\langle C_P^{ig}\rangle_H}{R} = \mathrm{MCPH(573.15,370.8;1.424,14.394E\text{-}3,\text{-}4.392E\text{-}6,0.0)} = 7.224$$

Then $$W_s(\text{isentropic}) = (\Delta H)_S = \langle C_P^{ig}\rangle_H (T_2 - T_1)$$

$$W_s(\text{isentropic}) = (7.224)(8.314)(370.8 - 573.15) = -12{,}153 \text{ J mol}^{-1}$$

(*b*) For ethylene,

$$T_c = 282.3 \text{ K} \qquad P_c = 50.4 \text{ bar} \qquad \omega = 0.087$$

At the initial state,

$$T_{r_1} = \frac{573.15}{282.3} = 2.030 \qquad P_{r_1} = \frac{45}{50.4} = 0.893$$

According to Fig. 3.14, the generalized correlations based on second virial coefficients should be satisfactory. The computational procedures of Eqs. (6.87), (6.88), (3.65), (3.66), (6.89) and (6.90) are represented by:

$$\frac{H_1^R}{RT_c} = \mathrm{HRB(2.030,0.893,0.087)} = -0.234$$

$$\frac{S_1^R}{R} = \mathrm{SRB(2.030,0.893,0.087)} = -0.097$$

Then, $$H_1^R = (-0.234)(8.314)(282.3) = -549 \text{ J mol}^{-1}$$

$$S_1^R = (-0.097)(8.314) = -0.806 \text{ J mol}^{-1}\text{ K}^{-1}$$

For an initial estimate of S_2^R, assume that $T_2 = 370.8$ K, the value determi in part (*a*). Then,

$$T_{r_2} = \frac{370.8}{282.3} = 1.314 \qquad P_{r_2} = \frac{2}{50.4} = 0.040$$

Whence, $$\frac{S_2^R}{R} = \mathrm{SRB(1.314,0.040,0.087)} = -0.0139$$

and $$S_2^R = (-0.0139)(8.314) = -0.116 \text{ J mol}^{-1}\text{ K}^{-1}$$

If the expansion process is isentropic, Eq. (6.94) becomes:

Whence,

$$\ln \frac{T_2}{573.15} = \frac{-26.576}{(C_P^{ig})_S}$$

or

$$T_2 = \exp\left(\frac{-26.576}{(C_P^{ig})_S} + 6.3511\right)$$

An iteration process exactly like that of part (*a*) yields the results

$$T_2 = 365.8 \text{ K} \qquad \text{and} \qquad T_{r_2} = 1.296$$

With this value of T_{r_2} and with $P_{r_2} = 0.040$,

$$\frac{S_2^R}{R} = \text{SRB}(1.296, 0.040, 0.087) = -0.0144$$

and

$$S_2^R = (-0.0144)(8.314) = -0.120 \text{ J mol}^{-1} \text{ K}^{-1}$$

This result is so little changed from the initial estimate that another recalculation of T_2 is unnecessary, and H_2^R is evaluated at the reduced conditions just established:

$$\frac{H_2^R}{RT_c} = \text{HRB}(1.296, 0.040, 0.087) = -0.0262$$

$$H_2^R = (-0.0262)(8.314)(282.3) = -61 \text{ J mol}^{-1}$$

By Eq. (6.93), $(\Delta H)_S = (C_P^{ig})_H(365.8 - 573.15) - 61 + 549$

Evaluation of $(C_P^{ig})_H$ as in part (*a*) with $T_2 = 365.8$ K gives:

$$(C_P^{ig})_H = 59.843 \text{ J mol}^{-1} \text{ K}^{-1}$$

Whence,

$$(\Delta H)_S = -11{,}920 \text{ J mol}^{-1}$$

and

$$W_s(\text{isentropic}) = (\Delta H)_S = -11{,}920 \text{ J mol}^{-1}$$

This differs from the ideal-gas value by less than 2%.