Bernoulli,s Equation

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Equation of Continuity

The **mass flow rate** is the mass that passes a given point per unit time. The flow rates at any two points must be **equal**, as long as no fluid is being added or taken away.

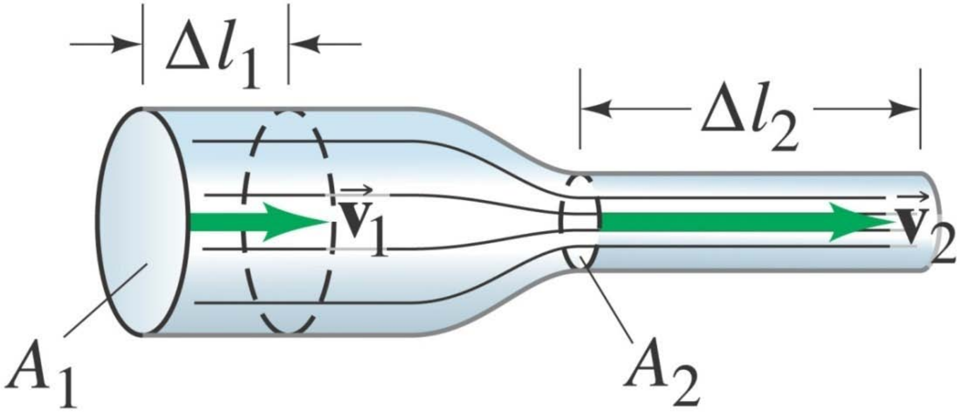
This gives us the **equation of continuity**:

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad (1)$$

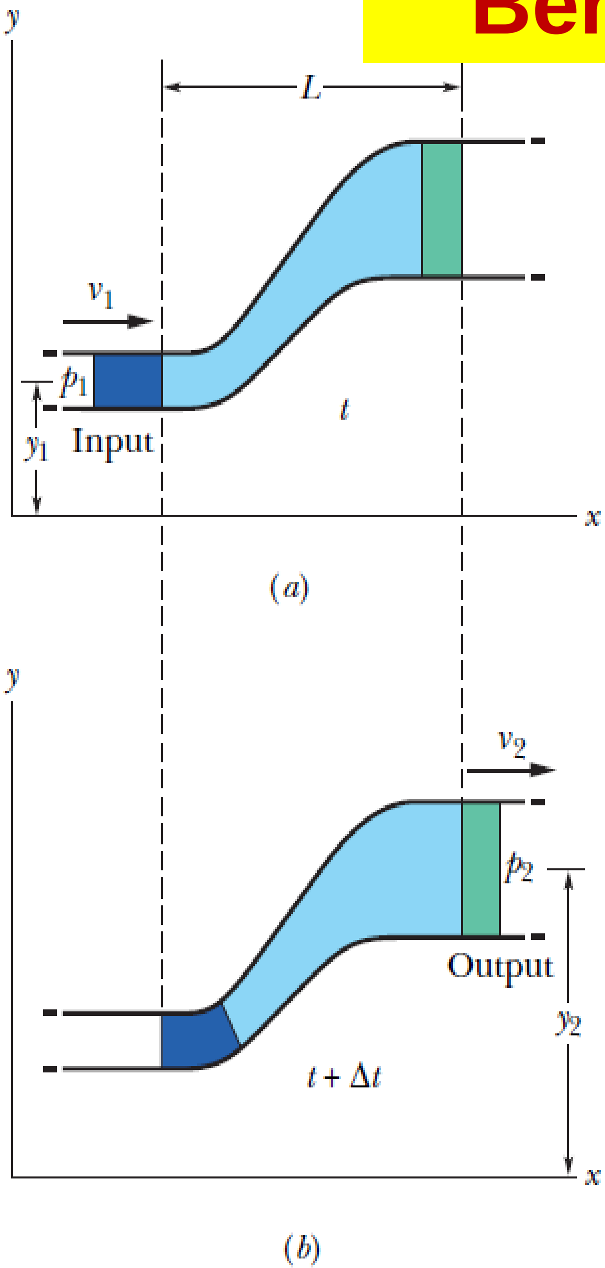
If the density doesn't change – typical for liquids – this simplifies to

$$A_1 v_1 = A_2 v_2$$

Where the pipe is **wider**, the flow is **slower**.



Bernoulli's Equation



In Figure below Fluid flows at a steady rate through a length L of a tube, from the input end at the left to the output end at the right. From time t in (a) to time t+Δt in (b), the amount of fluid shown in purple enters the input end and the equal amount shown in green emerges from the output end.

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2.$$

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant} \qquad \text{(Bernoulli's equation).}$$

If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.

This is the celebrated *Bernoulli equation—a very powerful tool in fluid mechanics. In 1738.* To use it correctly we must constantly remember the basic assumptions used in its derivation:

1. Viscous effects are assumed negligible
2. The flow is assumed to be steady
3. The flow is assumed to be incompressible
4. The equation is applicable along a streamline.

EXAMPLE 15.8 The Venturi Tube

The horizontal constricted pipe illustrated in Figure 15.21, known as a *Venturi tube*, can be used to measure the flow speed of an incompressible fluid. Let us determine the flow speed at point 2 if the pressure difference $P_1 - P_2$ is known.

Solution Because the pipe is horizontal, $y_1 = y_2$, and applying Equation 15.8 to points 1 and 2 gives

$$(1) \qquad P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

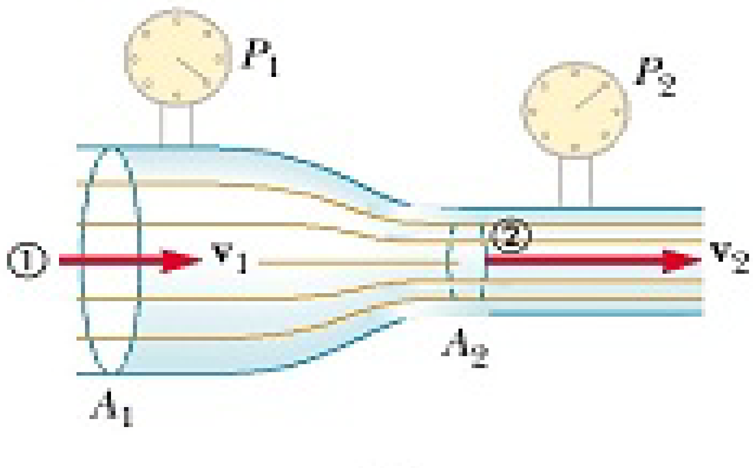


Figure 15.21 (a) Pressure P_1 is greater than pressure P_2 because $v_1 < v_2$. This device can be used to measure the speed of fluid flow. (b) A Venturi tube.

From the equation of continuity, $A_1 v_1 = A_2 v_2$, we find that

$$(2) \qquad v_1 = \frac{A_2}{A_1} v_2$$

Substituting this expression into equation (1) gives

$$P_1 + \tfrac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2{}^2 = P_2 + \tfrac{1}{2}\rho v_2{}^2$$

$$v_2 = A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1{}^2 - A_2{}^2)}}$$

We can use this result and the continuity equation to obtain an expression for v_1. Because $A_2 < A_1$, Equation (2) shows us that $v_2 > v_1$. This result, together with equation (1), indicates that $P_1 > P_2$. In other words, the pressure is reduced in the constricted part of the pipe. This result is somewhat analogous to the following situation: Consider a very crowded room in which people are squeezed together. As soon as a door is opened and people begin to exit, the squeezing (pressure) is least near the door, where the motion (flow) is greatest.