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مبادئ الهندسة الكيماوية

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Chapter1

Dimensions, Units, and Their Conversion

1.1 Units and Dimensions

Dimensions are our basic concepts of measurement such as length, time, mass, temperature, and so on; **units** are the means of expressing the dimensions, such as feet or centimeters for length, and hours or seconds for time.

In this lecture you will use the two most commonly used systems of units:

1. **SI**, formally called Le Systeme Internationale 'Unites, and informally called SI or more often (redundantly) the SI system of units.
2. **AE**, or American Engineering system of units.

Dimensions and their respective units are classified as fundamental or derived:

- **Fundamental** (or basic) dimensions/units are those that can be measured independently and are sufficient to describe essential physical quantities.
- **Derived** dimensions/units are those that can be developed in terms of the fundamental dimensions/units.

Tables 1.1 and 1.2 list both basic, derived, and alternative units in the SI and AE systems. Figure

1.1 illustrates the relation between the basic dimensions and some of the derived dimensions.

One of the best features of the SI system is that (except for time) units and their multiples and submultiples are related by standard factors designated by the **prefix** indicated in Table 1.3.

1.2 Operations with Units

The rules for handling units are essentially quite simple:

1.2.1 Addition, Subtraction, Equality

You can add, subtract, or equate numerical quantities only if the associated units of the quantities are the same. Thus, the operation

$$5 \text{ kilograms} + 3 \text{ joules}$$

cannot be carried out because the units as well as the dimensions of the two terms are different. The numerical operation

$$10 \text{ pounds} + 5 \text{ grams}$$

can be performed (because the dimensions are the same, mass) only after the units are transformed to be the same, either pounds, grams, or **ounces**, or some other mass unit.

Table 1.1 SI Units

Physical Quantity	Name of Unit	Symbol for Unit*	Definition of Unit
<i>Basic SI Units</i>			
Length	metre, meter	m	
Mass	kilogramme, kilogram	kg	
Time	second	s	
Temperature	kelvin	K	
Molar amount	mole	mol	
<i>Derived SI Units</i>			
Energy	joule	J	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2} \rightarrow \text{Pa} \cdot \text{m}^3$
Force	newton	N	$\text{kg} \cdot \text{m} \cdot \text{s}^{-2} \rightarrow \text{J} \cdot \text{m}^{-1}$
Power	watt	W	$\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3} \rightarrow \text{J} \cdot \text{s}^{-1}$
Density	kilogram per cubic meter		$\text{kg} \cdot \text{m}^{-3}$
Velocity	meter per second		$\text{m} \cdot \text{s}^{-1}$
Acceleration	meter per second squared		$\text{m} \cdot \text{s}^{-2}$
Pressure	newton per square meter, pascal		$\text{N} \cdot \text{m}^{-2}, \text{Pa}$
Heat capacity	joule per (kilogram · kelvin)		$\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$
<i>Alternative Units</i>			
Time	minute, hour, day, year	min, h, d, y	
Temperature	degree Celsius	°C	
Volume	litre, liter (dm ³)	L	
Mass	tonne, ton (Mg), gram	t, g	

Table 1.2 American Engineering (AE) System Units

Physical Quantity	Name of Unit	Symbol
<i>Some Basic Units</i>		
Length	foot	ft
Mass	pound (mass)	lb _m
Time	second, minute, hour, day	s, min, h (hr), day
Temperature	degree Rankine or degree Fahrenheit	°R or °F
Molar amount	pound mole	lb mol
<i>Derived Units</i>		
Force	pound (force)	lb _f
Energy	British thermal unit, foot pound (force)	Btu, (ft)(lb _f)
Power	horsepower	hp
Density	pound (mass) per cubic foot	lb _m /ft ³
Velocity	feet per second	ft/s
Acceleration	feet per second squared	ft/s ²
Pressure	pound (force) per square inch	lb _f /in. ² , psi
Heat capacity	Btu per pound (mass) per degree F	Btu/(lb _m)(°F)

Figure 1.1 Relation between the basic dimensions (in boxes) and various derived dimensions (in ellipses).

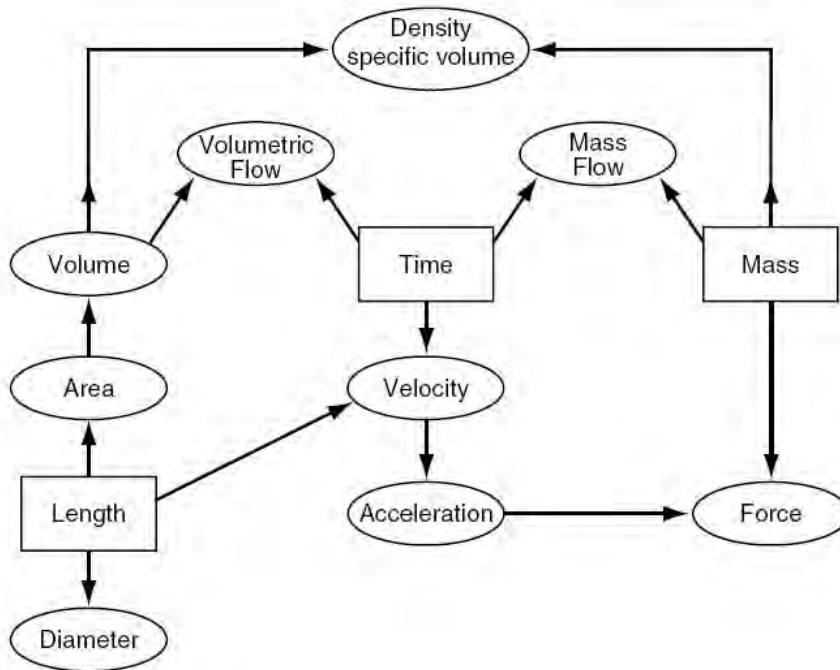


Table 1.3 SI Prefixes

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^9	giga	G	10^{-1}	deci	d
10^6	mega	M	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^2	hecto	h	10^{-6}	micro	μ
10^1	deka	da	10^{-9}	nano	n

1.2.2 Multiplication and Division

You can multiply or divide unlike units at will such as

$$50(\text{kg})(\text{m})/(\text{s})$$

but you cannot cancel or merge units unless they are identical. Thus, $3\text{m}^2/60\text{cm}$ can be converted to $3\text{m}^2/0.6\text{m}$, and then to 5m , but in m/s^2 , the units cannot be cancelled or combined.

Example 1.1

Add the following:

(a) 1 foot + 3 seconds

(b) 1 horsepower + 300 watts

Solution

The operation indicated by

$$1 \text{ ft} + 3 \text{ s}$$

has no meaning since the dimensions of the two terms are not the same. In the case of

$$1 \text{ hp} + 300 \text{ watts}$$

the dimensions are the same (energy per unit time), but the units are different. You must transform the two quantities into like units, such as horsepower or watts, before the addition can be carried out. Since $1 \text{ hp} = 746 \text{ watts}$,

$$746 \text{ watts} + 300 \text{ watts} = 1046 \text{ watts}$$

1.3 Conversion of Units and Conversion Factors

The procedure for converting one set of units to another is simply to multiply a number and its associated units by ratios termed **conversion factors** to arrive at the desired answer and its associated units.

If a plane travels at twice the speed of sound (assume that the speed of sound is 1100 ft/s), how fast is it going in miles per hour?

We formulate the conversion as follows

$$\frac{2 \times 1100 \text{ ft}}{\text{s}} \left| \frac{1 \text{ mi}}{5280 \text{ ft}} \right| \frac{60 \text{ s}}{1 \text{ min}} \left| \frac{60 \text{ min}}{1 \text{ hr}} \right|$$

$$\frac{\text{ft}}{\text{s}} \quad \frac{\text{mi}}{\text{s}} \quad \frac{\text{mi}}{\text{min}}$$

Example 1.2

- (a) Convert 2 km to miles. (b) Convert $400 \text{ in.}^3/\text{day}$ to cm^3/min .

Solution

(a) One way to carry out the conversion is to look up a direct conversion factor, namely $1.61 \text{ km} = 1 \text{ mile}$:

$$\frac{2 \text{ km}}{1.61 \text{ km}} \left| \frac{1 \text{ mile}}{1.61 \text{ km}} \right| = 1.24 \text{ mile}$$

Another way is to use conversion factors you know

$$\frac{2 \text{ km}}{1 \text{ km}} \left| \frac{10^5 \text{ cm}}{1 \text{ km}} \right| \left| \frac{1 \text{ in.}}{2.54 \text{ cm}} \right| \left| \frac{1 \text{ ft}}{12 \text{ in.}} \right| \left| \frac{1 \text{ mile}}{5280 \text{ ft.}} \right| = 1.24 \text{ mile}$$

$$(b) \frac{400 \text{ in.}^3}{\text{day}} \left| \left(\frac{2.54 \text{ cm}}{1 \text{ in.}} \right)^3 \right| \left| \frac{1 \text{ day}}{24 \text{ hr}} \right| \left| \frac{1 \text{ hr}}{60 \text{ min}} \right| = 4.55 \frac{\text{cm}^3}{\text{min}}$$

In part (b) note that not only are the numbers in the conversion of inches to centimeters raised to a power, but the units also are raised to the same power.

Example1.3

An example of a semiconductor is ZnS with a particle diameter of 1.8 nanometers. Convert this value to (a) dm (decimeters) and (b) inches.

Solution

$$(a) \frac{1.8 \text{ nm}}{1 \text{ nm}} \left| \frac{10^{-9} \text{ m}}{1 \text{ m}} \right| \frac{10 \text{ dm}}{1 \text{ m}} = 1.8 \times 10^{-8} \text{ dm}$$

$$(b) \frac{1.8 \text{ nm}}{1 \text{ nm}} \left| \frac{10^{-9} \text{ m}}{1 \text{ m}} \right| \frac{39.37 \text{ in.}}{1 \text{ m}} = 7.09 \times 10^{-8} \text{ in.}$$

In the AE system the conversion of terms involving pound mass and pound force deserves special attention. Let us start the discussion with Newton's Law:

$$F = Cma \tag{1.1}$$

Where:

F = force

C = a constant whose numerical value and its units depend on those selected for F, m,

and a, m = mass

a = acceleration

In the SI system in which the unit of force is defined to be the Newton (N) when 1 kg is accelerated at 1 m/s², a conversion factor $C = 1 \text{ N}/(\text{Kg})(\text{m})/\text{s}^2$ must be introduced to have the force be 1N:

$$F = \frac{1 \text{ N}}{\frac{(\text{kg})(\text{m})}{\text{s}^2}} \left| \frac{1 \text{ kg}}{\tilde{m}} \right| \left| \frac{1 \text{ m}}{\tilde{a}} \right| \frac{1 \text{ m}}{\text{s}^2} = 1 \text{ N} \tag{1.1}$$

Because the numerical value associated with the conversion factor is **1**, the conversion factor seems simple, even nonexistent, and the units are ordinarily ignored.

In the AE system an analogous conversion factor is required. If a mass of 1 lb_m is hypothetically accelerated at g ft/s², where g is the acceleration that would be caused by gravity (about 32.2 ft/s² depending on the location of the mass), we can make the force be 1 lb_f by choosing the proper numerical value and units for the conversion factor C:

$$F = \left(\frac{1(\text{lb}_f)(\text{s}^2)}{32.174(\text{lb}_m)(\text{ft})} \right) \left(\frac{1 \text{ lb}_m \left| \frac{\text{g ft}}{\text{s}^2} \right.}{\tilde{m} \quad \tilde{g}} \right) = 1 \text{ lb}_f \quad (1.2)$$

The inverse of the conversion factor with the numerical value **32.174** included is given the special symbol g_c (Note: in eq. [1.2], $g=32.2 \text{ ft/s}^2$)

$$g_c = 32.174 \frac{(\text{ft})(\text{lb}_m)}{(\text{s}^2)(\text{lb}_f)}$$

But never forget that the pound (**mass**) and pound (**force**) are not the same units in the **AE** system.

$$1 \text{ lbf} = 32.174 \text{ lbmft/s}^2$$

Example 1.4

What is the potential energy in $(\text{ft})(\text{lb}_f)$ of a 100 lb drum hanging 10 ft above the surface of the earth with reference to the surface of the earth?

Solution

Potential energy = $P = m gh$

Assume that the 100 lb means 100 lb mass; g = acceleration of gravity = 32.2 ft/s^2 . Figure E1.4 is a sketch of the system.

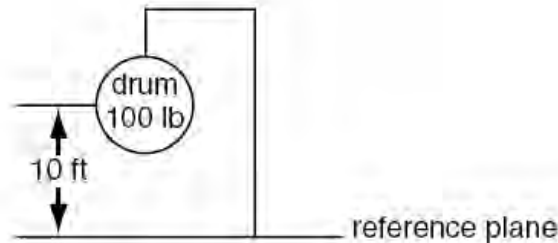


Figure E1.4

$$P = \frac{100 \text{ lb}_m \left| \frac{32.2 \text{ ft}}{\text{s}^2} \right| 10 \text{ ft} \left| \frac{(\text{s}^2)(\text{lb}_f)}{32.174(\text{ft})(\text{lb}_m)} \right.}{\quad \quad \quad} = 1000 (\text{ft})(\text{lb}_f)$$

Notice that in the ratio of 32.2 ft/s^2 divided by $32.174 [(\text{ft})(\text{lb}_m)] / [(\text{s}^2)(\text{lb}_f)]$, the numerical values are almost equal. Many engineers would solve the problem by saying that **100 lb × 10 ft = 1000**

(ft)(lb) without realizing that, in effect, they are canceling out the numbers in the g/g_c ratio, and that the lb in the solution means lb_f .

Example 1.5

In biological systems, production rate of glucose is $0.6 \mu\text{g mol}/(\text{mL})(\text{min})$. Determine the production rate of glucose for this system in the units of $\text{lb mol}/(\text{ft}^3)(\text{day})$.

Solution

Basis: 1 min

$$\begin{aligned} & \frac{0.6 \mu\text{g mol}}{(\text{mL})(\text{min})} \left| \frac{1 \text{ g mol}}{10^6 \mu\text{g mol}} \right| \left| \frac{1 \text{ lb mol}}{454 \text{ g mol}} \right| \left| \frac{1000 \text{ mL}}{1 \text{ L}} \right| \left| \frac{1 \text{ L}}{3.531 \times 10^{-2} \text{ ft}^3} \right| \left| \frac{60 \text{ min}}{\text{hr}} \right| \left| \frac{24 \text{ hr}}{\text{day}} \right| \\ & = 0.0539 \frac{\text{lb mol}}{(\text{ft}^3)(\text{day})} \end{aligned}$$

1.4 Dimensional Consistency (Homogeneity)

The concept of dimensional consistency can be illustrated by an equation that represents the pressure/volume/temperature behavior of a gas, and is known as van der Waals' equation.

$$\left(p + \frac{a}{V^2} \right) (V - b) = RT$$

Inspection of the equation shows that the constant **a** must have the units of **[(pressure)(volume)²]** for the expression in the first set of parentheses to be consistent throughout. If the units of pressure are **atm** and those of volume are **cm³**, **a** will have the units of **[(atm)(cm)⁶]**. Similarly, **b** must have the same units as **V**, or in this particular case the units of **cm³**.

Example 1.6

Your handbook shows that microchip etching roughly follows the relation

$$d = 16.2 - 16.2e^{-0.021t} \quad t < 200$$

where **d** is the depth of the etch in microns (micrometers, μm) and **t** is the time of the etch in seconds. What are the units associated with the numbers 16.2 and 0.021? Convert the relation so that **d** becomes expressed in inches and **t** can be used in minutes.

Solution

Both values of **16.2** must have the associated unit of microns (μm). The exponential must be dimensionless so that **0.021** must have the associated units of s^{-1} .

$$d_{in} = \frac{16.2 \mu\text{m}}{10^6 \mu\text{m}} \left| \frac{1 \text{ m}}{1 \text{ m}} \right| \frac{39.27 \text{ in.}}{1 \text{ m}} \left[1 - \exp \frac{-0.021}{s} \left| \frac{60s}{1 \text{ min}} \right| t_{\text{min}} \right]$$

$$= 6.38 \times 10^{-4} (1 - e^{-1.26t_{\text{min}}}) \text{ inches}$$

Nondimensional Groups:

As you proceed with the study of chemical engineering, you will find that groups of symbols may be put together, either by theory or based on experiment, that have no net units. Such collection of variables or parameters are called **dimensionless or nondimensional groups**. One example is the Reynolds number (group) arising in fluid mechanics.

$$\text{Reynolds number} = \frac{Dv\rho}{\mu} = N_{RE}$$

where D is the pipe diameter, say in cm; v is the fluid velocity, say in cm/s; ρ is the fluid density, say in g/cm³; and μ is the viscosity, say in centipoise, units that can be converted to g/(cm)(s). Introducing the consistent set of units for D, v, ρ , and μ into $Dv\rho/\mu$, you will find that all the units cancel out so that the numerical value of **1** is the result of the cancellation of the units.

$$\frac{\text{cm}}{\text{s}} \left| \frac{\text{cm}}{\text{s}} \right| \frac{\text{g}}{\text{cm}^3} \left| \frac{(\text{cm})(\text{s})}{\text{g}} \right|$$

Example 1.7

Explain without differentiating why the following differentiation cannot be correct:

$$\frac{d}{dx} \sqrt{1 + (x^2/a^2)} = \frac{2ax}{\sqrt{1 + (x^2/a^2)}}$$

where x is length and a is a constant.

Solution

- Observe that x and a must have the same units because the ratio x^2/a^2 must be dimensionless (because 1 is dimensionless).
- Thus, the left-hand side of the equation has units of $1/x$ (from d/dx). However, the right-hand side of the equation has units of x^2 (the product of ax).
- Consequently, something is wrong as the equation is not dimensionally consistent.

Questions

- Which of the following best represents the force needed to lift a heavy suitcase?
a. 25N b. 25 kN c. 250N d. 250 kN
- Pick the correct answer(s); a watt is
a. one joule per second b. equal to $1(\text{kg})(\text{m}^2)/\text{s}^2$ c. the unit for all types of power
d. all of the above e. none of the above
- Is kg/s a basic or derived unit in SI?
- Answer the following questions yes or no. Can you
a. divide ft by s? b. divide m by cm? c. multiply ft by s? d. divide ft by cm? e. divide m by (deg) K? f. add ft and s? g. subtract m and (deg) K h. add cm and ft? i. add cm and m^2 ?
j. add 1 and 2 cm?
- Why is it not possible to add 1 ft and 1ft^2 ?
- What is g_c ?
- Is the ratio of the numerator and denominator in a conversion factor equal to unity?
- What is the difference, if any, between pound force and pound mass in the AE system?
- Could a unit of force in the SI system be kilogram force?
- Contrast the procedure for converting units within the SI system with that for the AE system.
- What is the weight of a one pound mass at sea level? Would the mass be the same at the center of Earth? Would the weight be the same at the center of Earth?
- What is the mass of an object that weighs 9.80 kN at sea level?
- Explain what dimensional consistency means in an equation.
- Explain why the so-called dimensionless group has no net dimensions.
- If you divide all of a series of terms in an equation by one of the terms, will the resulting series of terms be dimensionless?
- How might you make the following variables dimensionless:
a. Length (of a pipe). b. Time (to empty a tank full of water).

Answers:

- (c)
- (a)
- Derived.
- (a) - (e) yes; (f) and (g) no; (h) and (i) no; (j) no.

5. The dimensions are not the same.
6. A conversion factor in the American Engineering system of units.
7. Yes.
8. lbf is force and lb_m is mass, and the dimensions are different.
9. The unit is not legal in SI.
10. In SI the magnitudes of many of the units are scaled on the basis of 10, in AE. Consequently, the units are often ignored in making conversion in SI.
11. (a) 1 lbf in the AE system of units; (b) yes; (c) no.
12. 1000 kg.
13. All additive terms on the right-hand side of an equation must have the same dimensions as those on the left-hand side.
14. All of the units cancel out.
15. Yes.
16. (a) Divide by the radius or diameter; (b) divide by the total time to empty the tank, or by a fixed unit of time.

Problems

1. Classify the following units as correct or incorrect units in the SI system:
a. nm b. K c. sec d. N/mm e. kJ/(s)(m³)
2. Add 1 cm and 1 m.
3. Subtract 3 ft from 4 yards.
4. Divide $3\text{ m}^{1.5}$ by $2\text{ m}^{0.5}$.
5. Multiply 2 ft by 4 lb.
6. What are the value and units of g_c in the SI system?
7. Electronic communication via radio travels at approximately the speed of light (186,000 miles/second). The edge of the solar system is roughly at Pluto, which is 3.6×10^9 miles from Earth at its closest approach. How many hours does it take for a radio signal from Earth to reach Pluto?
8. Determine the kinetic energy of one pound of fluid moving in a pipe at the speed of 3 feet per second.
9. Convert the following from AE to SI units:
a. $4\text{ lb}_m/\text{ft}$ to kg/m b. $1.00\text{ lb}_m/(\text{ft}^3)(\text{s})$ to $\text{kg}/(\text{m}^3)(\text{s})$
10. Convert the following $1.57 \times 10^{-2}\text{ g}/(\text{cm})(\text{s})$ to $\text{lb}_m/(\text{ft})(\text{s})$
11. Convert 1.1 gal to ft^3 .

12. Convert 1.1 gal to m³.

13. An orifice meter is used to measure the rate of flow of a fluid in pipes. The flow rate is related to the pressure drop by the following equation

$$u = c \sqrt{\frac{\Delta P}{\rho}}$$

Where u = fluid velocity

Δp = pressure drop 1 force per unit area²

ρ = density of the flowing fluid

c = constant

What are the units of c in the SI system of units?

14. The thermal conductivity k of a liquid metal is predicted via the empirical equation

$$k = A \exp (B/T)$$

where k is in J/(s)(m)(K) and A and B are constants. What are the units of A and B ?

Answers:

1. (a), (s), (d), (e) are correct.
2. Change units to get 101cm.
3. Change units to get 9ft.
4. 1.5 m.
5. 8(ft)(lb).
6. 1, dimensionless.
7. 5.38hr.
8. 0.14 (ft)(lb_f).
9. a. 5.96 kg/m; b. 16.0kg/(m³)(s)
10. 1.06 * 10⁻³lb_m/(ft)(s)
11. 0.15ft³
12. 4.16 * 10⁻³m³.
13. c is dimensionless
14. A has the same units as k ; B has the units of T

Supplementary Problems (ChapterOne):

Problem1

Convert the following quantities to the ones designated :

- 42 ft²/hr to cm²/s.
- 25 psig to psia.
- 100 Btu to hp-hr.

Solution

$$a. \frac{42.0 \text{ ft}^2}{\text{hr}} \left| \frac{1.0 \text{ m}}{3.2808 \text{ ft}} \right|^2 \left| \frac{10^4 \text{ cm}^2}{1.0 \text{ m}^2} \right| \left| \frac{1 \text{ hr}}{3600 \text{ s}} \right| = 10.8 \text{ cm}^2/\text{s}$$

$$b. \frac{100 \text{ Btu}}{1 \text{ Btu}} \left| \frac{3.93 \times 10^{-4} \text{ hp-hr}}{1 \text{ Btu}} \right| = 3.93 \times 10^{-2} \text{ hp-hr}$$

$$c. \frac{80.0 \text{ lb}_f}{(\text{lb}_f)(\text{s})^2} \left| \frac{32.174 (\text{lb}_m)(\text{ft})}{2.20 \text{ lb}_m} \right| \left| \frac{1 \text{ kg}}{2.20 \text{ lb}_m} \right| \left| \frac{1 \text{ m}}{3.2808 \text{ ft}} \right| \left| \frac{1 \text{ N}}{1 (\text{kg})(\text{m})(\text{s})^{-2}} \right| = 356 \text{ N}$$

Problem2

Convert the ideal gas constant : $R = 1.987 \frac{\text{cal}}{(\text{gmol})(\text{K})}$ to $\frac{\text{Btu}}{(\text{lb mol})(^\circ\text{R})}$

Solution

$$\frac{1.987 \text{ cal}}{(\text{gmol})(\text{K})} \left| \frac{1 \text{ Btu}}{252 \text{ cal}} \right| \left| \frac{454 \text{ gmol}}{1 \text{ lb mol}} \right| \left| \frac{1 \text{ K}}{1.8 ^\circ\text{R}} \right| = 1.98 \frac{\text{Btu}}{(\text{lb mol})(^\circ\text{R})}$$

Problem3

Mass flow through a sonic nozzle is a function of gas pressure and temperature. For a given pressure p and temperature T, mass flow rate through the nozzle is given by

$$m = 0.0549 p / (T)^{0.5} \quad \text{where } m \text{ is in lb/min, } p \text{ is in psia and } T \text{ is in } ^\circ\text{R}$$

- Determine what the units for the constant 0.0549 are.
- What will be the new value of the constant, now given as 0.0549, if the variables in the equation are to be substituted with SI units and m is calculated in SI units.

Solution

- Calculation of the constant.

The first step is to substitute known units into the equation.

$$\frac{\text{lb}_m}{\text{min}} = 0.0549 \frac{\text{lb}_f}{(\text{in}^2)(^\circ\text{R})^{0.5}}$$

$$\frac{\text{lb}_f}{(\text{in}^2)(^\circ\text{R})^{0.5}} \left| \frac{(\text{lb}_m)(\text{in})^2(^\circ\text{R})^{0.5}}{(\text{min})(\text{lb}_f)} \right. \text{-----} \left. \frac{(\text{lb}_m)}{(\text{min})} \right.$$

Units for the constant 0.0549 are $\frac{(\text{lb}_m)(\text{in})^2(^\circ\text{R})^{0.5}}{(\text{min})(\text{lb}_f)}$

- b. To determine the new value of the constant, we need to change the units of the constant to appropriate SI units using conversion factors.

$$\frac{0.0549 (\text{lb}_m)(\text{in}^2)(^\circ\text{R})^{0.5}}{(\text{lb}_f)(\text{min})} \left| \frac{(0.454 \text{ kf})}{(1 \text{ lb}_m)} \right| \frac{(14.7 \text{ lb}_f / \text{in}^2)}{101.3 \times 10^3 \text{ N/m}^2} \left| \frac{(1 \text{ min})}{(60 \text{ s})} \right| \frac{(\text{K})^{0.5}}{(1.8 \text{ }^\circ\text{R})^{0.5}} \left| \frac{(\text{p})}{(\text{T})^{0.5}} \right.$$

$$\mathbf{m} = 4.49 \times 10^{-8} (\text{m}) (\text{s}) (\text{K})^{0.5} \frac{(\text{p})}{(\text{T})^{0.5}}$$

Substituting pressure and temperature in SI units

$$\mathbf{m} = 4.49 \times 10^{-8} (\text{m}) (\text{s}) (\text{K})^{0.5} \frac{(\text{p})(\text{N/m}^2)}{(\text{T})^{0.5}(\text{K})^{0.5}} \left| \frac{1 \text{ kg}/(\text{m})(\text{s})^2}{1 \text{ N/m}^2} \right.$$

$$\mathbf{m} \frac{(\text{kg})}{(\text{s})} = 4.49 \times 10^{-8} \frac{(\text{p})}{(\text{T})^{0.5}} \quad \text{where p is in N/m}^2 \text{ and T is in K}$$

Problem4

An empirical equation for calculating the inside heat transfer coefficient, h_i , for the turbulent flow of liquids in a pipe is given by:

$$h_i = \frac{0.023 G^{0.8} K^{0.67} C_p^{0.33}}{D^{0.2} \mu^{0.47}}$$

where h_i = heat transfer coefficient, Btu/(hr)(ft)²(°F)

G = mass velocity of the liquid, lb_m/(hr)(ft)²

K = thermal conductivity of the liquid, Btu/(hr)(ft)(°F)

C_p = heat capacity of the liquid, Btu/(lb_m)(°F)

μ = Viscosity of the liquid, lb_m/(ft)(hr)

D = inside diameter of the pipe, (ft)

- Verify if the equation is dimensionally consistent.
- What will be the value of the constant, given as 0.023, if all the variables in the equation are inserted in SI units and h_i is in SI units.

Solution

a. First we introduce American engineering units into the equation:

$$h_i = \frac{0.023[(\text{lb}_m)/(\text{ft})^2(\text{hr})]^{0.80} [\text{Btu}/(\text{hr})(\text{ft})(^\circ\text{F})]^{0.67} [\text{Btu}/(\text{lb}_m)(^\circ\text{F})]^{0.33}}{(\text{ft})^{0.2} [\text{lb}_m/(\text{ft})(\text{hr})]^{0.47}}$$

$$h_i = \frac{0.023(\text{Btu})^{0.67} (\text{lb}_m)^{0.8}}{[(\text{lb}_m)^{0.33}(\text{lb}_m)^{0.47}]} \left| \frac{(\text{ft})^{0.47}}{[(\text{ft})^{1.6}(\text{ft})^{0.67}(\text{ft})^{0.2}]} \right| \left| \frac{(1)}{[(^\circ\text{F})^{0.67}(\text{ft})^{0.33}]} \right| \left| \frac{(\text{hr})^{0.47}}{[(\text{hr})^{0.8}(\text{hr})^{0.67}]} \right|$$

$$h_i = 0.023 \frac{\text{Btu}}{(\text{hr})(\text{ft})^2 (^\circ\text{F})}$$

The equation is dimensionally consistent.

b. The constant 0.023 is dimensionless; a change in units of the equation parameters will not have any effect on the value of this constant.

Chapter2

Moles, Density and Concentration

2.1 The Mole

In the SI system a mole is composed of 6.022×10^{23} molecules (**Avogadro's number**). To convert the number of moles to mass and the mass to moles, we make use of the **molecular weight** – the mass per mole:

$$\text{Molecular Weight (MW)} = \frac{\text{Mass}}{\text{Mole}}$$

Thus, the calculations you carry out are

$$\text{the g mol} = \frac{\text{mass in g}}{\text{molecular weight}}$$

$$\text{the lb mol} = \frac{\text{mass in lb}}{\text{molecular weight}}$$

and

$$\text{Mass in g} = (\text{MW}) (\text{gmol})$$

$$\text{Mass in lb} = (\text{MW}) (\text{lbmol})$$

Forexample

$$\frac{100.0 \text{ g H}_2\text{O}}{18.0 \text{ g H}_2\text{O}} \left| \frac{1 \text{ g mol H}_2\text{O}}{18.0 \text{ g H}_2\text{O}} \right. = 5.56 \text{ g mol H}_2\text{O}$$

$$\frac{6.0 \text{ lb mol O}_2}{1 \text{ lb mol O}_2} \left| \frac{32.0 \text{ lb O}_2}{1 \text{ lb mol O}_2} \right. = 192 \text{ lb O}_2$$

- ❖ **The atomic weight** of an element is the mass of an atom based on the scale that assigns a mass of exactly 12 to the carbon isotope ^{12}C .
- ❖ A **compound** is composed of more than one atom, and the molecular weight of the compound is nothing more than the sum of the weights of atoms of which it is composed.

Example 2.1

What is the molecular weight of the following cell of a superconductor material? (The figure represents one cell of a larger structure.)

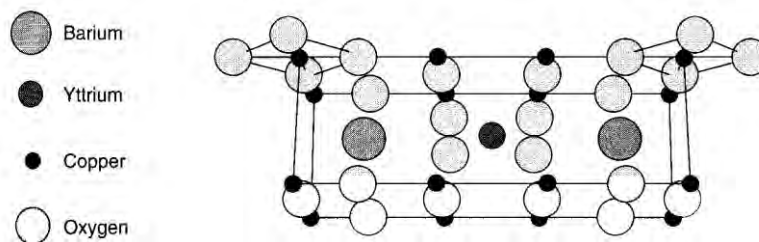


Figure E2.1

Solution

Element	Number of atoms	Atomic weights	Mass (g)
Ba	2	137.34	2(137.34)
Cu	16	63.546	16(63.546)
O	24	16.00	24(16.00)
Y	1	88.905	<u>1(88.905)</u>
		Total	1764.3

The molecular weight of the cell for each mole is 1764.3 g/gmol.

Example 2.2

If a bucket holds 2.00 lb of NaOH (MW=40), how many

- a) Pound moles of NaOH does it contain?
- b) Gram moles of NaOH does it contain?

Solution

$$(a) \frac{2.00 \text{ lb NaOH}}{1} \left| \frac{1 \text{ lb mol NaOH}}{40.0 \text{ lb NaOH}} \right. = 0.050 \text{ lb mol NaOH}$$

$$(b_1) \frac{2.00 \text{ lb NaOH}}{1} \left| \frac{1 \text{ lb mol NaOH}}{40.0 \text{ lb NaOH}} \right| \frac{454 \text{ g mol}}{1 \text{ lb mol}} = 22.7 \text{ g mol}$$

$$(b_2) \frac{2.00 \text{ lb NaOH}}{1} \left| \frac{454 \text{ g}}{1 \text{ lb}} \right| \frac{1 \text{ g mol NaOH}}{40.0 \text{ g NaOH}} = 22.7 \text{ g mol}$$

Example 2.3

How many pounds of NaOH (MW=40) are in 7.50 g mol of NaOH?

Solution

$$\frac{7.50 \text{ g mol NaOH}}{1} \left| \frac{1 \text{ lb mol}}{454 \text{ g mol}} \right| \frac{40.0 \text{ lb NaOH}}{1 \text{ lb mol NaOH}} = 0.661 \text{ lb NaOH}$$

2.2 Density

Density is the ratio of mass per unit volume, as for example, kg/m³ or lb/ft³. Density has both a numerical value and units. **Specific volume** is the inverse of density, such as cm³/g or ft³/lb.

$$\rho = \text{density} = \frac{\text{mass}}{\text{volume}} = \frac{m}{V}$$

$$\hat{V} = \text{specific volume} = \frac{\text{volume}}{\text{mass}} = \frac{V}{m}$$

For example, given that the density of n-propyl alcohol is 0.804 g/cm³, what would be the volume of 90.0 g of the alcohol? The calculation is

$$\frac{90.0 \text{ g}}{1} \left| \frac{1 \text{ cm}^3}{0.804 \text{ g}} \right. = 112 \text{ cm}^3$$

- ❖ In a packed bed of solid particles containing void spaces, the bulk density is

$$\rho_B = \text{bulk density} = \frac{\text{total mass of solids}}{\text{total empty bed volume}}$$

- ❖ A homogeneous mixture of two or more components, whether solid, liquid, or gaseous, is called a **solution**.

For some solutions, the density of the solution is

$$V = \sum_{i=1}^n V_i \quad \text{where } n = \text{number of components}$$

$$m = \sum_{i=1}^n m_i$$

$$\rho_{\text{solution}} = \frac{m}{V}$$

For others you cannot.

2.3 Specific Gravity الثقل النوعي

Specific gravity is commonly thought of as a dimensionless ratio.

$$\text{sp.gr. of } A = \text{specific gravity of } A = \frac{(\text{g/cm}^3)_A}{(\text{g/cm}^3)_{\text{ref}}} = \frac{(\text{kg/m}^3)_A}{(\text{kg/m}^3)_{\text{ref}}} = \frac{(\text{lb/ft}^3)_A}{(\text{lb/ft}^3)_{\text{ref}}}$$

- ◆ The reference substance for **liquids** and **solids** normally is **water**.
- ◆ The density of water is **1.000 g/cm³, 1000 kg/m³, or 62.43 lb/ft³ at 4°C**.
- ◆ The specific gravity of **gases** frequently is referred to **air**, but may be referred to other gases.

For Example If dibromopentane (DBP) has a specific gravity of 1.57, what is the density in (a) g/cm³? (b) lb_m/ft³? and (c) kg/m³?

$$(a) \quad \frac{1.57 \frac{\text{g DBP}}{\text{cm}^3}}{1.00 \frac{\text{g H}_2\text{O}}{\text{cm}^3}} \left| \frac{1.00 \frac{\text{g H}_2\text{O}}{\text{cm}^3}}{\text{cm}^3} \right. = 1.57 \frac{\text{g DBP}}{\text{cm}^3}$$

$$(b) \quad \frac{1.57 \frac{\text{lb DBP}}{\text{ft}^3}}{1.00 \frac{\text{lb H}_2\text{O}}{\text{ft}^3}} \left| \frac{62.4 \frac{\text{lb H}_2\text{O}}{\text{ft}^3}}{\text{ft}^3} \right. = 97.97 \frac{\text{lb DBP}}{\text{ft}^3}$$

$$(c) \quad \frac{1.57 \frac{\text{g DBP}}{\text{cm}^3}}{\text{cm}^3} \left| \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^3 \right| \frac{1 \text{ kg}}{1000 \text{ g}} = 1.57 \times 10^3 \frac{\text{kg DBP}}{\text{m}^3}$$

or

$$\frac{1.57 \frac{\text{kg DBP}}{\text{m}^3}}{1.00 \frac{\text{kg H}_2\text{O}}{\text{m}^3}} \left| \frac{1.00 \times 10^3 \text{ kg H}_2\text{O}}{\text{m}^3} \right. = 1.57 \times 10^3 \frac{\text{kg DBP}}{\text{m}^3}$$

Example2.4

If a 70% (by weight) solution of glycerol has a specific gravity of 1.184 at 15°C, what is the density of the solution in (a) g/cm³? (b) lbm/ft³? and (c) kg/m³?

Solution

(a) $(1.184 \text{ g glycerol/ cm}^3)/(1 \text{ g water/ cm}^3) * (1 \text{ g water/ cm}^3) = 1.184 \text{ g solution/cm}^3$.

(b) $(1.184 \text{ lb glycerol/ft}^3)/(1 \text{ lb water/ft}^3) * (62.4 \text{ lb water/ft}^3) = 73.9 \text{ lbsolution/ft}^3$.

(c) $(1.184 \text{ kg glycerol/m}^3)/(1 \text{ kg water/m}^3) * (1000 \text{ kg water/m}^3) = 1.184 * 10^3 \text{ kgsolution/m}^3$.

The specific gravity of petroleum products is often reported in terms of a hydrometer scale called °API (American Petroleum Institute). The equation for the API scale is

$$^{\circ}\text{API} = \frac{141.5}{\text{sp.gr.}_{60^{\circ}\text{F}}} - 131.5 \quad (\text{API gravity}) \quad (2.1)$$

or

$$\text{sp.gr.}_{60^{\circ}} = \frac{141.5}{^{\circ}\text{API} + 131.5} \quad (2.2)$$

$60^{\circ}\text{F} = 15^{\circ}\text{C}$ Note: $T^{\circ}\text{F} = 1.8 T^{\circ}\text{C} + 32$ $T^{\circ}\text{C} = (T^{\circ}\text{F} - 32)/1.8$

The **volume** and therefore the **density** of petroleum products vary with **temperature**, and the petroleum industry has established 60 °F as the standard temperature for volume and API gravity.

Example2.5

In the production of a drug having a molecular weight of 192, the exit stream from the reactor flows at a rate of 10.5 L/min. The drug concentration is 41.2% (in water), and the specific gravity of the solution is 1.024. Calculate the concentration of the drug (in kg/L) in the exit stream, and the flow rate of the drug in kgmol/min.

Solution

Take 1 kg of the exit solution as a basis for convenience.

Basis: 1 kg solution

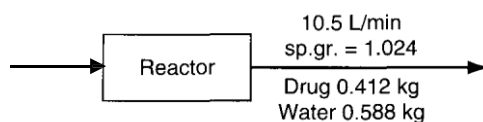


Figure E2.5

$$\text{density of solution} = \frac{1.024 \frac{\text{g soln}}{\text{cm}^3}}{1.000 \frac{\text{g H}_2\text{O}}{\text{cm}^3}} \left| \frac{1.000 \frac{\text{g H}_2\text{O}}{\text{cm}^3}}{1.000 \frac{\text{g H}_2\text{O}}{\text{cm}^3}} \right| = 1.024 \frac{\text{g soln}}{\text{cm}^3}$$

$$\frac{0.412 \text{ kg drug}}{1.000 \text{ kg soln}} \left| \frac{1.024 \text{ g soln}}{1 \text{ cm}^3} \right| \left| \frac{1 \text{ kg}}{10^3 \text{ g}} \right| \left| \frac{10^3 \text{ cm}^3}{1 \text{ L}} \right| = 0.422 \text{ kg drug/L soln}$$

To get the flow rate, take a different basis, namely 1 minute.

Basis: 1 min = 10.5 L solution

$$\frac{10.5 \text{ L soln}}{1 \text{ min}} \left| \frac{0.422 \text{ kg drug}}{1 \text{ L soln}} \right| \frac{1 \text{ kg mol drug}}{192 \text{ kg drug}} = 0.023 \text{ kg mol/min}$$

2.4 FlowRate

For continuous processes the **flow rate** of a process stream is the rate at which material is transported through a pipe. The **mass flow rate** (\dot{m}) of a process stream is the **mass (m)** transported through a line per unit **time(t)**.

$$\dot{m} = \frac{m}{t}$$

The **volumetric flow rate (F)** of a process stream is the **volume (V)** transported through a line per unit **time**.

$$F = \frac{V}{t}$$

The **molar flow rate** (\dot{n}) of a process stream is the number of **moles (n)** of a substance transported through a line per unit **time**.

$$\dot{n} = \frac{n}{t}$$

2.5 Mole Fraction and Mass (Weight) Fraction

- ☒ **Mole fraction** is simply the number of moles of a particular compound in a mixture or solution divided by the total number of moles in the mixture or solution.
- ☒ This definition holds for **gases, liquids, and solids**.
- ☒ Similarly, the **mass (weight) fraction** is nothing more than the **mass (weight)** of the compound divided by the total mass (weight) of all of the compounds in the mixture or solution.

Mathematically, these ideas can be expressed as

$$\text{mole fraction of } A = \frac{\text{moles of } A}{\text{total moles}}$$

$$\text{mass (weight) fraction of } A = \frac{\text{mass of } A}{\text{total mass}}$$

Mole percent and **mass (weight) percent** are the respective fractions times **100**.

Example 2.6

An industrial-strength drain cleaner contains 5 kg of water and 5 kg of NaOH. What are the mass (weight) fractions and mole fractions of each component in the drain cleaner container?

Solution

Basis: 10 kg of totalsolution

Component	kg	Weight fraction	Mol. Wt.	kg mol	Mole fraction
H ₂ O	5.00	$\frac{5.00}{10.0} = 0.500$	18.0	0.278	$\frac{0.278}{0.403} = 0.69$
NaOH	5.00	$\frac{5.00}{10.00} = 0.500$	40.0	0.125	$\frac{0.125}{0.403} = 0.31$
Total	10.00	1.000		0.403	1.00

The kilogram moles are calculated as follows:

$$\frac{5.00 \text{ kg H}_2\text{O}}{18.0 \text{ kg H}_2\text{O}} \left| \frac{1 \text{ kg mol H}_2\text{O}}{18.0 \text{ kg H}_2\text{O}} \right. = 0.278 \text{ kg mol H}_2\text{O}$$

$$\frac{5.00 \text{ kg NaOH}}{40.0 \text{ kg NaOH}} \left| \frac{1 \text{ kg mol NaOH}}{40.0 \text{ kg NaOH}} \right. = 0.125 \text{ kg mol NaOH}$$

Adding these quantities together gives the total kilogrammoles.

Example2.7

In normal living cells, the nitrogen requirement for the cells is provided from protein metabolism (i.e., consumption of the protein in the cells). When individual cells are commercially grown, (NH₄)₂SO₄ is usually used as the source of nitrogen. Determine the amount of (NH₄)₂SO₄ consumed in a fermentation medium in which the final cell concentration is 35 g/L in a 500 L volume of the fermentation medium. Assume that the cells contain 9 wt.% N, and that (NH₄)₂SO₄ is the only nitrogen source.

Solution

Basis: 500 L solution containing 35 g/L

$$\frac{500 \text{ L}}{1} \left| \frac{35 \text{ g cell}}{\text{L}} \right| \left| \frac{0.09 \text{ g N}}{1 \text{ g cell}} \right| \left| \frac{1 \text{ g mol N}}{14 \text{ g N}} \right. \times \left. \left| \frac{1 \text{ g mol (NH}_4\text{)}_2\text{SO}_4}{2 \text{ g mol N}} \right| \left| \frac{132 \text{ g (NH}_4\text{)}_2\text{SO}_4}{1 \text{ g mol (NH}_4\text{)}_2\text{SO}_4} \right. = 7425 \text{ g (NH}_4\text{)}_2\text{SO}_4$$

2.6 Analyses of Multicomponent Solutions and Mixtures

The **composition of gases** will always be assumed to be given in **mole percent** or **fraction** unless specifically stated otherwise.

The **composition of liquids and solids** will be given by **mass (weight) percent** or **fraction** unless otherwise specifically stated.

For Example Table below lists the detailed composition of dry air (composition of air 21% O₂ and 79% N₂). Calculate the average molecular weight of air?

Basis 100 mol of air

Component	Moles = percent	Mol. wt.	Lb or kg	Weight %
O ₂	21.0	32	672	23.17
N ₂	<u>79.0</u>	28.2	<u>2228</u>	<u>76.83</u>
Total	100		2900	100.00

The average molecular weight is 2900 lb/100 lb mol = 29.0, or 2900 kg/100 kg mol = 29

2.7 Concentration

Concentration generally refers to the quantity of some substance per unit volume.

- Mass per unit volume (lb of solute/ft³ of solution, g of solute/L, lb of solute/barrel, kg of solute/m³).
- Moles per unit volume (lb mol of solute/ft³ of solution, gmol of solute/L, gmol of solute/cm³).
- Parts per million (**ppm**); parts per billion (**ppb**), a method of expressing the concentration of extremely dilute solutions; **ppm is equivalent to a mass(weight) fraction for solids and liquids** because the total amount of material is so much higher order of magnitude than the amount of solute; it is a **mole fraction for gases**.
- Parts per million by volume (ppmv) and parts per billion by volume (ppbv)
- Other methods of expressing concentration with which you may be familiar are molarity (g mol/L), molality (mole solute/kg solvent), and normality (equivalents/L).

Example 2.8

The current Occupational Safety and Health Administration (OSHA) 8-hour limit for Hydrogen cyanide (HCN) (boils at 25.6 °C) (MW = 27.03) in a air is 10.0 ppm. A lethal dose of HCN in air is (from the Merck Index) 300 mg/kg of air at room temperature. How many mg HCN/kg air is 10 ppm? What fraction of the lethal dose is 10.0 ppm?

Solution

Basis: 1 kg mol of the air/HCN mixture

$$\text{The 10.0 ppm is } \frac{10.0 \text{ g mol HCN}}{10^6 (\text{air} + \text{HCN}) \text{ g mol}} = \frac{10.0 \text{ g mol HCN}}{10^6 \text{ g mol air}}$$

$$\text{a. } \frac{10.0 \text{ g mol HCN}}{10^6 \text{ g mol air}} \left| \frac{27.03 \text{ g HCN}}{1 \text{ g mol HCN}} \right| \left| \frac{1 \text{ g mol air}}{29 \text{ g air}} \right| \left| \frac{1000 \text{ mg HCN}}{1 \text{ g HCN}} \right| \times \frac{1000 \text{ g air}}{1 \text{ kg air}} = 9.32 \text{ mg HCN/kg air}$$

$$\text{b. } \frac{9.32}{300} = 0.031$$

Example 2.9

A solution of HNO₃ in water has a specific gravity of 1.10 at 25°C. The concentration of the HNO₃ is 15 g/L of solution. What is the

- Mole fraction of HNO₃ in the solution?

b. ppm of HNO₃ in the solution?

Solution

Basis: 1 L of solution

$$\text{Density} = 1.1 \times 1 \text{ g/cm}^3 = 1.1 \text{ g/cm}^3 \text{ (density of solution)}$$

$$\frac{15 \text{ g HNO}_3}{1 \text{ L soln}} \left| \frac{1 \text{ L}}{1000 \text{ cm}^3} \right| \left| \frac{1 \text{ cm}^3}{1.10 \text{ g soln}} \right| = 0.01364 \frac{\text{g HNO}_3}{\text{g soln}}$$

Basis: 100 g solution

The mass of water in the solution is: $100 - 1.364 = 98.636 \text{ g H}_2\text{O}$.

	g	MW	gmol	mol fraction
HNO ₃	1.364	63.02	0.02164	0.00394
H ₂ O	98.636	18.016	<u>5.475</u>	<u>0.99606</u>
Total			5.4966	1

b. $\frac{0.01364}{1} = \frac{13,640}{10^6}$ or 13,640 ppm

Example 2.10

Sulfur trioxide (SO₃) can be absorbed in sulfuric acid solution to form more concentrated sulfuric acid. If the gas to be absorbed contains 55% SO₃, 41% N₂, 3% SO₂, and 1% O₂, how many parts per million of O₂ are there in the gas? What is the composition of the gas on a N₂ free basis?

Solution

(a) $\frac{1 \text{ mol O}_2}{100 \text{ mol gas}} \Rightarrow \frac{10^4 \text{ mol O}_2}{10^6 \text{ mol gas}}$ or 10⁴ ppm

(b) Basis: 100 mol gas

answer

<u>Comp.</u>	<u>% = mol</u>	<u>mol fr.</u>	<u>or mol %</u>
SO ₃	55	0.932	93.2
SO ₂	3	0.051	5.1
O ₂	<u>1</u>	<u>0.017</u>	<u>1.7</u>
Total	59	1.000	100.0

Example 2.11

To avoid the possibility of explosion in a vessel containing gas having the composition of 40% N₂, 45% O₂, and 15% CH₄, the recommendation is to dilute the gas mixture by adding an equal amount of pure N₂. What is the final mole fraction of each gas?

Solution

The basis is 100 moles of initial gas

Chemical Engineering principles– First Year/ ChapterTwo

Composition	Original Mixture mol%	After Addition N ₂	Final Mixture Mole Fraction
N ₂	40 → +100	140	140/200 = 0.70
O ₂	45	45	45/200 = 0.23
CH ₄	15	15	15/200 = 0.07
Total	100	200	1.00

Example 2.12

Calculate the empirical formula of an organic compound with the following mass analysis: carbon, 26.9%; hydrogen, 2.2%; and oxygen as the only other element present.

Solution

Basis: 100 g of compound

	C	H	O
Mass (<i>m</i>) combining / g	26.9	2.2	70.9
Molar mass (<i>M</i>) / g mol ⁻¹	12	1	16
Number of moles combining (mass ÷ molar mass)	26.9 / 12 = 2.24	2.2 / 1 = 2.20	70.9 / 16 = 4.43
Ratio of number of moles	2.24 / 2.20 = 1.02	2.20 / 2.20 = 1.00	4.43 / 2.20 = 2.01
Simplest ratio	1	1	2

The empirical formula of this organic compound is C₁H₁O₂.

Questions

1. Answer the following questions true or false:
 - a. The pound mole is comprised of 2.73×10^{26} molecules
 - b. The kilogram mole is comprised of 6.022×10^{26} molecules.
 - c. Molecular weight is the mass of a compound or element per mole.
2. What is the molecular weight of acetic acid (CH₃COOH)?
3. For numbers such as 2 mL of water + 2 mL of ethanol, does the sum equal to 4 mL of the solution?
4. Answer the following questions true or false:
 - a. The inverse of the density is the specific volume.
 - b. Density of a substance is the mass per unit volume.
 - c. The density of water is less than the density of mercury.

5. A cubic centimeter of mercury has a mass of 13.6 g at Earth's surface. What is the density of mercury?
6. What is the approximate density of water at room temperature in kg/m^3 ?
7. For liquid HCN, a handbook gives: $\text{sp. gr. } 10^\circ\text{C}/4^\circ\text{C} = 1.2675$. What does this statement mean?
8. Answer the following questions true or false:
 - a. The density and specific gravity of mercury are the same.
 - b. Specific gravity is the ratio of two densities.
 - c. If you are given the value of a referenced density, you can determine the density of a substance of interest by multiplying by the specific gravity.
 - d. The specific gravity is a dimensionless quantity.
9. A mixture is reported as 15% water and 85% ethanol. Should the percentages be deemed to be by mass, mole, or volume?
10. Answer the following questions true or false:
 - a) In engineering practice the compositions of liquids and solids are usually denoted in weight (mass) fraction or percent.
 - b) In engineering practice the composition of gases is usually denoted in mole fraction or percent.
 - c) e. A pseudo-average molecular weight can be calculated for a mixture of pure components whether solid, liquid, or gases.
11. Do parts per million denote a concentration that is a molar ratio?
12. Does the concentration of a component in a mixture depend on the amount of the mixture?
13. Pick the correct answer. How many ppm are there in 1 ppb? (a) 1000, (b) 100, (c) 1, (d) 0.1, (e) 0.01, (f) 0.001?
14. How many ppb are there in 1 ppm?
15. Does 50 ppm represent an increase of five times a value of 10 ppm?

Answers:

1. (a) T; (b) T; (c) T
2. 60.05
3. No
4. (a) T; (b) T; (c) T
5. 13.6g/cm^3
6. 1000kg/m^3

7. The statement means that the density at 10°C of liquid HCN is 1.2675 times the density of water at 4°C .
8. (a) F – the units differ; (b) T; (c) T; (d) F.
9. Mass
10. (a) T; (b) T; (c) T
11. For gases but not for liquids or solids.
12. No
13. 0.001
14. 1000
15. No (4 times)

Problems

1. Convert the following:
 - a) 120 g mol of NaCl to g.
 - b) 120 g of NaCl to g mol.
 - c) 120 lb mol of NaCl to lb.
 - d) 120 lb of NaCl to lb mol.
2. Convert 39.8 kg of NaCl per 100 kg of water to kg mol of NaCl per kg mol of water.
3. How many lb mol of NaNO_3 are there in 100 lb?
4. The density of a material is 2 kg/m^3 . What is its specific volume?
5. An empty 10 gal tank weighs 4.5 lb. What is the total weight of the tank plus the water when it is filled with 5 gal of water?
6. If you add 50 g of sugar to 500 mL of water, how do you calculate the density of the sugar solution?
7. For ethanol, a handbook gives: sp. gr. $60^{\circ}\text{F} = 0.79389$. What is the density of ethanol at 60°F ?
8. The specific gravity of steel is 7.9. What is the volume in cubic feet of a steel ingot weighing 4000 lb?
9. The specific gravity of a solution is 0.80 at 70°F . How many cubic feet will be occupied by 100 lb of the solution at 70°F ?
10. A solution in water contains $1.704\text{ kg of HNO}_3/\text{kg H}_2\text{O}$, and the solution has a specific gravity of 1.382 at 20°C . What is the mass of HNO_3 in kg per cubic meter of solution at 20°C ?

- Forty gal/min of a hydrocarbon fuel having a specific gravity of 0.91 flows into a tanktruck with a load limit of 40,000 lb of fuel. How long will it take to fill the tank in the truck?
- Pure chlorine enters a process. By measurement it is found that 2.4 kg of chlorine pass into the process every 3.1 minutes. Calculate the molar flow rate of the chlorine in kgmol/hr.
- Commercial sulfuric acid is 98% H_2SO_4 and 2% H_2O . What is the molar ratio of H_2SO_4 to H_2O ?
- A compound contains 50% sulfur and 50% oxygen by mass. Is the empirical formula of the compound (1) SO , (2) SO_2 , (3) SO_3 , or (4) SO_4 ?
- How many kg of activated carbon (a substance used in removing trace impurities) must be mixed with 38 kg of sand so that the final mixture is 28% activated carbon?
- A gas mixture contains 40 lb of O_2 , 25 lb of SO_2 , and 30 lb of SO_3 . What is the composition of the mixture in mole fractions?
- Saccharin, an artificial sweetener that is 3000 times sweeter than sucrose, is composed of 45.90% carbon, 2.73% hydrogen, 26.23% oxygen, 7.65% nitrogen, and 17.49% sulfur. Is the molecular formula of saccharin (a) $\text{C}_{14}\text{H}_{10}\text{O}_6\text{N}_2\text{S}_2$, (b) $\text{C}_5\text{H}_7\text{O}_3\text{NS}$, (c) $\text{C}_8\text{H}_9\text{O}_2\text{NS}$, and (d) $\text{C}_7\text{H}_5\text{O}_3\text{NS}$?
- A mixture of gases is analyzed and found to have the following composition: CO_2 12.0%, CO 6.0%, CH_4 27.3%, H_2 9.9% and N_2 44.8%. How much will 3 lbmol of this gas weigh?
- Aliquid mixture of n-butane, n-pentane, and n-hexane has the following composition: n- C_4H_{10} 50%, n- C_5H_{12} 30%, and n- C_6H_{14} 20%. For this mixture, calculate:
 - The weight fraction of each component.
 - The mole fraction of each component.
 - The mole percent of each component.
 - The average molecular weight of the mixture.
- How many mg/L is equivalent to a 1.2% solution of a substance in water?

Answers:

- (a) 7010 g; (b) 2.05 g mol; (c) 7010 lb; (d) 2.05 lbmol
- 0.123 kg mol NaCl/kg mol H_2O
- 1.177 lbmol
- $0.5\text{m}^3/\text{kg}$
- 46.2 lb
- Measure the mass of water (should be about 500g) and add it to 50g. Measure the volume of the solution (will not be 450 mL). Divide the mass by the volume.

7. 0.79389 g/cm³ (assuming the density of water is also at 60°F)
8. 8.11 ft³
9. 2 ft³
10. 870 kg HNO₃/m³ solution.
11. 132 min
12. 0.654 kgmol/hr
13. 9
14. SO₂ 15.
- 14.8 kg
16. O₂ 0.62; SO₂ 0.19; SO₃ 0.19
17. (d)
18. 72.17 lb
19. (a) C₄: 0.50, C₅: 0.30, C₆: 0.20; (b) C₄: 0.57, C₅: 0.28, C₆: 0.15; (c) C₄: 57, C₅: 28, C₆: 15;
(d) 66.4 kg/kgmol
20. 12000 mg/L

Supplementary Problems (ChapterTwo):

Problem1

Calcium carbonate is a naturally occurring white solid used in the manufacture of lime and cement. Calculate the number of lb mols of calcium carbonate in:

- a. 50 g mol of CaCO₃.
- b. 150 kg of CaCO₃.
- c. 100 lb of CaCO₃.

Solution

$$\text{a. } \frac{50 \text{ g mol CaCO}_3}{1 \text{ g mol CaCO}_3} \left| \frac{100 \text{ g CaCO}_3}{454 \text{ g CaCO}_3} \right| \frac{1 \text{ lb CaCO}_3}{100 \text{ lb CaCO}_3} = 0.11 \text{ lb mol}$$

$$\text{b. } \frac{150 \text{ kg CaCO}_3}{1 \text{ kg CaCO}_3} \left| \frac{2.205 \text{ lb CaCO}_3}{100 \text{ lb CaCO}_3} \right| = 3.30 \text{ lb mol}$$

$$\text{c. } \frac{100 \text{ lb CaCO}_3}{100 \text{ lb CaCO}_3} = 1.00 \text{ lb mol CaCO}_3$$

Problem2

Silver nitrate (lunar caustic) is a white crystalline salt, used in marking inks, medicine and chemical analysis. How many kilograms of silver nitrate (AgNO_3) are there in :

- a. 13.0 lb mol AgNO_3 .
- b. 55.0 g mol AgNO_3

Solution

$$\begin{aligned} \text{a. } & \frac{13.0 \text{ lb mol AgNO}_3}{1 \text{ lb mol AgNO}_3} \left| \frac{170 \text{ lb AgNO}_3}{1 \text{ lb mol AgNO}_3} \right| \left| \frac{1 \text{ kg}}{2.205 \text{ lb}} \right| = 1002 \text{ kg or } 1000 \text{ kg} \\ \text{b. } & \frac{55.0 \text{ g mol AgNO}_3}{1 \text{ g mol AgNO}_3} \left| \frac{170 \text{ g AgNO}_3}{1 \text{ g mol AgNO}_3} \right| \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| = 9.35 \text{ kg} \end{aligned}$$

Problem3

Phosphoric acid is a colorless deliquescent acid used in the manufacture of fertilizers and as a flavoring agent in drinks. For a given 10 wt % phosphoric acid solution of specific gravity 1.10 determine:

- a. the mol fraction composition of this mixture.
- b. the volume (in gallons) of this solution which would contain 1 g mol H_3PO_4 .

Solution

a. Basis: 100 g of 10 wt% solution

	g	MW	g mol	mol fr
H_3PO_4	10	97.97	0.102	0.020
H_2O	90	18.01	5.00	0.980

b. Specific gravity = $\frac{\rho_{\text{soln}}}{\rho_{\text{ref}}}$ The ref. liquid is water

The density of the solution is $\frac{1.10 \text{ g soln/cm}^3 \text{ soln}}{1.00 \text{ g H}_2\text{O/cm}^3} \left| \frac{1.00 \text{ g H}_2\text{O/cm}^3}{1.00 \text{ g H}_2\text{O/cm}^3} \right| = 1.10 \frac{\text{g soln}}{\text{cm}^3}$

$$\frac{1 \text{ cm}^3 \text{ soln}}{1.10 \text{ g soln}} \left| \frac{1 \text{ g soln}}{0.1 \text{ g H}_3\text{PO}_4} \right| \left| \frac{97.97 \text{ g H}_3\text{PO}_4}{1 \text{ g mol H}_3\text{PO}_4} \right| \left| \frac{264.2 \text{ gal}}{10^6 \text{ cm}^3} \right| = 0.24 \text{ gal/g mol}$$

Problem4

The density of a liquid is 1500 kg/m³ at 20 °C.

- a. What is the specific gravity 20°C/4°C of this material.
- b. What volume (ft³) does 140 lb_m of this material occupy at 20°C.

Solution

Assume the reference substance is water which has a density of 1000 kg/m³ at 4°C.

$$\text{a. Specific gravity} = \frac{\rho_{\text{soln}}}{\rho_{\text{ref}}} = \frac{(\text{kg/m}^3)_{\text{soln}}}{(\text{kg/m}^3)_{\text{ref}}} = \frac{1500 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = 1.50$$

$$\text{b. } \frac{1 \text{ m}^3 \text{ liquid}}{1500 \text{ kg}} \left| \frac{1 \text{ kg}}{2.20 \text{ lb}} \right| \frac{35.31 \text{ ft}^3}{1 \text{ m}^3} \left| \frac{140 \text{ lb}_m}{140 \text{ lb}_m} \right| = 1.50 \text{ ft}^3$$

Problem5

The 1993 Environmental Protection Agency (EPA) regulation contains standards for 84 chemicals and minerals in drinking water. According to the EPA one of the most prevalent of the listed contaminants is naturally occurring antimony. The maximum contaminant level for antimony and nickel has been set at 0.006 mg/L and 0.1 mg/L respectively.

A laboratory analysis of your household drinking water shows the antimony concentration to be 4 ppb (parts per billion) and that of nickel to be 60 ppb. Determine if the drinking water is safe with respect to the antimony and nickel levels.

Assume density of water to be 1.00 g/cm³

Solution

Antimony

$$\frac{0.006 \text{ mg Sb}}{1 \text{ L soln}} \left| \frac{1 \text{ L soln}}{1000 \text{ cm}^3 \text{ soln}} \right| \frac{1 \text{ cm}^3 \text{ soln}}{1.00 \text{ g H}_2\text{O}} \left| \frac{1 \text{ g}}{1000 \text{ mg}} \right| = \frac{6 \text{ g Sb}}{10^9 \text{ g soln}} = 6 \text{ ppb}$$

Nickel

$$\frac{0.1 \text{ mg Ni}}{1 \text{ L soln}} \left| \frac{1 \text{ L soln}}{1000 \text{ cm}^3 \text{ soln}} \right| \frac{1 \text{ cm}^3 \text{ soln}}{1.0 \text{ g H}_2\text{O}} \left| \frac{1 \text{ g}}{1000 \text{ mg}} \right| = \frac{9 \text{ g Ni}}{10^9 \text{ g soln}} = 100 \text{ ppb}$$

House hold drinking water contains less than the EPA mandated tolerance levels of antimony and nickel. Drinking water is therefore safe.

Problem6

Wine making involves a series of very complex reactions most of which are performed by microorganisms. The starting concentration of sugars determines the final alcohol content and sweetness of the wine. The specific gravity of the starting stock is therefore adjusted to achieve desired quality of wine.

A starting stock solution has a specific gravity of 1.075 and contains 12.7 wt% sugar. If all the sugar is assumed to be $C_{12}H_{22}O_{11}$, determine

- kg sugar/kg H_2O
- lb solution/ft³ solution
- g sugar/L solution

Solution

Basis: 100 kg starting stock solution

$$a. \frac{12.7 \text{ kg sugar}}{100 \text{ kg soln}} \left| \frac{100 \text{ kg solution}}{87.3 \text{ kg } H_2O} \right. = .145 \frac{\text{kg sugar}}{\text{kg } H_2O}$$

$$b. \frac{1.075 \text{ g soln/cm}^3}{1.0 \text{ g } H_2O/\text{cm}^3} \left| \frac{1.00 \text{ g } H_2O/\text{cm}^3}{1.00 \text{ g } H_2O/\text{cm}^3} \right| \left| \frac{1 \text{ lb}}{454 \text{ g}} \right| \left| \frac{2.832 \times 10^4 \text{ cm}^3}{\text{ft}^3} \right. = 67.1 \frac{\text{lb soln}}{\text{ft}^3 \text{ soln}}$$

$$c. \frac{1.075 \text{ g soln/cm}^3}{1.0 \text{ g } H_2O/\text{cm}^3} \left| \frac{1.0 \text{ g } H_2O/\text{cm}^3}{1.0 \text{ g } H_2O/\text{cm}^3} \right| \left| \frac{12.7 \text{ g sugar}}{100 \text{ g soln}} \right| \left| \frac{1000 \text{ cm}^3}{1 \text{ L}} \right. = 136 \frac{\text{g sugar}}{\text{L soln}}$$

Problem7

How many ppb are there in 1 ppm? Does the system of units affect your answer? Does it make any difference if the material for which the ppb are measured is a gas, liquid, or solid?

Solution

a) **1000**

b) **No**

c) **Yes, because for solids and liquids the ratio in ppb is mass whereas for gases the ratio is in moles.**

Chapter 3

Choosing a Basis

- ❖ A **basis** is a reference chosen by you for the calculations you plan to make in any particular problem, and a proper choice of basis frequently makes the problem much easier to solve.
- ❖ The **basis** may be a **period of time** such as **hours**, or a **given mass of material**, such as **5 kg** of CO₂, or some other convenient quantity.
- ❖ For **liquids** and **solids** in which a **mass (weight)** analysis applies, a convenient basis is often **1 or 100 lb or kg**; similarly, **1 or 100 moles** is often a good choice for **gas**.

Example 3.1

A gas mixture 10.0% H₂, 40.0% CH₄, 30.0% CO, and 20.0% CO₂, what is the average molecular weight of the gas?

Solution

Basis: 100 kg mol or lb mol of gas

Component	Percent = kg mol or lb mol	Mol wt.	Kg or lb
CO ₂	20.0	44.0	880
CO	30.0	28.0	840
CH ₄	40.0	16.04	642
H ₂	10.0	2.02	20
Total	100.0		2382

$$\text{Average molecular weight} = \frac{2382 \text{ kg}}{100 \text{ kg mol}} = 23.8 \text{ kg/kg mol}$$

Other Method for Solution:

$$\text{Average molecular weight} = 0.2 * 44 + 0.3 * 28 + 0.4 * 16.04 + 0.1 * 2.02 = 23.8 \text{ kg/kgmol}$$

Example 3.2

A liquefied mixture has the following composition: (Butane) n-C₄H₁₀ 50% (MW=58), (Pentane) n-C₅H₁₂ 30% (MW=72), and (hexane) n-C₆H₁₄ 20% (MW=86). For this mixture, calculate: (a) mole fraction of each component. (b) Average molecular weight of the mixture.

Solution

Chemical Engineering principles– First Year/ ChapterThree

Basis: 100 kg

	% = kg	wt fr	MW	kg mol	mol fr
n - C ₄ H ₁₀	50	0.50	58	0.86	0.57
n - C ₅ H ₁₂	30	0.30	72	0.42	0.28
n - C ₆ H ₁₄	20	0.20	86	0.23	0.15
	100	1.00		1.51	1.00

$$\text{Average molecular weight} = \frac{\text{total mass}}{\text{total mol}} = \frac{100 \text{ kg}}{1.51 \text{ kg mol}} = 66$$

Example 3.3

A medium-grade bituminous coal analyzes as follows:

Component	Percent
S	2
N	1
O	6
Ash	11
Water	3
Residuum	77

The residuum is C and H, and the molar ratio in the residuum is H/C=9. Calculate the weight (mass) fraction composition of the coal with the ash and the moisture omitted (ash – and moisture– free).

Solution

Take as a basis 100 kg of coal because then percent = kilograms.

Basis: 100 kg of coal

The sum of the S + N + O + ash + water is $2 + 1 + 6 + 11 + 3 = 23 \text{ kg}$

We need to determine the individual kg of **C** and of **H** in the **77 kg** total residuum.

To determine the kilograms of C and H, you have to select a new basis.

Basis: 100 kg mol (Because the H/C ratio is given in terms of moles, not weight)

Component	Mole fraction	kg mol	Mol. wt.	kg	Mass fraction
H	$\frac{9}{1+9} = 0.90$	90	1.008	90.7	0.43
C	$\frac{1}{1+9} = \frac{0.10}{1.00}$	$\frac{10}{100}$	12	$\frac{120}{210.7}$	$\frac{0.57}{1.00}$

H: (77kg) (0.43) = 33.15kg

C: (77kg) (0.57) = 43.85kg

Finally, we can prepare a table summarizing the results on the basis of **1 kg of the coal ash-free and water-free**.

Component	kg	Wt. fraction
C	43.85	0.51
H	33.15	0.39
S	2	0.02
N	1	0.01
O	<u>6</u>	<u>0.07</u>
Total	86.0	1.00

Supplementary Problems (ChapterThree):

Problem1

1 mol of gas containing O₂ 20%, N₂ 78%, and SO₂ 2%, find the composition of the gas on an SO₂–free basis, meaning gas without the SO₂ init.

Solution

Basis: 1.00 mol gas

Components	Mol fraction	Mol	Mol SO ₂ free	Mol fraction SO ₂ free
O ₂	0.20	0.20	0.20	0.20
N ₂	0.78	0.78	0.78	0.80
SO ₂	0.02	0.02		
	1.00	1.00	0.98	1.00

Problem2

InaternaryalloysuchasNd_{4.5}Fe₇₇B_{18.5}theaveragegrainsizeisabout30nm.Byreplacing0.2 atoms of Fe with atoms of Cu, the grain size can be reduced (improved) to 17nm.

- (a) What is the molecular formula of the alloy after adding the Cu to replace theFe?
- (b) What is the mass fraction of each atomic species in the improvedalloy?

Solution

Basis: 100 g mol (or atoms) ofNd_{4.5}Fe₇₇B_{18.5}

- (a) The final alloy isNd_{4.5}Fe_{76.8}B_{18.5}Cu_{0.2}.
- (b) Use a table to calculate the respective massfractions.

Component	Original g mol	Final g mol	MW	g	Mass fraction
Nd	4.5	4.5	144.24	649.08	0.126
Fe	77	76.8	55.85	4289.28	0.833
B	18.5	18.5	10.81	199.99	0.039
Cu		0.2	63.55	12.71	0.002
Total	100.0	100.0		5151.06	1.000

Problem 3 (Basic Principles.... Book, Page87)

Readeachofthefollowingproblemsandselectasuitablebasisforsolvingeachone.Donotsolve theproblems.

- a. Youhave130kgofgasofthefollowingcomposition:40%N₂,30%CO₂,and30%CH₄ina tank. What is the average molecular weight of thegas?

- b. You have 25 lb of a gas of the following composition: CH_4 80%, C_2H_4 10%, and C_2H_6 10%. What is the average molecular weight of the mixture? What is the weight (mass) fraction of each of the components in the mixture?
- c. The proximate and ultimate analysis of coal is given in the following table. What is the composition of the “Volatile combustible material” (VCM)? Present your answer in the form of the mass percent of each element in the VCM.

Proximate Analysis (%)		Ultimate Analysis (%)	
Moisture	3.2	Carbon	79.90
Volatile combustible material	21.0	Hydrogen	4.85
Fixed carbon	69.3	Sulfur	0.69
Ash	6.5	Nitrogen	1.30
		Ash	6.50
		Oxygen	6.76
Total	100.0	Total	100.00

- d. A fuel gas is reported to analyze, on a mole basis, 20% methane, 5% ethane, and the remainder C_2H_2 and O_2 . Calculate the analysis of the fuel gas on a mass percentage basis.

Solution

- (a) A gas requires a convenient basis of 1 or 100 g moles or kg moles (if use SI units).
- (b) A gas requires a convenient basis of 1 or 100 lb moles (if use AE units).
- (c) Use 1 or 100 kg of coal, or 1 or 100 lb of coal because the coal is a solid and mass is a convenient basis.
- (d) Use 1 or 100 moles (SI or AE) as a convenient basis as you have a gas.

Problem 4 (Basic Principles.... Book, Page 88)

Choose a basis for the following problem: Chlorine usage at a water treatment plant averages 134.2 lb/day. The average flow rate of water leaving the plant is 10.7 million gal/day. What is the average chlorine concentration in the treatment water leaving the plant (assuming no reaction of the chlorine), expressed in milligrams per liter?

Solution

Pick one day as a basis which is equivalent to what is given - - two numbers:

- (a) 134.2 lb Cl_2 (b) 10.7×10^6 gal water.

Chapter 4 Temperature

Temperature

- * **Temperature** is a measure of the energy (mostly kinetic) of the molecules in a system. This definition tells us about the amount of energy.
- * Other scientists prefer to say that **Temperature** is a property of the state of thermal equilibrium of the system with respect to other systems because temperature tells us about the capability of a system to transfer energy (as heat).

☒ Four types of temperature:

Two based on a relative scale, **degrees Fahrenheit (°F)** and **Celsius (°C)**, and two based on an absolute scale, **degree Rankine (°R)** and **Kelvin (K)**.

☒ Temperature Conversion

$$\Delta^{\circ}\text{F} = \Delta^{\circ}\text{R}$$

$$\Delta^{\circ}\text{C} = \Delta\text{K}$$

Also, the $\Delta^{\circ}\text{C}$ is larger than the $\Delta^{\circ}\text{F}$

$$\frac{\Delta^{\circ}\text{C}}{\Delta^{\circ}\text{F}} = 1.8 \quad \text{or} \quad \Delta^{\circ}\text{C} = 1.8 \Delta^{\circ}\text{F}$$

$$\frac{\Delta\text{K}}{\Delta^{\circ}\text{R}} = 1.8 \quad \text{or} \quad \Delta\text{K} = 1.8 \Delta^{\circ}\text{R}$$

Also, because of the temperature difference between boiling water and ice (Celsius: $100^{\circ}\text{C} - 0^{\circ}\text{C} = 100^{\circ}\text{C}$; Fahrenheit: $212^{\circ}\text{F} - 32^{\circ}\text{F} = 180^{\circ}\text{F}$), the following relationships hold:

$$\Delta^{\circ}\text{C} = 1.8000 \Delta^{\circ}\text{F} \quad \text{and} \quad \Delta\text{K} = 1.8000 \Delta^{\circ}\text{F}$$

The proper meaning of the symbols $^{\circ}\text{C}$, $^{\circ}\text{F}$, **K**, and $^{\circ}\text{R}$, as either the temperature or the unit temperature difference, must be interpreted from the context of the equation or sentence being examined.

Suppose you have the relation:

$$T_{\circ\text{F}} = a + bT_{\circ\text{C}}$$

What are the units of **a** and **b**? The units of **a** must be $^{\circ}\text{F}$ for consistency. The correct units for **b** must involve the conversion factor ($1.8 \Delta^{\circ}\text{F} / \Delta^{\circ}\text{C}$), the factor that converts the size of an interval on one temperature scale

$$T_{\circ\text{F}} = a_{\circ\text{F}} + \left(\frac{1.8 \Delta^{\circ\text{F}}}{\underbrace{\Delta^{\circ\text{C}}}_b} \right) T_{\circ\text{C}}$$

Unfortunately, the units for b are usually ignored; just the value of b (1.8) is employed.

★ The relations between °C, °F, K, and °R are:

$$T_{\circ\text{R}} = T_{\circ\text{F}} \left(\frac{1 \Delta^{\circ\text{R}}}{1 \Delta^{\circ\text{F}}} \right) + 460^{\circ\text{R}} \quad \text{Or} \quad \boxed{T_{\circ\text{R}} = T_{\circ\text{F}} + 460}$$

$$T_{\text{K}} = T_{\circ\text{C}} \left(\frac{1 \Delta\text{K}}{1 \Delta^{\circ\text{C}}} \right) + 273 \text{ K} \quad \text{Or} \quad \boxed{T_{\text{K}} = T_{\circ\text{C}} + 273}$$

$$T_{\circ\text{F}} - 32^{\circ\text{F}} = T_{\circ\text{C}} \left(\frac{1.8 \Delta^{\circ\text{F}}}{1 \Delta^{\circ\text{C}}} \right) \quad \text{Or} \quad \boxed{T_{\circ\text{F}} = 1.8 T_{\circ\text{C}} + 32}$$

Example 4.1

Convert 100 °C to (a) K, (b) °F, and (c) °R.

Solution

$$(a) (100 + 273)^{\circ\text{C}} \frac{1 \Delta\text{K}}{1 \Delta^{\circ\text{C}}} = 373 \text{ K}$$

or with suppression of the Δ symbol,

$$(100 + 273)^{\circ\text{C}} \frac{1 \text{ K}}{1^{\circ\text{C}}} = 373 \text{ K}$$

$$(b) (100^{\circ\text{C}}) \frac{1.8 \Delta^{\circ\text{F}}}{1 \Delta^{\circ\text{C}}} + 32^{\circ\text{F}} = 212^{\circ\text{F}}$$

$$(c) (212 + 460)^{\circ\text{F}} \frac{1 \Delta^{\circ\text{R}}}{1 \Delta^{\circ\text{F}}} = 672^{\circ\text{R}}$$

or

$$(373 \text{ K}) \frac{1.8 \Delta^{\circ\text{R}}}{1 \Delta\text{K}} = 672^{\circ\text{R}}$$

Example 4.2

The heat capacity of sulfuric acid has the units J/(g mol)(°C), and is given by the relation

$$\text{Heat capacity} = 139.1 + 1.56 * 10^{-1} T$$

where T is expressed in °C. Modify the formula so that the resulting expression has the associated units of Btu/(lbmol) (°R) and T is in °R.

Solution

$$T_{\circ F} = 1.8 T_{\circ C} + 32 \quad \longrightarrow \quad T_{\circ C} = (T_{\circ F} - 32)/1.8$$

$$T_{\circ R} = T_{\circ F} + 460 \quad \longrightarrow \quad T_{\circ F} = T_{\circ R} - 460$$

$$\therefore T_{\circ C} = [T_{\circ R} - 460 - 32]/1.8$$

$$\text{heat capacity} = \left\{ 139.1 + 1.56 \times 10^{-1} \left[\overbrace{(T_{\circ R} - 460 - 32)}^{T_{\circ C}} \frac{1}{1.8} \right] \right\} \times \underbrace{\frac{1 \text{ J}}{(\text{g mol})(\circ\text{C})} \left| \frac{1 \text{ Btu}}{1055 \text{ J}} \right| \frac{454 \text{ g mol}}{1 \text{ lb mol}} \left| \frac{1 \circ\text{C}}{1.8 \circ\text{R}} \right|}_{\text{conversion factors}} =$$

$$= 23.06 + 2.07 \times 10^{-2} T_{\circ R}$$

Note the suppression of the Δ symbol in the conversion between $\circ\text{C}$ and $\circ\text{R}$.

Problems

1. Complete the following table with the proper equivalent temperatures:

$\circ\text{C}$	$\circ\text{F}$	K	$\circ\text{R}$
-40	_____	_____	_____
_____	77.0	_____	_____
_____	_____	698	_____
_____	_____	_____	69.8

2. The heat capacity of sulfur is $C_p = 15.2 + 2.68T$, where C_p is in $\text{J}/(\text{g mol})(\text{K})$ and T is in K . Convert this expression so that C_p is in $\text{cal}/(\text{g mol})(\circ\text{F})$ with T in $\circ\text{F}$.

Answers:

1.

$\circ\text{C}$	$\circ\text{F}$	K	$\circ\text{R}$
-40.0	-40.0	233	420
25.0	77.0	298	537
425	796	698	1256
-234	-390	38.8	69.8

2. $C_p = 93.2 + 0.186T_{\circ F}$

Supplementary Problems (Chapter Four):

Problem 1

Complete the table below with the proper equivalent temperatures.

$\circ\text{C}$	$\circ\text{F}$	K	$\circ\text{R}$
-40.0	-----	-----	-----
-----	77.0	-----	-----
-----	-----	698	-----
-----	-----	-----	69.8

Solution

The conversion relations to use are:

$^{\circ}\text{F}$	=	$1.8\ ^{\circ}\text{C}$	+	32
K	=	$^{\circ}\text{C}$	+	273
$^{\circ}\text{R}$	=	$^{\circ}\text{F}$	+	460
$^{\circ}\text{R}$	=	$1.8\ \text{K}$		

$^{\circ}\text{C}$	$^{\circ}\text{F}$	K	$^{\circ}\text{R}$
- 40.0	- 40.0	233	420
25.0	77.0	298	437
425	797	698	1257
- 235	-390	38.4	69.8

Problem2

The specific heat capacity of toluene is given by following equation

$$C_p = 20.869 + 5.293 \times 10^{-2} T \quad \text{where } C_p \text{ is in Btu/(LB mol) } (^{\circ}\text{F}) \text{ and } T \text{ is in } ^{\circ}\text{F}$$

Express the equation in cal/(g mol) (K) with T in K.

Solution

First, conversion of the units for the overall equation is required.

$$C_p = \frac{[20.869 + 5.293 \times 10^{-2} (T_{\circ\text{F}})] \text{ Btu}}{1 \text{ (lb mol) } (^{\circ}\text{F})} \left| \frac{252 \text{ cal}}{1 \text{ Btu}} \right| \left| \frac{1 \text{ lb mol}}{454 \text{ g mol}} \right| \left| \frac{1.8 \text{ } ^{\circ}\text{F}}{1 \text{ K}} \right|$$

$$= [20.869 + 5.293 \times 10^{-2} (T_{\circ\text{F}})] \frac{\text{cal}}{(\text{g mol}) (\text{K})}$$

Note that the coefficients of the equation remain unchanged in the new units for this particular conversion. The T of the equation is still in $^{\circ}\text{F}$, and must be converted to kelvin.

$$T_{\circ\text{F}} = (T_{\text{K}} - 273) 1.8 + 32$$

$$C_p = 20.69 + 5.293 \times 10^{-2} [(T_{\text{K}} - 273) 1.8 + 32]$$

Simplifying $C_p = -3.447 + 9.527 \times 10^{-2} T_{\text{K}}$

Chapter5

Pressure

5.1 Pressure and Its Units

Pressure is defined as “the normal (perpendicular) force per unit area (Figure 5.1). The pressure at the bottom of the static (nonmoving) column of mercury exerted on the sealing plate is

$$p = \frac{F}{A} = \rho gh + p_0 \quad \dots 5.1$$

Where p = pressure at the bottom of the column of the fluid, F = force, A = area, ρ = density of fluid, g = acceleration of gravity, h = height of the fluid column, and p_0 = pressure at the top of the column of fluid

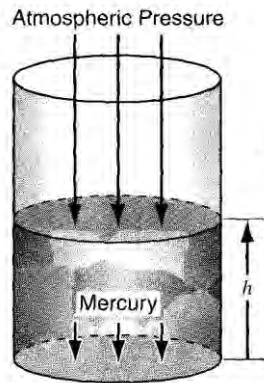


Figure 5.1 Pressure is the normal force per unit area. Arrows show the force exerted on the respective areas

For Example, suppose that the cylinder of fluid in Figure 5.1 is a column of mercury that has an area of 1 cm^2 and is 50 cm high. The density of the Hg is 13.55 g/cm^3 . Thus, the force exerted by the mercury alone on the 1 cm^2 section of the bottom plate by the column of mercury is

$$F = \frac{13.55 \text{ g}}{\text{cm}^3} \left| \frac{980 \text{ cm}}{\text{s}^2} \right| \left| \frac{50 \text{ cm}}{1} \right| \left| \frac{1 \text{ cm}^2}{1} \right| \left| \frac{1 \text{ kg}}{1000 \text{ g}} \right| \left| \frac{1 \text{ m}}{100 \text{ cm}} \right| \left| \frac{1(\text{N})(\text{s}^2)}{1(\text{kg})(\text{m})} \right|$$

$$= 6.64 \text{ N} \qquad P=F/A \quad \dots \quad F= P \cdot A = \rho g h \cdot A$$

The pressure on this section of the plate covered by the mercury is the force per unit area of the mercury plus the pressure of the atmosphere

$$p = \frac{6.64 \text{ N}}{1 \text{ cm}^2} \left| \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^2 \right| \left| \frac{(1 \text{ m}^2)(1 \text{ Pa})}{(1 \text{ N})} \right| \left| \frac{1 \text{ kPa}}{1000 \text{ Pa}} \right| + p_0 = 66.4 \text{ kPa} + p_0$$

If we had started with units in the AE system, the pressure would be computed as [the density of mercury is 845.5 lb_m/ft³]

$$p = \frac{845.5 \text{ lb}_m}{1 \text{ ft}^3} \left| \frac{32.2 \text{ ft}}{\text{s}^2} \right| \frac{50 \text{ cm}}{2.54 \text{ cm}} \left| \frac{1 \text{ in.}}{12 \text{ in.}} \right| \frac{1 \text{ ft}}{32.174(\text{ft})(\text{lb}_m)} + p_0$$

$$= 1388 \frac{\text{lb}_f}{\text{ft}^2} + p_0$$

5.2 Measurement of Pressure

Pressure, like temperature, can be expressed using either an **absolute** or a **relative scale**.

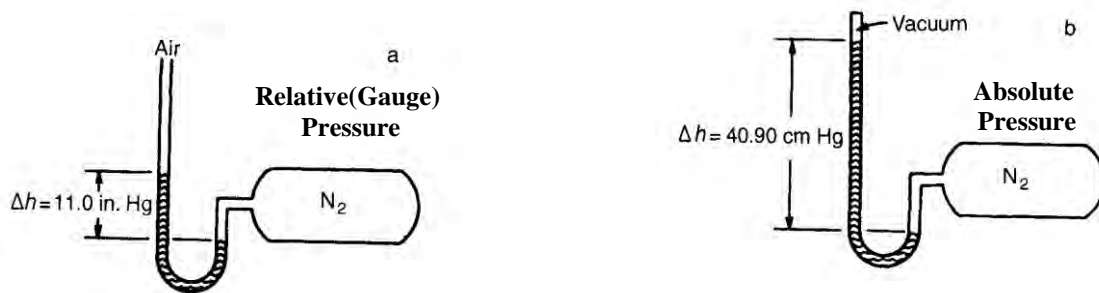


Figure 5.2 (a) **Open-end manometer** showing a pressure above atmospheric pressure. (b) **Manometer** measuring an **absolute pressure**.

The relationship between **relative** and **absolute pressure** is given by the following expression:

$$\text{Gauge Pressure} + \text{Barometer Pressure (atmospheric)} = \text{Absolute Pressure} \quad \dots 5.2$$

$$P_{\text{vacuum}} = P_{\text{atmospheric}} - P_{\text{absolute}}$$

- ☒ The **standard atmosphere** is defined as the pressure (in a standard gravitational field) equivalent to 1 atm or 760 mm Hg at 0°C or other equivalent.

The **standard atmosphere** is equal to

- ◆ 1.00 atmospheres (atm)
- ◆ 33.91 feet of water (ftH₂O)
- ◆ 14.7 pounds (force) per square inch absolute (psia)
- ◆ 29.92 inches of mercury (in.Hg)
- ◆ 760.0 millimeters of mercury (mmHg)
- ◆ 1.013 * 10⁵ pascal (Pa) or newtons per square meter (N/m²); or 101.3 kPa

For Example, convert 35 psia to inches of mercury and kPa.

$$\frac{35 \text{ psia}}{14.7 \text{ psia}} \left| \frac{29.92 \text{ in. Hg}}{14.7 \text{ psia}} \right| = 71.24 \text{ in Hg} \quad \text{And,} \quad \frac{35 \text{ psia}}{14.7 \text{ psia}} \left| \frac{101.3 \text{ kPa}}{14.7 \text{ psia}} \right| = 241 \text{ kPa}$$

For Example. What is the equivalent pressure to 1 kg/cm^2 (i.e., kgf/cm^2) in pascal ($g = 9.8 \text{ m/s}^2$)
 $[1 \text{ kg/cm}^2] * [9.8 \text{ m/s}^2] * [(100 \text{ cm/1 m})^2] = 9.8 * 10^4 \text{ N/m}^2$ (or Pa)

Example5.1

What is the equivalent pressure to 60 Gpa (gigapascal) in

- (a) atmospheres (b) psia (c) inches of Hg (d) mm of Hg

Solution

Basis: 60 GPa

- (a) $\frac{60 \text{ GPa}}{1 \text{ GPa}} \left| \frac{10^6 \text{ kPa}}{1 \text{ GPa}} \right| \frac{1 \text{ atm}}{101.3 \text{ kPa}} = 0.59 \times 10^6 \text{ atm}$
- (b) $\frac{60 \text{ GPa}}{1 \text{ GPa}} \left| \frac{10^6 \text{ kPa}}{1 \text{ GPa}} \right| \frac{14.696 \text{ psia}}{101.3 \text{ kPa}} = 8.70 \times 10^6 \text{ psia}$
- (c) $\frac{60 \text{ GPa}}{1 \text{ GPa}} \left| \frac{10^6 \text{ kPa}}{1 \text{ GPa}} \right| \frac{29.92 \text{ in. Hg}}{101.3 \text{ kPa}} = 1.77 \times 10^7 \text{ in. Hg}$
- (d) $\frac{60 \text{ GPa}}{1 \text{ GPa}} \left| \frac{10^6 \text{ kPa}}{1 \text{ GPa}} \right| \frac{760 \text{ mm Hg}}{101.3 \text{ kPa}} = 4.50 \times 10^8 \text{ mm Hg}$

Example5.2

The pressure gauge on a tank of CO₂ used to fill soda-water bottles reads 51.0 psi. At the same time the barometer reads 28.0 in. Hg. What is the absolute pressure in the tank in psia? See Figure E5.2.

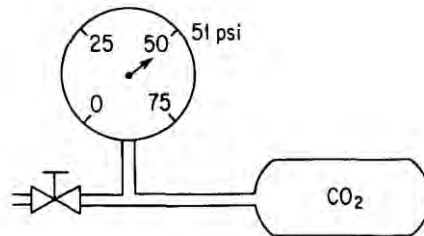


Figure E5.2

Solution

$$\text{Atmospheric pressure} = \frac{28.0 \text{ in. Hg}}{29.92 \text{ in Hg}} \left| \frac{14.7 \text{ psia}}{1 \text{ in Hg}} \right| = 13.76 \text{ psia}$$

The absolute pressure in the tank is

$$51.0 \text{ psia} + 13.76 \text{ psia} = 64.8 \text{ psia}$$

Example 5.3

Small animals such as mice can live (although not comfortably) at reduced air pressures down to 20 kPa absolute. In a test, a mercury manometer attached to a tank, as shown in Figure E5.3, reads 64.5 cm Hg and the barometer reads 100 kPa. Will the mice survive?

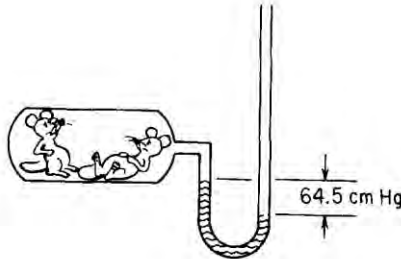


Figure E5.3

Solution

You are expected to realize from the figure that the tank is **below atmospheric pressure** because the **left leg** of the manometer is **higher** than the **right leg**, which is open to the atmosphere.

Consequently, to get the **absolute pressure** you **subtract** the **64.5 cm Hg** from the **barometer reading**.

The **absolute pressure** in the tank is

$$100 \text{ kPa} - \frac{64.5 \text{ cm Hg}}{76.0 \text{ cm Hg}} \times 101.3 \text{ kPa} = 100 - 86 = 14 \text{ kPa absolute}$$

The mice probably will **not survive**.

5.3 Differential Pressure Measurements

When the columns of fluids are at equilibrium (see Figure 5.3), the relationship among ρ_1, ρ_2, ρ_3 , and the heights of the various columns of fluid is as follows:

$$P_1 + \rho_1 d_1 g = P_2 + \rho_2 d_2 g + \rho_3 d_3 g \quad \dots 5.3$$

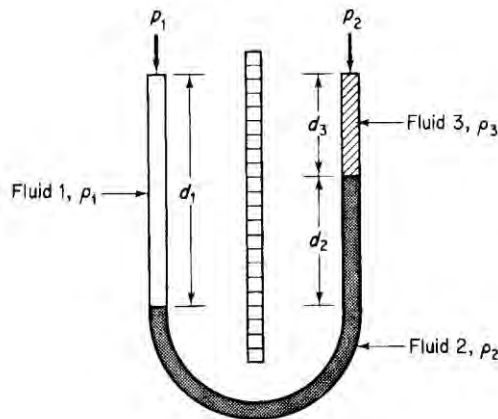


Figure 5.3 Manometer with three fluids.

Note

If fluids 1 and 3 are gases, and fluid 2 is mercury, the density of the gas is somuch less than that of mercury that you can **ignore** the term involving the gas in Equation (5.3) for practical applications.

- ★ Can you show for the case in which $\rho_1 = \rho_3 = \rho$ that the manometer expression reduces to the differential manometer equation:

$$P_1 - P_2 = (\rho_2 - \rho) g d_2 \quad \dots 5.4$$

Example 5.4

In measuring the flow of fluid in a pipeline as shown in Figure E5.4, a differential manometer was used to determine the pressure difference across the orifice plate. The flow rate was to be calibrated with the observed pressure drop (difference). Calculate the **pressure drop $p_1 - p_2$** in pascals for the manometer reading in Figure E5.4.

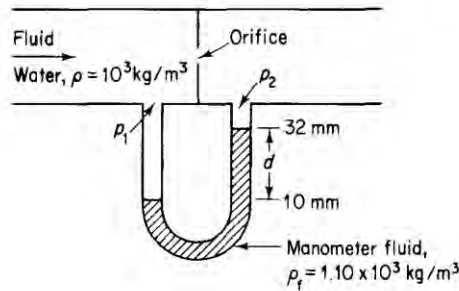


Figure E5.4

Solution

In this problem you **cannot ignore the water density** above the manometer fluid.

$$\begin{aligned}
 p_1 - p_2 &= (\rho_f - \rho)gd \\
 &= \frac{(1.10 - 1.00)10^3 \text{ kg}}{\text{m}^3} \left| \frac{9.807 \text{ m}}{\text{s}^2} \right| \left| \frac{(22)(10^{-3})\text{m}}{(\text{kg})(\text{m})} \right| \frac{1(\text{N})(\text{s}^2)}{1(\text{N})} \frac{1(\text{Pa})(\text{m}^2)}{1(\text{N})} \\
 &= 21.6 \text{ Pa}
 \end{aligned}$$

Example 5.5

Air is flowing through a duct under a draft of 4.0 cm H₂O. The barometer indicates that the atmospheric pressure is 730 mm Hg. What is the absolute pressure of the air in inches of mercury? See Figure E5.5

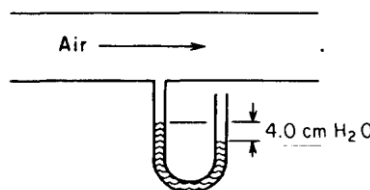


Figure E5.5

Solution

In this problem you can ignore the gas density above the manometer fluid and the air above the open end of the manometer.

$$\text{Atmospheric pressure} = \frac{730 \text{ mm Hg}}{760 \text{ mm Hg}} \left| \frac{29.92 \text{ in. Hg}}{29.92 \text{ in. Hg}} \right| = 28.7 \text{ in. Hg}$$

Next, convert 4.0 cm H₂O to in. Hg:

$$\frac{4.0 \text{ cm H}_2\text{O}}{2.54 \text{ cm}} \left| \frac{1 \text{ in.}}{12 \text{ in.}} \right| \left| \frac{29.92 \text{ in. Hg}}{33.91 \text{ ft H}_2\text{O}} \right| = 0.12 \text{ in. Hg}$$

Since the reading is 4.0 cm H₂O draft (**under atmospheric**), the absolute reading in uniform units is

$$28.7 \text{ in. Hg} - 0.12 \text{ in. Hg} = 28.6 \text{ in. Hg absolute}$$

Questions

- Figure SAT5.1Q2 shows four closed containers completely filled with water. Order the containers from the one exerting the highest pressure to the lowest on their respective base.

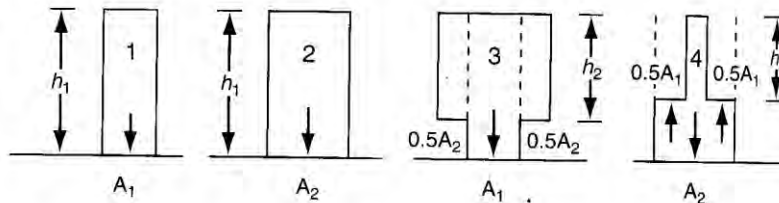


Figure SAT5.1Q2

- Answer the following questions true or false:
 - Atmospheric pressure is the pressure of the air surrounding us and changes from day to day
 - The standard atmosphere is a constant reference atmosphere equal to 1.000 atm or the equivalent pressure in other units.
 - Absolute pressure is measured relative to a vacuum.
 - Gauge pressure is measured upward relative to atmospheric pressure.
 - Vacuum and draft pressures are measured downward from atmospheric pressure.
 - You can convert from one type of pressure measurement to another using the standard atmosphere.
 - A manometer measures the pressure difference in terms of the height of fluid(s) in the manometer tube.

3. What is the equation to convert gauge pressure to absolute pressure?
4. What are the values and units of the standard atmosphere for six different methods of expressing pressure?
5. What is the equation to convert vacuum pressure to absolute pressure?

Answers:

1. 3 is the highest pressure; next are 1 and 2, which are the same; and 4 is last. The decisions are made by dividing the weight of water by the base area.
2. All are true
3. Gauge pressure + barometric pressure = absolute pressure
4. See lectures
5. Barometric pressure - vacuum pressure = absolute pressure

Problems

1. Convert a pressure of 800 mm Hg to the following units:
 - a. psia b. kPa c. atm d. ft H₂O
2. Your textbook lists five types of pressures: atmospheric pressure, barometric pressure, gauge pressure, absolute pressure, and vacuum pressure.
 - a. What kind of pressure is measured by the device in Figure SAT5.2P2A?

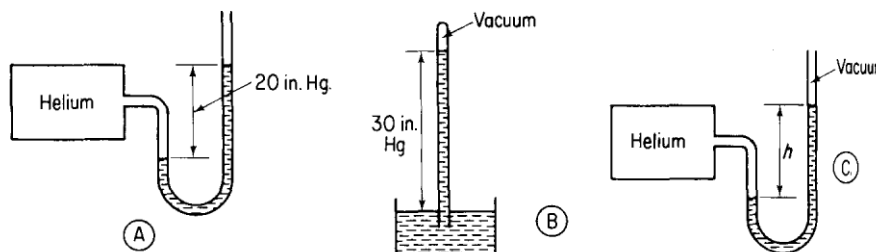


Figure SAT5.2P2A

- b. What kind of pressure is measured by the device in Figure SAT5.2P2B?
 - c. What would be the reading in Figure SAT5.2P2C assuming that the pressure and temperature inside and outside the helium tank are the same as in parts (a) and (b)?
3. A evaporator shows a reading of 40 kPa vacuum. What is the absolute pressure in the evaporator in kPa?
4. A U-tube manometer filled with mercury is connected between two points in a pipeline. If the manometer reading is 26 mm of Hg, calculate the pressure difference in kPa between the points when (a) water is flowing through the pipeline, and (b) also when air at atmospheric pressure and 20°C with a density of 1.20 kg/m³ is flowing in the pipeline.

5. A Bourdon gauge and a mercury manometer are connected to a tank of gas, as shown in Figure SAT5.3P2. If the reading on the pressure gauge is 85 kPa, what is h in centimeters of Hg?

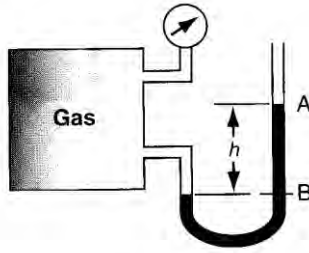


Figure SAT5.3P2

Answers:

- (a) 15.5; (b) 106.6; (c) 1.052; (d) 35.6
- (A) Gauge pressure; (B) barometric pressure, absolute pressure; (C) 50 in.Hg
- In the absence of a barometric pressure value, assume 101.3 kPa. The absolute pressure is 61.3 kPa.
- The Hg is static. (a) 3.21 kPa; (b) 3.47 kPa
- 63.8 cm Hg

Supplementary Problems (Chapter Five):

Problem 1

A solvent storage tank, 15.0 m high contains liquid styrene (sp. gr. 0.909). A pressure gauge is fixed at the base of the tank to be used to determine the level of styrene.

- Determine the gage pressure when the tank is full of styrene.
- If the tank is to be used for storage of liquid hexane (sp. gr. 0.659), will the same pressure gage calibration be adequate? What is the risk in using the same calibration to determine the level of hexane in the tank.
- What will be the new pressure with hexane to indicate that the tank is full.

Solution

- The liquid in full tank will exert a gage pressure at the bottom equal to 15.0 m of styrene. The tank has to operate with atmospheric pressure on it and in it, or it will break on expansion at high pressure or collapse at lower pressure.

$$\begin{aligned}
 p &= h \rho g \\
 &= 15.0 \text{ m} \left| \frac{0.909 \text{ g styrene/cm}^3}{1.0 \text{ g H}_2\text{O/cm}^3} \right| \left| \frac{1.0 \text{ g H}_2\text{O/cm}^3}{1 \text{ g/cm}^3} \right| \left| \frac{10^3 \text{ kg/m}^3}{1 \text{ g/cm}^3} \right| \left| \frac{9.80 \text{ m/s}^2}{1 \text{ (kg)(m)}^{-1}\text{(s)}^{-2}} \right| \left| \frac{1 \text{ Pa}}{1 \text{ (kg)(m)}^{-1}\text{(s)}^{-2}} \right| \\
 &= 134 \times 10^3 \text{ Pa} = \mathbf{134 \text{ kPa gage}}
 \end{aligned}$$

b. Hexane is a liquid of specific gravity lower than that of styrene; therefore a tank full of hexane would exert a proportionally lower pressure. If the same calibration is used the tank may overflow while the pressure gage was indicating only a partially full tank.

c. New $p = h \rho g$

$$= 15.0 \text{ m} \frac{0.659 \text{ g hexane/cm}^3}{1.0.0 \text{ g H}_2\text{O/cm}^3} \left| \frac{1.0 \text{ g H}_2\text{O/cm}^3}{10^3 \text{ kg/m}^3} \right| \frac{9.8 \text{ m/s}^2}{1(\text{kg})(\text{m})^{-1}(\text{s})^{-2}}$$

$$= 96900 \text{ Pa} = 96.9 \text{ kPa}$$

Problem2

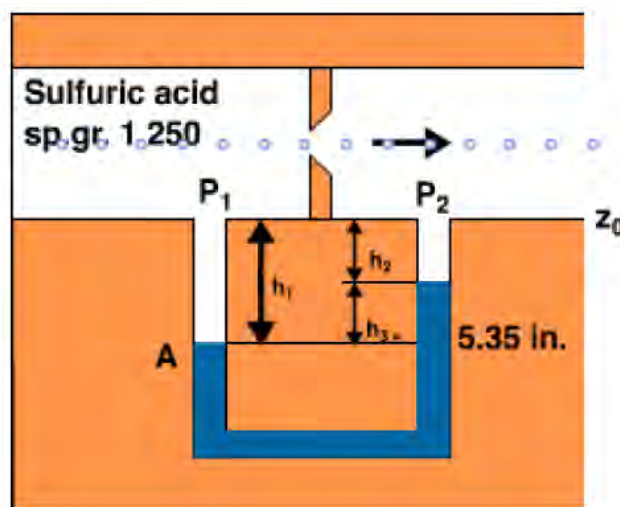
A U-tube manometer is used to determine the pressure drop across an orifice meter. The liquid flowing in the pipe line is a sulfuric acid solution having a specific gravity (60°/60°) of 1.250. The manometer liquid is mercury, with a specific gravity (60°/60°) of 13.56. The manometer reading is 5.35 inches, and all parts of the system are at a temperature of 60°F. What is the pressure drop across the orifice meter in psi.

Solution

First we calculate density of acid and mercury.

$$\rho_{\text{acid}} = \frac{1.250}{1.728 \times 10^3 \text{ in}^3} \left| \frac{62.4 \text{ lb/ft}^3}{1 \text{ ft}^3} \right| = 0.0451 \text{ lb/in}^3$$

$$\rho_{\text{Hg}} = \frac{13.56}{1.728 \times 10^3 \text{ in}^3} \left| \frac{62.4 \text{ lb/ft}^3}{1 \text{ ft}^3} \right| = 0.490 \text{ lb/in}^3$$



	<i>left column</i>	<i>right column</i>	
At z_0	$p_1 + \rho_a h_1 g$	$= p_2 + \rho_a h_2 g + \rho_{Hg} h_3 g$	
	$p_1 - p_2 + \rho_a (h_1 - h_2) g$	$= \rho_{Hg} h_3 g$	
	$p_1 - p_2 + \rho_a h_3 g$	$= \rho_{Hg} h_3 g$	
	$p_1 - p_2$	$= (\rho_{Hg} - \rho_a) h_3 g$	

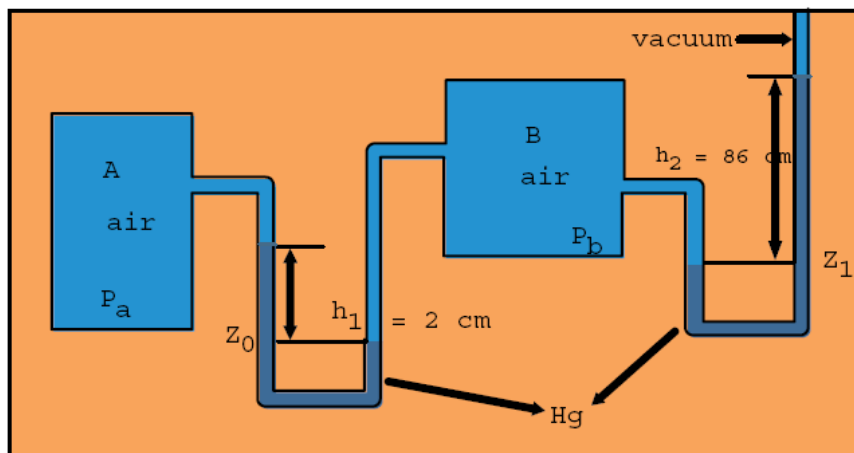
$$p_1 - p_2 = \frac{(0.490 - 0.0451) \text{ lbf}}{\text{in}^2} \times \frac{(5.35) \text{ in}}{32.174 \text{ (ft)(lb}_m\text{)/(s}^2\text{)(lbf)}} = 2.38 \text{ lbf/in}^2 \text{ (psi)}$$

Problem3

The pressure difference between two air tanks A and B is measured by a U - tube manometer, with mercury as the manometer liquid. The barometric pressure is 700 mm Hg.

- a. What is the absolute pressure in the tank A ?
- b. What is the gauge pressure in the tank A ?

Solution



a. At Z_0 $p_a + h_1 \rho_{Hg} g = p_b$ (neglecting the effect of air in the U - tube) (1)

at Z_1 $p_b = h_2 \rho_{Hg} g$ (2)

Eliminate p_b from the equations

$$p_a + h_1 \rho_{Hg} g = h_2 \rho_{Hg} g$$

$$p_a = (h_2 - h_1) \rho_{Hg} g$$

$$= 840 \text{ mm Hg absolute}$$

The pressure measured by this manometer system is the absolute pressure because the reference (pressure above the mercury) in the vertical tube is a vacuum.

b. $p_a = 840 - 700 = 140 \text{ mm Hg}$

Chapter6

Introduction to Material Balances

6.1 The Concept of a Material Balance

A **material balance** is nothing more than the application of the law of the **conservation of mass**:

“Matter is neither created nor destroyed”

6.2 Open and Closed Systems

a. System

By **system** we mean any arbitrary portion of a whole **process** that you want to consider for analysis. You can define a **system** such as a **reactor**, a **section of a pipe**. Or, you can define the **limits of the system** by drawing the **system boundary**, namely a line that encloses the portion of the process that you want to analyze.

b. Closed System

Figure 6.1 shows a two-dimensional view of a three-dimensional vessel holding **1000 kg of H₂O**.

Note that material neither enters nor leaves the vessel, that is, no material crosses the system boundary. Changes can take place **inside the system**, but for a **closed system**, **no mass exchange occurs with the surroundings**.

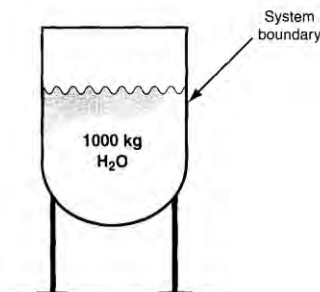


Figure 6.1 A closed system.

c. Open System

Figure 6.2 is an example of an **open system** (also called a **flow system**) because material crosses the system boundary.

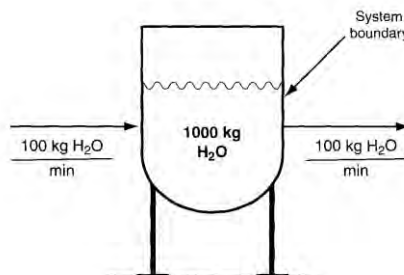


Figure 6.2 An open steady–state system.

6.3 Steady-State and Unsteady-State Systems

a. Steady–State System

Because the rate of addition of water is equal to the rate of removal, the amount of water in the vessel shown in **Figure 6.2** remains constant at its original value (**1000kg**). We call such a process or system a **steady–state process** or a **steady–state system** because

1. The **conditions** inside the process (specifically the amount of water in the vessel in Figure 6.2) **remain unchanged with time**, and
2. The **conditions** of the flowing streams **remain constant with time**.

- ★ Thus, in a **steady-state process**, by definition all of the conditions in the process (e.g., **temperature, pressure, mass of material, flow rate, etc.**) remain constant with time. A **continuous process** is one in which material enters and/or leaves the system without interruption.

b. Unsteady–State System

Because the amount of water in the system **changes with time** (**Figure 6.3**), the process and system are deemed to be an **unsteady–state (transient) process or system**.

- ★ For an **unsteady-state process**, not all of the **conditions** in the process (e.g., **temperature, pressure, mass of material, etc.**) remain constant with time, and/or the **flows** in and out of the system can **vary with time**.

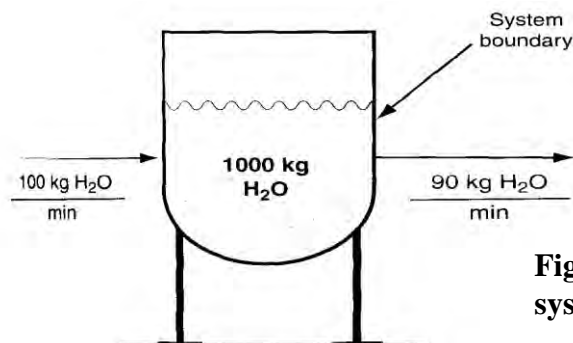


Figure 6.3 Initial conditions for an open unsteady–state system with accumulation.

- ★ Figure 6.4 shows the system after 50 minutes of accumulation (Fifty minutes of accumulation at 10 kg/min amounts to 500 kg of total accumulation).

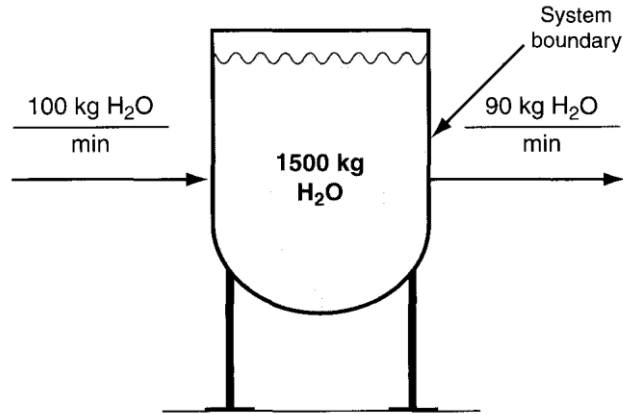


Figure 6.4 The condition of the open unsteady–state system with accumulation after 50 minutes.

* Figures 6.5 and 6.6 demonstrate negative accumulation.

Note that the amount of water in the system decreases with time at the rate of 10 kg/min. Figure 6.6 shows the system after 50 minutes of operation.

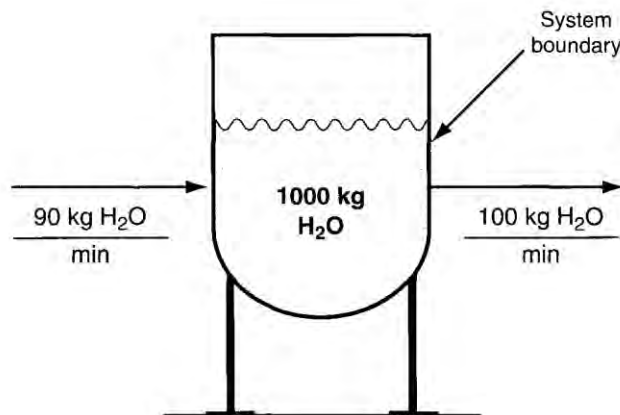


Figure 6.5 Initial conditions for an unsteady–state process with negative accumulation.

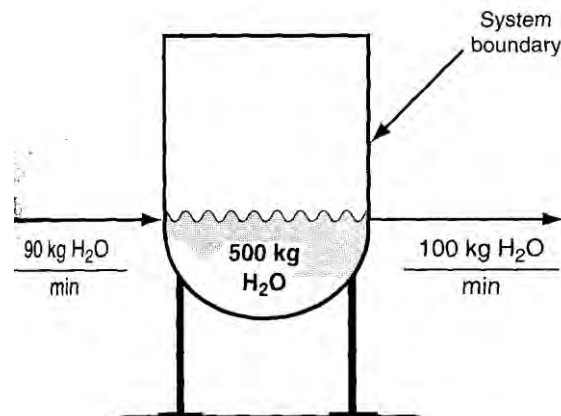


Figure 6.6 Condition of the open unsteady–state system with negative accumulation after 50 minutes.

* The material balance for a single component process is

$$\left\{ \begin{array}{l} \text{Accumulation of material} \\ \text{within the system} \end{array} \right\} = \left\{ \begin{array}{l} \text{Total flow into} \\ \text{the system} \end{array} \right\} - \left\{ \begin{array}{l} \text{Total flow out} \\ \text{of the system} \end{array} \right\} \dots 6.1$$

Equation 6.1 can apply to moles or any quantity that is conserved. As an example, look at Figure 6.7 in which we have converted all of the mass quantities in Figure 6.2 to their equivalent values in moles.

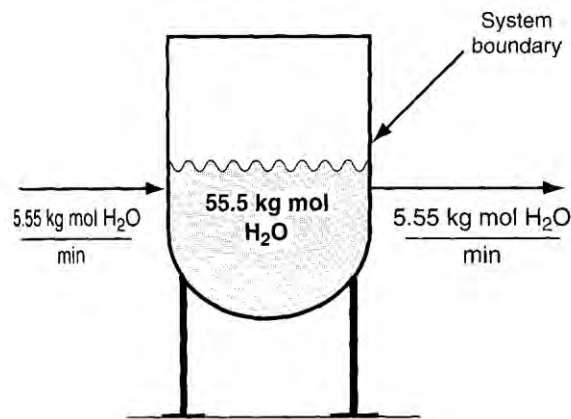


Figure 6.7 The system in Figure 6.2 with the flow rates shown in kgmol.

If the process is in the steady state, the accumulation term by definition is zero, and Equation 6.1 simplifies to a famous truism

$$\text{What goes in must come out (In = Out)} \dots 6.2$$

If you are analyzing an unsteady-state process, the accumulation term over a time interval can be calculated as

$$\{\text{Accumulation}\} = \left\{ \begin{array}{l} \text{Final material} \\ \text{in the system} \end{array} \right\} - \left\{ \begin{array}{l} \text{Initial material} \\ \text{in the system} \end{array} \right\} \quad (6.3)$$

The times you select for the final and initial conditions can be anything, but you usually select an interval such as **1 minute** or **1 hour** rather than specific times.

* When you combine Equations 6.1 and 6.3 you get the general material balance for a component in the system in the absence of reaction

$$\left\{ \begin{array}{l} \text{Final material} \\ \text{in the system} \\ \text{at } t_2 \end{array} \right\} - \left\{ \begin{array}{l} \text{Initial material} \\ \text{in the system} \\ \text{at } t_1 \end{array} \right\} = \left\{ \begin{array}{l} \text{Flow into} \\ \text{the system} \\ \text{from } t_1 \text{ to } t_2 \end{array} \right\} - \left\{ \begin{array}{l} \text{Flow out of} \\ \text{the system} \\ \text{from } t_1 \text{ to } t_2 \end{array} \right\} \dots 6.4$$

Example 6.1

Will you save money if instead of buying premium 89 octane gasoline at \$1.269 per gallon that has the octane you want, you blend sufficient 93 octane supreme gasoline at \$1.349 per gallon with 87 octane regular gasoline at \$1.149 per gallon?

Solution

Choose a basis of **1 gallon of 89 octane gasoline**, the desired product. The system is the gasoline tank.

- For simplicity, assume that **no gasoline exists** in the tank at the start of the blending, and **one gallon exists** in the tank at the end of the blending.
- This arrangement corresponds to an **unsteady-state process**. Clearly it is an **open system**.

The **initial number of gallons** in the system is **zero** and the **final number of gallons** is **one**.

Let **x** = the number of gallons of **87 octane** gasoline added, and

y = the number of gallons of **93 octane** added to the blend.

Since **x + y = 1** is the total flow into the tank,

$$\therefore y = 1 - x$$

According to Equation (6.4) the balance on the octane number is

$$\left. \begin{array}{c} \text{Accumulation} \\ \frac{89 \text{ octane}}{1 \text{ gal}} \left| \frac{1 \text{ gal}}{1 \text{ gal}} - 0 \right. \end{array} \right| = \left. \begin{array}{c} \text{Inputs} \\ \frac{87 \text{ octane}}{1 \text{ gal}} \left| \frac{x \text{ gal}}{1 \text{ gal}} + \frac{93 \text{ octane}}{1 \text{ gal}} \left| \frac{(1 - x) \text{ gal}}{1 \text{ gal}} \right. \right. \end{array} \right|$$

The solution is **x = 2/3 gal** and thus **y = 1 - x = 1/3 gal**.

The cost of the blended gasoline is **(2/3) (\$1.149) + (1/3) (\$1.349) = \$1.216**

A value less than the cost of the 89 octane gasoline (\$1.269).

6.4 Multiple Component Systems

Suppose the input to a vessel contains **more than one component**, such as 100 kg/min of a 50% water and 50% sugar (sucrose, C₁₂H₂₂O₁₁, MW=342.3) mixture (see Figure 6.8). The mass balances with respect to the **sugar and water**, balances that we call **component balances**.

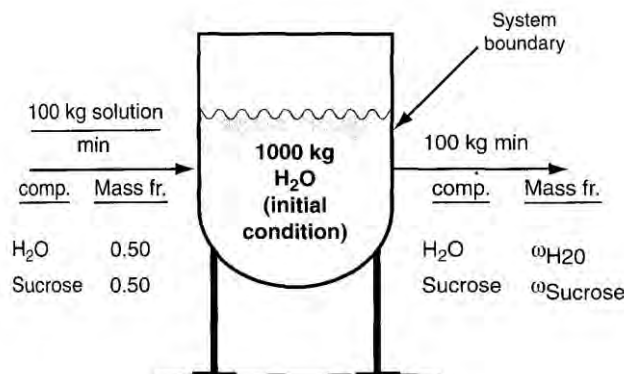


Figure 6.8 An open system involving two components.

For Example, look at the mixers shown in Figure 6.9, an apparatus that mixes two streams to increase the concentration of NaOH in a dilute solution. **The mixer is a steady–state open system.** Initially the mixer is empty, and after 1 hour it is empty again.

Basis = 1 hour for convenience. As an alternate to the **basis** we selected you could select **$F_1 = 9000 \text{ kg/hr}$ as the basis, or $F_2 = 1000 \text{ kg/hr}$ as the basis**; the numbers for this example would not change – just the **units** would change. Here are the components and total balances in **kg**:

Balances	Flow in		Flow out	Accum.
	F_1	F_2		
NaOH	450	500	950	= 0
H ₂ O	8,550	500	9,050	= 0
Total	9,000	1,000	10,000	= 0

We can convert the kg shown in Figure 6.9 to kg moles by dividing each compound by its respective molecular weight (NaOH = 40 and H₂O = 18).

$$\begin{array}{l} \text{NaOH: } \frac{450}{40} = 11.25 \quad \frac{500}{40} = 12.50 \quad \frac{950}{40} = 23.75 \\ \text{H}_2\text{O: } \frac{8550}{18} = 475 \quad \frac{500}{18} = 27.78 \quad \frac{9050}{18} = 502.78 \end{array}$$

Then the component and total balances in **kg molar**:

Balances	Flow in		Flow out	Accum.
	F_1	F_2		
NaOH	11.25	12.50	23.75	= 0
H ₂ O	475	27.78	502.78	= 0
Total	486.25	40.28	536.53	= 0

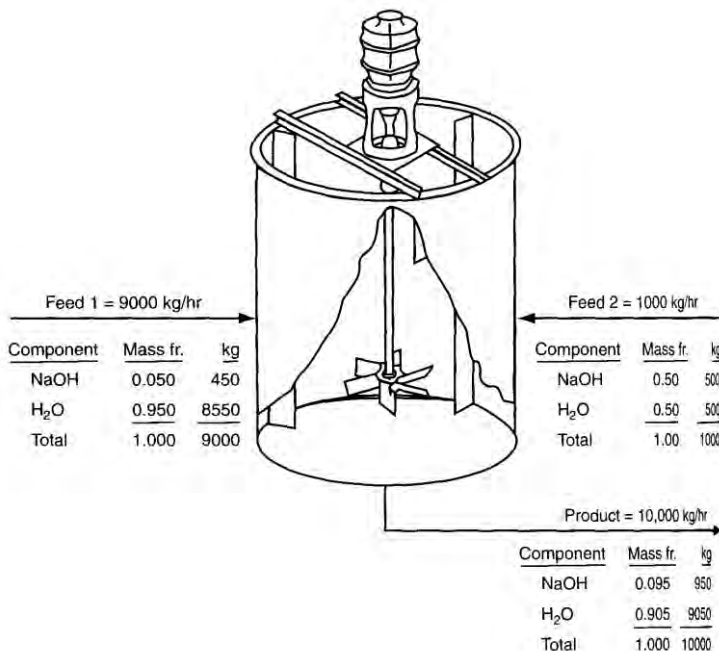


Figure 6.9 Mixing of a dilute stream of NaOH with a concentrated stream of NaOH. Values below the stream arrows are based on 1 hour of operation.

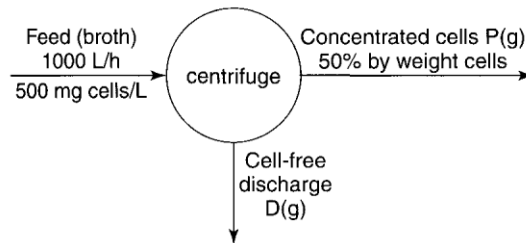
Example6.2

Centrifuges are used to separate particles in the range of 0.1 to 100 μm in diameter from a liquid using centrifugal force. Yeast cells are recovered from a broth (a liquid mixture containing cells) using a tubular centrifuge (a cylindrical system rotating about a cylindrical axis). Determine the amount of the cell-free discharge per hour if 1000 L/hr is fed to the centrifuge, the feed contains 500 mg cells/L, and the product stream contains 50 wt. % cells. Assume that the feed has a density of 1 g/cm³.

Solution

This problem involves a **steady state, open (flow) system without reaction**.

Basis = 1 hour



FigureE6.2

M.B. on cells

$$\text{In (mass)} = \text{Out(mass)}$$

$$\frac{1000 \text{ L feed}}{1 \text{ L feed}} \left| \frac{500 \text{ mg cells}}{1 \text{ L feed}} \right| \frac{1 \text{ g}}{1000 \text{ mg}} = \frac{0.5 \text{ g cells}}{1 \text{ g } P} \left| P \text{ g} \right|$$

$$P = 1000 \text{ g}$$

M.B. on fluid

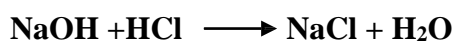
$$\text{In (mass)} = \text{Out(mass)}$$

$$\frac{1000 \text{ L}}{1 \text{ L}} \left| \frac{1000 \text{ cm}^3}{1 \text{ L}} \right| \frac{1 \text{ g fluid}}{1 \text{ cm}^3} = \frac{1000 \text{ g } P}{1 \text{ g } P} \left| \frac{0.50 \text{ g fluid}}{1 \text{ g } P} \right| + D \text{ g fluid}$$

$$D = (10^6 - 500) \text{ g}$$

6.5 Accounting for Chemical Reactions in Material Balances

Chemical reaction in a system requires the augmentation of Equation 6.4 to take into account the effect of the reaction. To illustrate this point, look at Figure 6.10, which shows a steady-state system in which HCl reacts with NaOH by the following reaction:



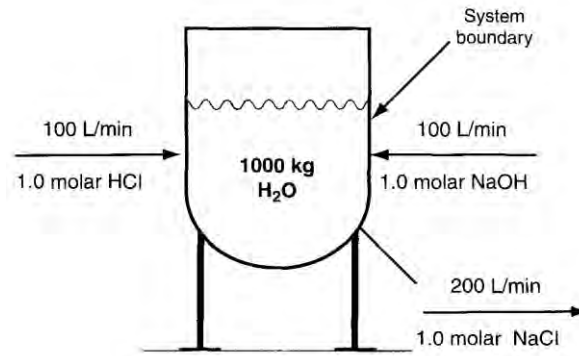


Figure 6.10 Reactor for neutralizing HCl with NaOH.

Equation 6.4 must be augmented to include terms for the **generation** and **consumption** of components by the **chemical reaction** in the system as follows

$$\left\{ \begin{array}{c} \text{Accumulation} \\ \text{within the} \\ \text{system} \end{array} \right\} = \left\{ \begin{array}{c} \text{Input} \\ \text{through} \\ \text{the system} \\ \text{boundaries} \end{array} \right\} - \left\{ \begin{array}{c} \text{Output} \\ \text{through} \\ \text{the system} \\ \text{boundaries} \end{array} \right\} + \left\{ \begin{array}{c} \text{Generation} \\ \text{within the} \\ \text{system} \end{array} \right\} - \left\{ \begin{array}{c} \text{Consumption} \\ \text{within the} \\ \text{system} \end{array} \right\} \quad \dots 6.5$$

6.6 Material Balances for Batch and Semi-Batch Processes

- ◆ A **batch process** is used to process a fixed amount of material each time it is operated. **Initially**, the material to be processed is charged into the system. After processing of the material is complete, the products are removed.
- ◆ **Batch processes** are used **industrially** for specialty processing applications (e.g., producing **pharmaceutical products**), which typically operate at relatively **low production rates**.
- ◆ Look at Figure 6.11a that illustrates what occurs at the start of a batch process, and after thorough mixing, the final solution remains in the system (Figure 6.11b).

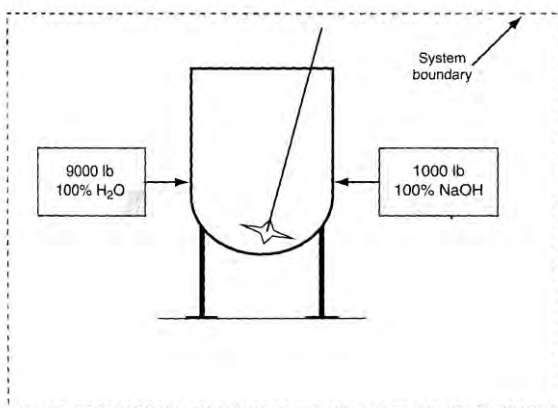


Figure 6.11a The initial state of a batch mixing process.

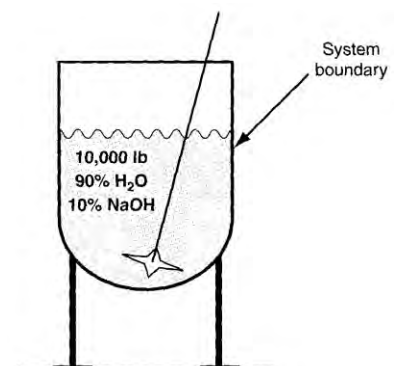


Figure 6.11b The final state of a batch mixing process.

- ◆ We can summarize the **hypothetical operation** of the **batch** as a flow system (opensystem) as follows (**Figure6.12**):

Final conditions: All values =0

Flowsout:

NaOH =
 1,000lb
H₂O =
9,000lb
 Total = 10,000lb

Initial conditions: All value =0

Flows in:

NaOH =
 1,000lb
H₂O =
9,000lb
 Total 10,000lb

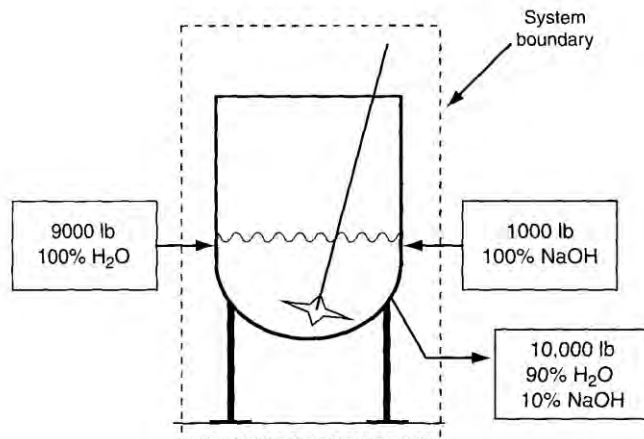


Figure 6.12 The batch process in Figure 6.11 represented as an opensystem.

- ☒ In a **semi-batch process** material **enters** the process during its operation, but **doe snot leave**. Instead, mass is allowed to accumulate in the process vessel. Product is **withdrawn** only after the process is over.
- ☒ A **Figure6.13** illustrates a semi-batch mixing process. Initially the vessel is empty (**Figure 6.13a**). **Figure 6.13b** shows the semi-batch system after **1 hour** of operation. **Semi-batch processes** are **open** and **unsteady –state**.
- ☒ Only flows **enter** the systems, and **none leave**, hence the system is an **unsteady state** –one that you can treat as having **continuous flows**, as follows:

Finalconditions:

NaOH = 1,000 lb
H₂O = 9,000lb
 Total = 10,000 lb

Flows out: All values =0

Flows in:

NaOH =
1,000lbH₂O =
9,000lb
Total = 10,000lb

Initial conditions: All values =0