الجامعة التكنولوجية

قسم الهندسة الكيمياوية

المرحلة الاولى

الفيزياء

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Physics in general:

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Motion:

In this chapter, you will learn more about motion, a field of study called *kinematics*. You will become familiar with concepts such as velocity, acceleration and displacement. For now, the focus is on how things move, not what causes them to move. Later, you will study *dynamics*, which centers on forces and how they affect motion. Dynamics and kinematics make up *mechanics*, the study of force and motion.

Two key concepts in this chapter are velocity and acceleration. Velocity is how fast something is moving (its speed) **and** in what direction it is moving. Acceleration is the rate of change in velocity. In this chapter, you will have many opportunities to learn about velocity and acceleration and how they relate. To get a feel for these concepts, you can experiment by using the two simulations on the right. These simulations are versions of the tortoise and hare race. In this classic parable, the steady tortoise always wins the race. With your help, though, the hare stands a chance. (After all, this is your physics course, not your literature course.)

In the first simulation, the tortoise has a head start and moves at a constant velocity of three meters per second to the right. The hare is initially stationary; it has zero velocity. You set its acceleration □ in other words, how much its velocity changes each second. The acceleration you set is constant throughout the race. Can you set the acceleration so that the hare crosses the finish line first and wins the race? To try, click on Interactive 1, enter an acceleration value in the entry box in the simulation, and press GO to see what happens. Press RESET if you want to try again. Try acceleration values up to 10 meters per second squared. (At this acceleration, the velocity increases by 10 meters per second every second. Values larger than this will cause the action to occur so rapidly that the hare may quickly disappear off the screen.) It does not really matter if you can cause the hare to beat this rather fast-moving tortoise. However, we do want you to try a few different rates of acceleration and see how they affect the hare’s velocity. Nothing particularly tricky is occurring here; you are simply observing two basic properties of motion:

velocity and acceleration.

In the second simulation, the race is a round trip. To win the race, a contestant needs to go around the post on the right and then return to the starting line. The tortoise has been given a head start in this race. When you start the simulation, the tortoise has already rounded the post and is moving at a constant velocity on the homestretch back to the finish line. In this simulation, when you press GO the hare starts off moving quickly to the right. Again, you supply a value for its acceleration. The challenge is to supply a value for the hare's acceleration so that it turns around at the post and races back to beat the tortoise. We have given you a fair number of concepts in this introduction. These fundamentals are the foundation of the study of motion, and you will learn much more about them shortly. The definition of displacement is precise: the direction and length of the **shortest** path from the **initial** to the **final** position of an object's motion. As you may recall from your mathematics courses, the shortest path between two points is a straight line. Physicists use arrows to indicate the direction of displacement. In the illustrations to the right, the arrow points in the direction of the mouse's displacement. Physicists use the Greek letter Δ (delta) to indicate a change or difference. A change in position is displacement, and since x represents position, we write Δx to indicate displacement. You see this notation, and the equation for calculating displacement, to the right. In the equation, x_f represents the final position (the subscript f stands for final) and x_i represents the initial position (the subscript i stands for initial). Displacement is a vector. A vector is a quantity that must be stated in terms of its direction and its magnitude. Magnitude means the size or amount. "Move five meters to the right" is a description of a vector. Scalars, on the other hand, are quantities that are stated solely in terms of magnitude, like "a dozen eggs." There is no direction for a quantity of eggs, just an amount.

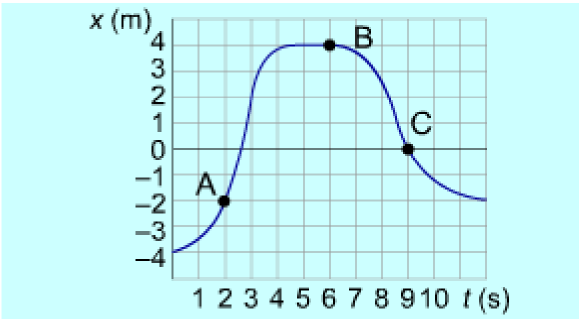
In one dimension, a positive or negative sign is enough to specify a direction. As mentioned, numbers to the right of the origin are positive, and those to the left are negative. This means displacement to the right is positive, and to the left it is negative. For instance, you can see in Example 1 that the mouse's car starts at the position +3.0 meters and moves to the left to the position −1.0 meters. (We measure the position at the middle of the car.) Since it moves to the left 4.0 meters, its displacement is −4.0 meters. Displacement measures the distance solely between the beginning and end of motion. We can use dance to illustrate this point. Let's say you are dancing and you take three steps forward and two steps back. Although you moved a total of five steps, your displacement after this maneuver is one step forward. It would be better to use signs to describe the dance directions, so we could describe forward as “positive” and backwards as “negative.” Three steps forward and two steps back yield a displacement of positive one step. Since displacement is in part a measure of distance, it is measured with units of length. Meters are the SI unit for displacement

Velocity: Speed and direction.

You are familiar with the concept of speed. It tells you how fast something is going: 55 miles per hour (mi/h) is an example of speed. The speedometer in a car measures speed but does not indicate direction. When you need to know both speed and direction, you use velocity. Velocity is a vector. It is the measure of how fast **and** in which direction the motion is occurring. It is represented by v. In this section, we focus on average velocity, which is represented by v with a bar over it, as shown in Equation 1. A police officer uses the concepts of both speed and velocity in her work. She might issue a ticket to a motorist for driving 36 mi/h (58 km/h) in a school zone; in this case, speed matters but direction is irrelevant. In another situation, she might be told that a suspect is fleeing **north** on I-405 at 90 mi/h (149 km/h); now velocity is important because it tells her both how fast and in what direction.

To calculate an object’s average velocity, divide its displacement by the time it takes to move that displacement. This time is called the elapsed time, and is represented by □*t*. The direction for velocity is the same as for the displacement. For instance, let’s say a car moves positive 50 mi (80 km) between the hours of 1 P.M. and 3 P.M. Its displacement is positive 50 mi, and two hours elapse as it moves that distance. The car’s average velocity equals +50 miles divided by two hours, or +25 mi/h (+40 km/h). Note that the direction is positive because the displacement was positive. If the displacement were negative, then the velocity would also be negative. At this point in the discussion, we are intentionally ignoring any variations in the car’s velocity. Perhaps the car moves at constant speed, or change direction. In other words, their velocity can change. For example, if you drop an egg off a 40-story building, the egg’s velocity will change: It will move faster as it falls. Someone on the building’s 39th floor would see it pass by with a different velocity than would someone on the 30th. When we use the word “instantaneous,” we describe an object’s velocity at a particular instant. In Concept 1, you see a snapshot of a toy mouse car at an instant when it has a velocity of positive six meters per second.

The fable of the tortoise and the hare provides a classic example of instantaneous versus average velocity. As you may recall, the hare seemed faster because it could achieve a greater instantaneous velocity than could the tortoise. But the hare's long naps meant that its average velocity was less than that of the tortoise, so the tortoise won the race. When the average velocity of an object is measured over a very short elapsed time, the result is close to the instantaneous velocity. The shorter the elapsed time, the closer the average and instantaneous velocities. Imagine the egg falling past the 39th floor window in the example we mentioned earlier, and let's say you wanted to determine its instantaneous velocity at the midpoint of the window. You could use a stopwatch to time how long it takes the egg to travel from the top to the bottom of the window. If you then divided the height of the window by the elapsed time,

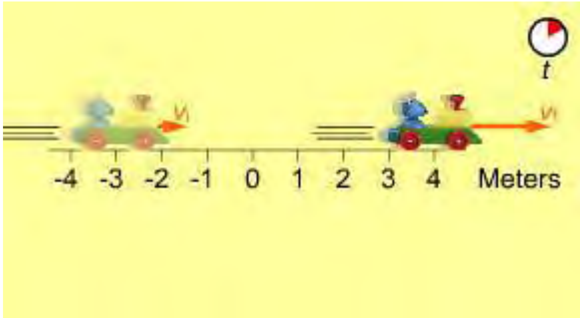


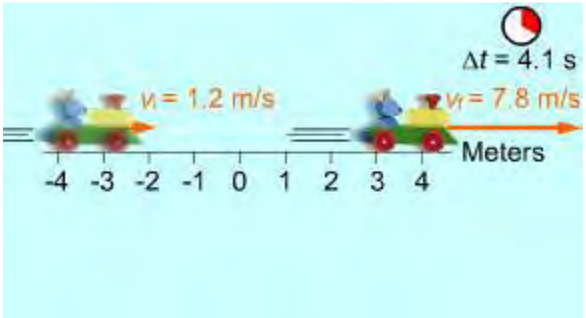
Acceleration: Change in velocity.

When an object's velocity changes, it accelerates. Acceleration measures the **rate** at which an object speeds up, slows down or changes direction. Any of these variations constitutes a change in velocity. The letter *a* represents acceleration. Acceleration is a popular topic in sports car commercials. In the commercials, acceleration is often expressed as how fast a car can go from zero to 60 miles per hour (97 km/h, or 27 m/s). For example, a current model Corvette® automobile can reach 60 mi/h in 4.9 seconds. There are even hotter cars than this in production.

To calculate average acceleration, divide the change in instantaneous velocity by the elapsed time, as shown in Equation 1. To calculate the acceleration of the Corvette, divide its change in velocity, from 0 to 27 m/s, by the elapsed time of 4.9 seconds. The car accelerates at an average rate of 5.5 m/s per second. We typically express this as 5.5 meters per second squared, or 5.5 m/s2. (This equals 18 ft/s2, and with this observation we will cease stating values in both measurement systems, in order to simplify the expression of numbers.) Acceleration is measured in units of length divided by time squared. Meters per second squared (m/s2) express acceleration in SI units. Let's assume the car accelerates at a constant rate; this means that each second the Corvette moves 5.5 m/s faster. At one second, it is moving at 5.5 m/s; at two seconds, 11 m/s; at three seconds, 16.5 m/s; and so forth. The car's velocity increases by 5.5 m/s every second. Since acceleration measures the change in **velocity**, an object can accelerate even while it is moving at a constant **speed**. For instance, consider a car moving around a curve. Even if the car's speed remains constant, it accelerates because the change in the car's direction means its velocity (speed plus direction) is changing. Acceleration can be positive or negative. If the Corvette uses its brakes to go from +60 to 0 mi/h in 4.9 seconds, its velocity is decreasing just as fast as it was increasing before. This is an example of negative acceleration.

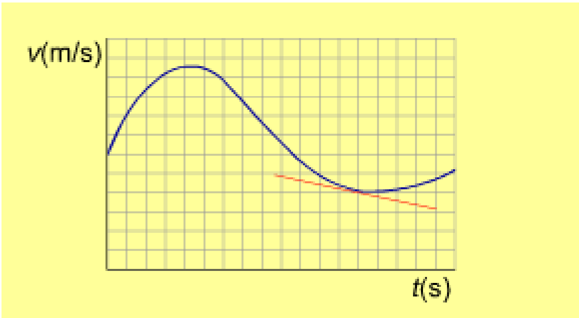
You may want to think of negative acceleration as "slowing down," but be careful! Let's say a train has an initial velocity of **negative** 25 m/s and that changes to **negative** 50 m/s. The train is moving at a faster rate (speeding up) but it has negative acceleration. To be precise, its negative acceleration causes an increasingly negative velocity. Velocity and acceleration are related but distinct values for an object. For example, an object can have **positive** velocity and **negative** acceleration. In this case, it is slowing down. An object can have zero velocity, yet be accelerating. For example, when a ball bounces off the ground, it experiences a moment of zero velocity as its velocity changes

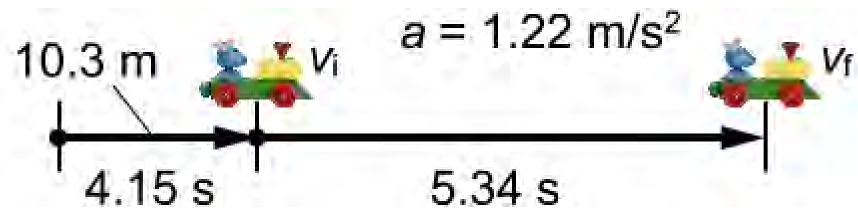




Instantaneous acceleration: Acceleration at a particular moment.

You have learned that velocity can be either average or instantaneous. Similarly, you can determine the average acceleration or the instantaneous acceleration of an object. We use the mouse in Concept 1 on the right to show the distinction between the two. The mouse moves toward the trap and then wisely turns around to retreat in a hurry. The illustration shows the mouse as it moves toward and then hurries away from the trap. It starts from a rest position and moves to the right with increasingly positive velocity, which means it has a positive acceleration for an interval of time. Then it slows to a stop when it sees the trap, and its positive velocity decreases to zero (this is negative acceleration). It then moves back to the left with increasingly negative velocity (negative acceleration again). If you would like to see this action occur again in the Concept 1 graphic, press the refresh button in your browser. We could calculate an average acceleration, but describing the mouse's motion with instantaneous acceleration is a more informative description of that motion. At some instants in time, it has positive acceleration and at other instants, negative acceleration. By knowing its acceleration and its velocity at an instant in time, we can determine whether it is moving toward the trap with increasingly positive velocity, slowing its rate of approach, or moving away with increasingly negative velocity. Instantaneous acceleration is defined as the change in velocity divided by the elapsed time as the elapsed time approaches zero. This concept is stated mathematically in Equation 1 on the right. Earlier, we discussed how the slope of the tangent at any point on a position-time graph equals the instantaneous velocity at that point. We can apply similar reasoning here to conclude that the instantaneous acceleration at any point on a velocity-time graph equals the slope of the tangent, as shown in Equation 2. Why? Because slope equals the rate of change, and acceleration is the rate of change of velocity. In Example 1, we show a graph of the velocity of the mouse as it approaches the trap and then flees. You are asked to determine the sign of the instantaneous acceleration at four points; you can do so by considering the slope of the tangent to the velocity graph at each point.





Variables

Part 1: Constant velocity

displacement	$\Delta x = 10.3$ m
elapsed time	$\Delta t = 4.15$ s
velocity	v

Part 2: Constant acceleration

initial velocity	$v_i = v$ (calculated above)
acceleration	$a = 1.22$ m/s^2
elapsed time	$\Delta t = 5.34$ s
final velocity	v_f

What is the strategy?

What is the strategy?

1. Use the definition of velocity to find the velocity of the mouse car before it accelerates. The velocity is constant during the first part of the journey.
2. Use the definition of acceleration and solve for the final velocity.

Physics principles and equations

The definitions of velocity and acceleration will prove useful. The velocity and acceleration are constant in this problem. In this and later problems, we use the definitions for average velocity and acceleration without the bars over the variables.

$$v = \Delta x/\Delta t$$

$$a = \Delta v/\Delta t = (v_f - v_i)/\Delta t$$

Step-by-step solution

We start by finding the velocity before the engine fires.

Step	Reason
1. $v = \Delta x/\Delta t$	definition of velocity
2. $v = (10.3\ \text{m})/(4.15\ \text{s})$	enter values
3. $v = 2.48\ \text{m/s}$	divide

Next we find the final velocity using the definition of acceleration. The initial velocity is the same as the velocity we just calculated.

Step	Reason
4. $a = (v_f - v_i)/\Delta t$	definition of acceleration
5. $1.22\ \text{m/s}^2 = \dfrac{v_f - 2.48\ \text{m/s}}{5.34\ \text{s}}$	enter given values, and velocity from step 3
6. $6.51\ \text{m/s} = v_f - 2.48\ \text{m/s}$	multiply by 5.34 s
7. $v_f = 8.99\ \text{m/s}$	solve for v_f

Strategy

First, we will discuss our strategy for this derivation. That is, we will describe our overall plan of attack. These strategy points outline the major steps of the derivation.

1. We start with the definition of acceleration and rearrange it. It includes the terms for initial and final velocity, as well as elapsed time.

2. We derive another equation involving time that can be used to eliminate the time variable from the acceleration equation. The condition of constant acceleration will be crucial here.

3. We eliminate the time variable from the acceleration equation and simplify. This results in an equation that depends on other variables, but not time.

Physics principles and equations

Since the acceleration is constant, the velocity increases at a constant rate. This means the average velocity is the sum of the initial and final

$$\bar{v} = (v_i + v_f)/2$$

velocities divided by two.

We will use the definition of acceleration,

$a = (vf \;\square\; vi)/t$

We will also use the definition of average velocity,

$$\bar{v} = \Delta x / t$$

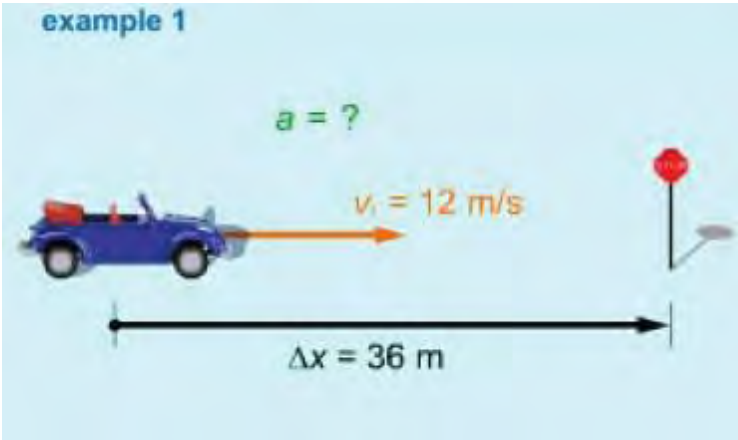
Step-by-step derivation

We start the derivation with the definition of average acceleration, solve it for the final velocity and do some algebra. This creates an equation with the square of the final velocity on the left side, where it appears in the equation we want to derive.

Step	Reason
1. $a = (v_f - v_i)/t$	definition of average acceleration
2. $v_f = v_i + at$	solve for final velocity
3. $v_f^2 = (v_i + at)^2$	square both sides
4. $v_f^2 = v_i^2 + 2v_i at + a^2t^2$	expand right side
5. $v_f^2 = v_i^2 + at(2v_i + at)$	factor out at
6. $v_f^2 = v_i^2 + at(v_i + v_i + at)$	rewrite $2v_i$ as a sum
7. $v_f^2 = v_i^2 + at(v_i + v_f)$	substitution from equation 2

The equation we just found is the basic equation from which we will derive the desired motion equation. But it still involves the time variable t - multiplied by a sum of velocities. In the **next** stage of the derivation, we use two different ways of expressing the average velocity to develop a second equation involving time multiplied by velocities. We will subsequently use that second equation to eliminate time from the equation above.

Step	Reason
8. $\bar{v} = \frac{v_i + v_f}{2}$	average velocity is average of initial and final velocities
9. $\bar{v} = \frac{\Delta x}{t}$	definition of average velocity
10. $\frac{v_i + v_f}{2} = \frac{\Delta x}{t}$	set right sides of 8 and 9 equal
11. $t(v_i + v_f) = 2\Delta x$	rearrange equation



What acceleration will stop the car exactly at the stop sign?

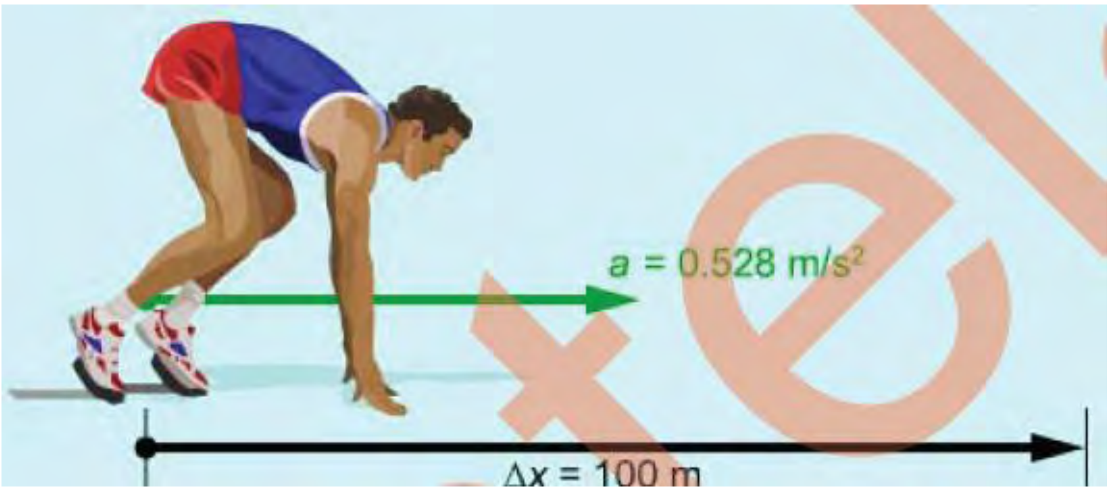
$v_f^2 = v_i^2 + 2a\Delta x$

$a = (v_f^2 - v_i^2)/2\Delta x$

$$a = \frac{(0.0\ \text{m/s})^2 - (12\ \text{m/s})^2}{2\,(36\ \text{m})}$$

$a = -144/72\ \text{m/s}^2$

$a = -2.0\ \text{m/s}^2$

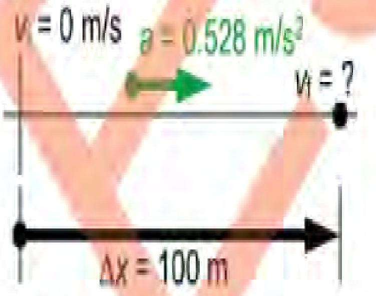


What is the runner's velocity at the end of a 100-meter dash?

You are asked to calculate the final velocity of a sprinter running a 100-meter dash. List the variables that you know and the one you are asked for, and then consider which equation you might use to solve the problem. You want an equation with just one unknown variable, which in this problem is the final velocity.

The sprinter's initial velocity is not explicitly stated, but he starts motionless, so it is zero m/s.

Draw a diagram



Variables

displacement	$\Delta x = 100$ m
acceleration	$a = 0.528$ m/s^2
initial velocity	$v_i = 0.00$ m/s
final velocity	v_f

What is the strategy?

1. Choose an appropriate equation based on the values you know and the one you want to find.
2. Enter the known values and solve for the final velocity.

Physics principles and equations

Based on the known and unknown values, the equation below is appropriate. We know all the variables in the equation except the one we are asked to find, so we can solve for it.

$$v_f^2 = v_i^2 + 2a\Delta x$$

Now we use a second motion equation containing the two velocities, substitute known values, and simplify. This gives us two equations with the two unknowns we want to find.

Step	Reason
4. $\Delta x = ½(v_i + v_f)t$	second motion equation
5. $11.8 \text{ m} = ½(v_i + v_f)(3.14 \text{ s})$	substitute known values
6. $7.52 \text{ m/s} = v_i + v_f$	multiply by 2, divide by 3.14 s

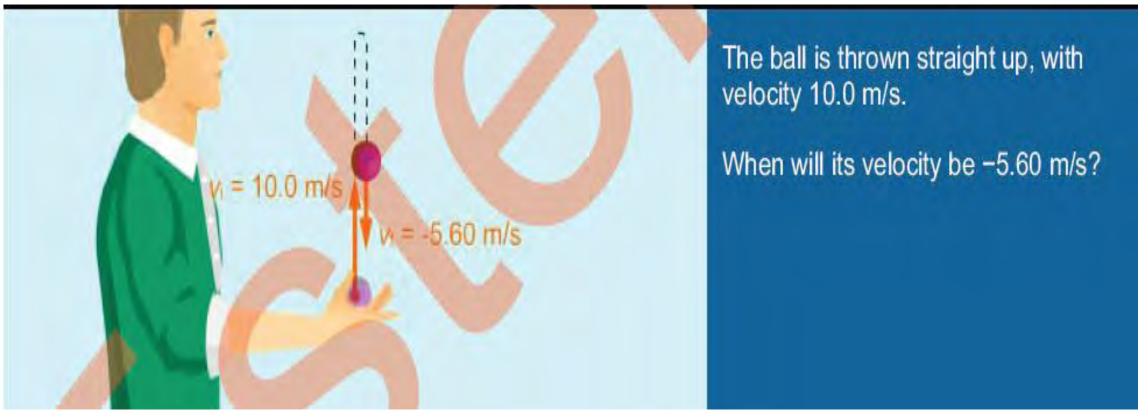
Now we solve the two equations.

Step	Reason
7. $7.52 \text{ m/s} = v_i + v_i + 3.80 \text{ m/s}$	substitute equation 3 into equation 6
8. $v_i = 1.86 \text{ m/s}$	solve for v_i
9. $v_f = v_i + 3.80 \text{ m/s} = 5.66 \text{ m/s}$	from equation 3

There are other ways to solve this problem. For example, you could use the equation

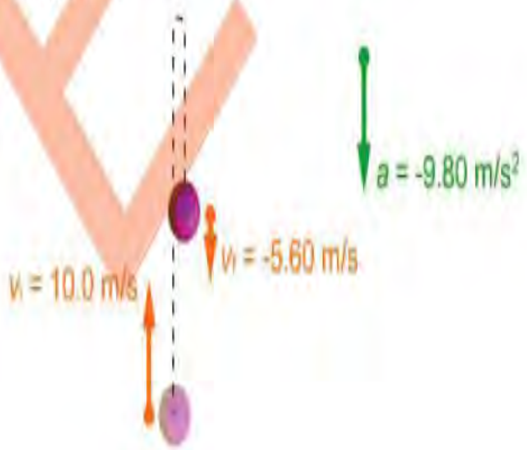
$$\Delta x = v_i t + ½at^2$$

to find the initial velocity from the displacement, acceleration, and elapsed time. Then you could use the equation $v_f = v_i + at$ to solve for the final velocity.



As you see above, a ball is tossed straight up into the air with an initial velocity of positive 10.0 meters per second. You are asked to figure out how long it will take before its velocity is **negative** 5.60 m/s. The ball will have this velocity when it is falling back to the ground.

Draw a diagram



Variables

Be careful with the signs for acceleration and velocity. We use the common convention that upward quantities are positive, and downward negative. The magnitude of the acceleration is the free-fall acceleration constant $g = 9.80\ \text{m/s}^2$.

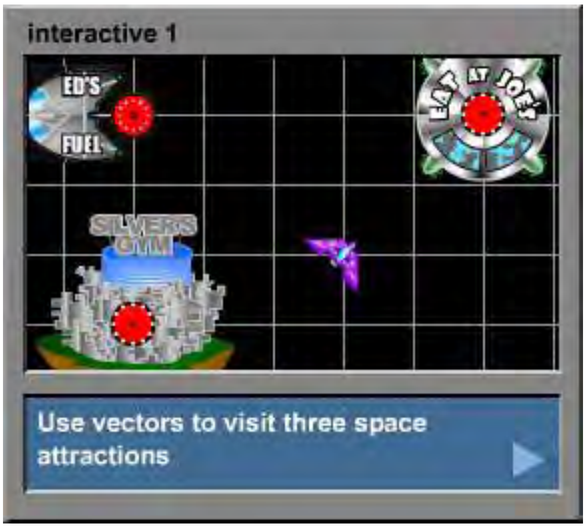
initial velocity	$v_i = 10.0\ \text{m/s}$
final velocity	$v_f = -5.60\ \text{m/s}$
acceleration	$a = -9.80\ \text{m/s}^2$
elapsed time	t

What is the strategy?

1. Choose an appropriate motion equation for the knowns and unknowns.
2. Solve for the elapsed time.

Knowing "how far" or "how fast" can often be useful, but "which way" sometimes proves even more valuable. If you have ever been lost, you understand that direction can be the most important thing to know. Vectors describe "how much" **and** "which way," or, in the terminology of physics, magnitude and direction. You use vectors frequently, even if you are not familiar with the term. "Go three miles northeast" or "walk two blocks north, one block east" are both vector descriptions. Vectors prove crucial in much of physics. For example, if you throw a ball up into the air, you need to understand that the initial velocity of the ball points "up" while the acceleration due to the force of gravity points "down." In this chapter, you will learn the fundamentals of vectors: how to write them and how to combine them using operations such as addition and subtraction. On the right, a simulation lets you explore vectors, in this case displacement vectors. In the simulation, you are the pilot of a small spaceship. There are three locations nearby that you want to visit: a refueling station, a diner, and the local gym. To reach any of these locations, you describe the displacement vector of the spaceship by setting its x (horizontal) and y (vertical) components. In other words, you set how far horizontally you want to travel, and how far vertically. This is a common way to express a two-dimensional vector.

There is a grid on the drawing to help you determine these values. You, and each of the places you want to visit, are at the intersection of two grid lines. Each square on the grid is one kilometer across in each direction. Enter the values, press GO, and the simulation will show you traveling in a straight line along the displacement vector according to the values you set. See if you can reach all three places. You can do this by entering displacement values to the nearest kilometer, like (3, 4) km. To start over at any time, press RESET.

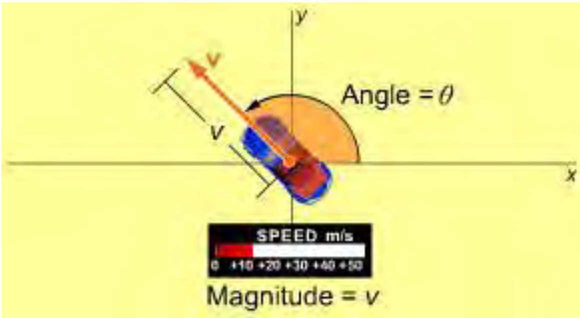


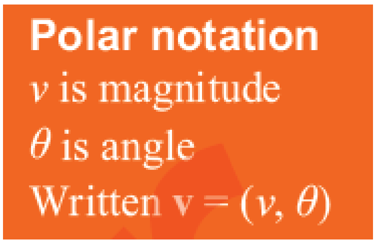
Scalar: A quantity that states only an amount.

Scalar quantities state an amount: "how much" or "how many." At the right is a picture of a dozen eggs. The quantity, a dozen, is a scalar. Unlike vectors, there is no direction associated with a scalar no up or down, no left or right just one quantity, the amount. A scalar is described by a single number, together with the appropriate units. Temperature provides another example of a scalar quantity; it gets warmer and colder, but at any particular time and place there is no "direction" to temperature, only a value. Time is another commonly used scalar.

Speed and distance are yet other scalars. A speed like 60 kilometers per hour says how fast but not which way. Distance is a scalar since it tells you how far away something is, but not the direction.

direction, a negative angle a clockwise direction. For example, 90° represents a quarter turn **counterclockwise** from the positive *x* axis. In other words, a vector with a 90° angle points straight up. We could also specify this angle as □270°. The radian is another unit of measurement for angles that you may have seen before. We will use degrees to specify angles unless we specifically note that we are using radians. (Radians do prove essential at times.)





Write the velocity vector of the car in polar notation.

$\mathbf{v} = (v, \theta)$

$\mathbf{v} = (5\text{ m/s}, 135^o)$

Rectangular notation: Defining a vector by its components.

Often what we know, or want to know, about a particular vector is not its overall magnitude and direction, but how far it extends horizontally and vertically. On a graph, we represent the horizontal direction as x and the vertical direction as y. These are called *Cartesian coordinates*. The x *component* of a vector indicates its extent in the horizontal dimension and the y *component* its extent in the vertical dimension.

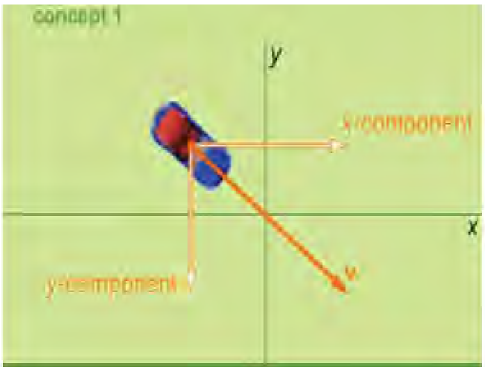
Rectangular notation is a way to describe a vector using the components that make up the vector. In rectangular notation, the x and y components of a vector are written inside parentheses. A vector that extends a units along the x axis and b units along the y axis is written as (a, b). For instance (3, 4) is a vector that extends positive three in the x direction and positive four in the y direction from its starting point.

The components of vectors are scalars with the direction indicated by their sign: x components point right (positive) or left (negative), and y components point up (positive) or down (negative). You see the x and y components of a car's velocity vector in Concept 1 at the right, shown as "hollow" vectors. The x and y values define the vector, as they provide direction and magnitude.

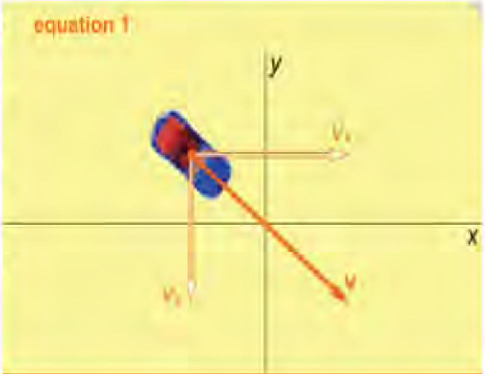
For a vector $\mathbf{A}$, the x and y components are sometimes written as A_x and A_y. You see this notation used for a velocity vector $\mathbf{v}$ in Equation 1 and Example 1 on the right.

Consider the car shown in Example 1 on the right. Its velocity has an x component v_x of 17 m/s and a y component v_y of −13 m/s. We can write the car's velocity vector as (17, −13) m/s.

A vector can extend in more than two dimensions: z represents the third dimension. Sometimes z is used to represent distance toward or away from you. For instance, your computer monitor's width is measured in the x dimension, its height with y and your distance from the monitor with z. If you are reading this on a computer monitor and punch your computer screen, your fist would be moving in the z dimension. (We hope



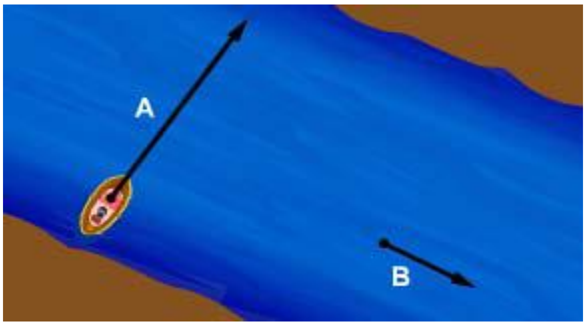
Vector components and rectangular notation
x component and y component



Rectangular notation
v_x is horizontal component

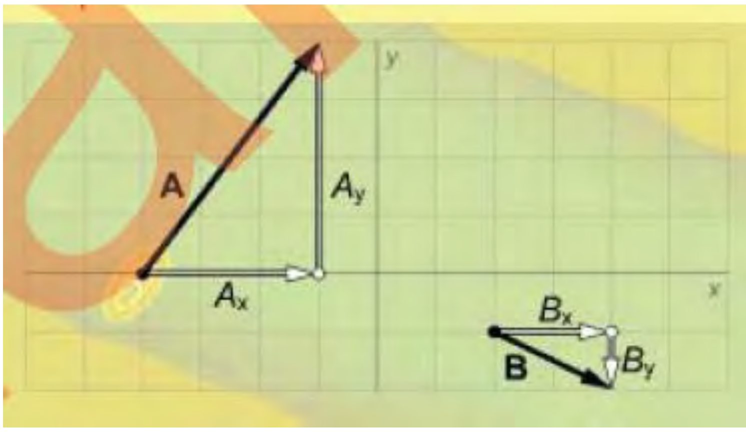
Adding and subtracting vectors by components

You can combine vectors graphically, but it may be more precise to add up their components. You perform this operation intuitively outside physics. If you were a dancer or a cheerleader, you would easily understand the following choreography: "Take two steps forward, four steps to the right and one step back." These are vector instructions. You can add them to determine the overall result. If asked how far **forward** you are after this dance move, you would say "one step," which is two steps forward plus one step back. You realize that your progress forward or back is unaffected by steps to the left or right. You correctly process left/right and forward/back separately. If a physics-oriented dance instructor asked you to describe the results of your “dancing vector” math, you would say, “One step forward, four steps to the right.” You have just learned the basics of vector addition, which is reasonably straightforward: Break the vector into its components and add each component independently. In physics though, you concern yourself with more than dance steps. You might want to add the vector (20, □40, 60) to (10, 50, 10). Let’s assume the units for both vectors are meters. As with the dance example, each component is added independently. You add the first number in each set of parentheses: 20 plus 10 equals 30, so the sum along the *x* axis is 30. Then you add □40 and 50 for a total of 10 along the *y* axis. The sum along the *z* axis is 60 plus 10, or 70. The vector sum is (30, 10, 70) meters. If following all this in the text is hard, you can see another problem worked in Example 1 on the right. Although we use displacement vectors in much of this discussion since they may be the most intuitive to understand, it is important to note that all types of vectors can be added or subtracted. You can add two velocity vectors, two acceleration vectors, two force vectors and so on. As illustrated in the example problem, where two velocity vectors are added, the process is identical for any type of vector. Vector subtraction works similarly to addition when you use components. For example, (5, 3) minus (2, 1) equals 5 minus 2, and 3 minus 1; the result is the vector (3, 2).



Adding and subtracting vectors by components

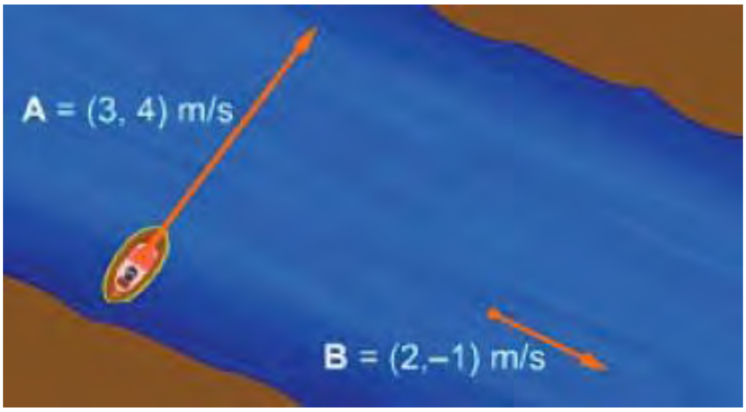
Add (or subtract) each component separately



Adding and subtracting vectors by components

$$\mathbf{A} + \mathbf{B} = (A_x + B_x, A_y + B_y)$$
$$\mathbf{A} - \mathbf{B} = (A_x - B_x, A_y - B_y)$$

A, **B** = vectors
A_x, A_y = **A** components
B_x, B_y = **B** components



The boat has the velocity A in still water. Calculate its velocity as the sum of A and the velocity B of the river's current.

$$\mathbf{v} = \mathbf{A} + \mathbf{B}$$

$$\mathbf{v} = (3, 4)\ \text{m/s} + (2, -1)\ \text{m/s}$$

$$\mathbf{v} = (3 + 2, 4 + (-1))\ \text{m/s}$$

$$\mathbf{v} = (5, 3)\ \text{m/s}$$

equation 1

Multiplying a rectangular vector by a scalar

$$s\mathbf{r} = (sr_x, sr_y)$$

s = a scalar

r = a vector

r_x, r_y = **r** components

example 1

What is the displacement d of the plane after 5.0 seconds?

$\mathbf{d} = (5.0\ \text{s})\mathbf{v}$

$\mathbf{d} = (5.0\ \text{s})\ (12\ \text{m/s},\ 15\ \text{m/s})$

$\mathbf{d} = (\ (5.0\ \text{s})(12\ \text{m/s}),\ (5.0\ \text{s})(15\ \text{m/s})\ )$

$\mathbf{d} = (60,\ 75)\ \text{m}$

3.10 - Multiplying polar vectors by a scalar

Multiplying a vector represented in polar notation by a positive scalar requires only one multiplication operation: Multiply the magnitude of the vector by the scalar. The angle is unchanged.

Let's say there is a vector of magnitude 50 km with an angle of $30°$. You are asked to multiply it by positive three. This situation is shown in Example 1 to the right. Since you are multiplying by a positive scalar, the angle stays the same at $30°$, and so the answer is 150 km at $30°$.

If you multiply a vector by a negative scalar, multiply its magnitude by the absolute value of the scalar (that is, ignore the negative sign). Then change the direction of the vector by $180°$ so that it points in the opposite direction. In polar notation, since the magnitude is always positive, you add $180°$ to the vector's angle to take its opposite. The result of multiplying (50 km, $30°$) by negative three is (150 km, $\mathbf{210°}$).

If adding $180°$ would result in an angle greater than $360°$, then subtract $180°$ instead. For instance, in reversing an angle of $300°$, subtract $180°$ and express the result as $120°$ rather than $480°$. The two results are identical, but $120°$ is easier to understand.

concept 1

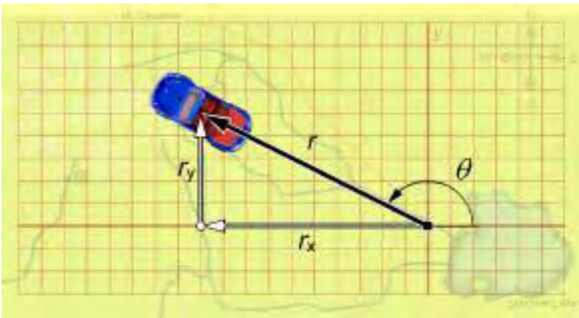
Multiplying polar vector by positive scalar

Multiply vector's magnitude by scalar

Angle unchanged

and the y component by multiplying 3.0 km by sin 35°. Here, x = (3.0 km)(0.82) and y = (3.0 km)(0.57), so the vector in rectangular coordinates is (2.5, 1.7) km. Using the same method with the other vector, 2.0 km at □15° equals (1.9, □0.52) km. The positive x component and negative y component indicate that this vector points down and to the right, the correct direction for a vector with an angle of □15°. We began this section by asking you how you would add these two vectors. Our work has made this an easier problem: (2.5, 1.7) plus (1.9, □0.52) equals (4.4, 1.2). The units are kilometers.

The x and y components can be positive or negative. For instance, the x component will be negative when the cosine is negative, which it is for angles between 90° and 270°. This corresponds to vectors that have an x component which points to the left. The y component will be positive when the sine is positive (between 0° and 180°, the vector has an upward y component) and negative when the sine is negative (between 180° and 360°, the vector has a downward y component). Since it is easy to err, it is a good practice to compare directions and the signs of the components. In Example 1, the negative x component is correct, since the car is moving to the left. If we had calculated a negative y component, we have erred in our calculations, since the car is clearly moving "up" in the positive y direction.



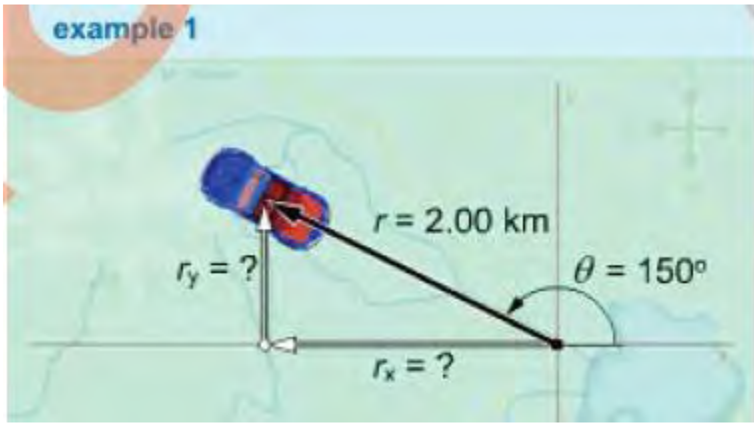
Converting a vector from polar to rectangular notation

To express (r, θ) as (r_x, r_y)

$$r_x = r\cos\theta$$
$$r_y = r\sin\theta$$

r = magnitude, θ = angle

r_x, r_x = components of vector



What is the displacement vector r of the car in rectangular notation?

$r_x = r\cos\theta = (2.00\text{ km})(\cos 150^\circ)$

$r_x = -1.73\text{ km}$

$r_y = r\sin\theta = (2.00\text{ km})(\sin 150^\circ)$

$r_y = 1.00\text{ km}$

$\mathbf{r} = (x, y) = (-1.73, 1.00)\text{ km}$

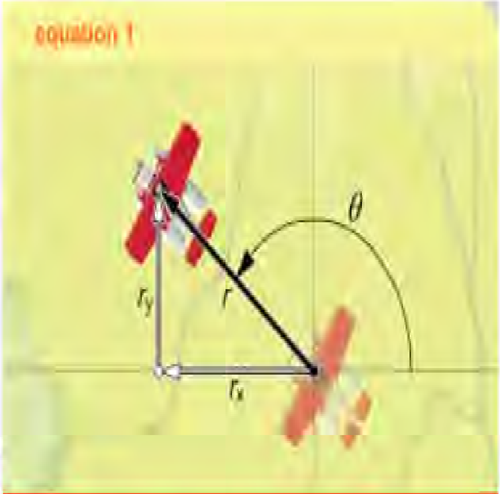
3.12 - Converting vectors from rectangular to polar notation

n some counties of the United States, the main roads travel either east-west or north-south. If you wanted to drive from one town to another, the roads might force you to ravel 40 km west and then 30 km north. On the other hand, if you had a plane, you ould fly in a straight line between the two towns, which would be a shorter distance. ou would need to know the angle at which to fly and the distance. We work this roblem out in Example 1, but before that, we review the concepts necessary to solve he problem.

o determine the angle and distance, you need to convert from rectangular to polar oordinates. You would use trigonometry to do so. The x and y components represent he legs of a triangle. You need to determine the length of the hypotenuse and the angle he hypotenuse makes with the positive x axis.

n Equation 1 on the right, you see that the Pythagorean theorem is used to calculate he hypotenuse when the two legs are known. The magnitude of the vector (the ypotenuse) is represented with r, and the two legs, called r_x and r_y here, are the omponents of the vector. In the example, the distance in kilometers is the square root f $(-40\text{ km})^2 + (30\text{ km})^2$. That works out to 50 km.

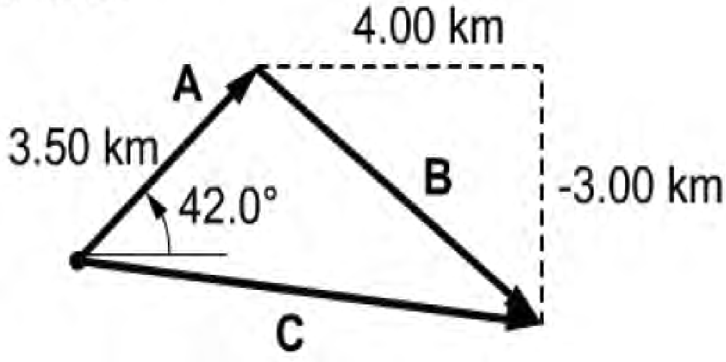
low you can determine the angle, which we represent as θ. As you may recall, the angent function relates the base and height of a right triangle to the angle between the ypotenuse and the base (in this case, the x axis). The angle θ is the arctangent of the



Converting rectangular to polar

To express (r_x, r_y) as (r, θ)

Draw a diagram



Variables

We use **A** to indicate the first vector, **B** for the second vector, and **C** for their sum.

	polar notation	rectangular notation
vector **A**	(3.50 km, 42.0°)	(A_x, A_y)
vector **B**	not needed	(4.00, −3.00) km
vector sum **C**	(C, θ)	(C_x, C_y)

What is the strategy?

1. Convert the first vector **A** to rectangular notation.
2. Add vectors **A** and **B** by adding their components. This will give you the resulting displacement **C** in rectangular notati
3. Convert **C** to polar notation. Check to make sure the angle is in the right quadrant.

Mathematics principles

Polar to rectangular conversion

$$r_x = r\cos\theta$$

$$r_y = r\sin\theta$$

Rectangular to polar conversion

$$r = \sqrt{r_x^2 + r_y^2}$$

$$\theta = \arctan(r_y / r_x)$$

Adding vectors

$$\mathbf{A} + \mathbf{B} = (A_x + B_x, A_y + B_y)$$

Step-by-step solution

We start by converting vector **A** to rectangular notation.

Step	Reason
1. $A_x = A \cos \theta$	x component of vector
2. $A_x = (3.50 \text{ km})(\cos 42.0°)$	enter values
3. $A_x = 2.60 \text{ km}$	cosine, multiplication
4. $A_y = A \sin \theta$	y component of vector
5. $A_y = (3.50 \text{ km})(\sin 42.0°)$	enter values
6. $A_y = 2.34 \text{ km}$	sine, multiplication
7. $\mathbf{A} = (2.60, 2.34) \text{ km}$	combine components

Mathematics principles

Polar to rectangular conversion

$$r_x = r \cos \theta$$

$$r_y = r \sin \theta$$

Rectangular to polar conversion

$$r = \sqrt{r_x^2 + r_y^2}$$

$$\theta = \arctan(r_y / r_x)$$

Adding three vectors

$$\mathbf{A} + \mathbf{B} + \mathbf{C} = (A_x + B_x + C_x, A_y + B_y + C_y)$$

Step-by-step solution

First, we convert vector $\mathbf{A}$ to rectangular notation.

Step	Reason
1. $A_x = A \cos \theta$	*x* component of vector
2. $A_x = (6.10\text{ N})(\cos 55.0°)$	enter values
3. $A_x = 3.50\text{ N}$	evaluate
4. $A_y = A \sin \theta$	*y* component of vector
5. $A_y = (6.10\text{ N})(\sin 55.0°)$	enter values
6. $A_y = 5.00\text{ N}$	evaluate
7. $\mathbf{A} = (3.50, 5.00)\text{ N}$	combine components

We use steps similar to those above to convert **B** to rectangular notation.

Step	Reason
8. $B_x = (9.00\text{ N})(\cos 274°)$	enter values
9. $B_x = 0.628\text{ N}$	evaluate
10. $B_y = (9.00\text{ N})(\sin 274°)$	enter values
11. $B_y = –8.98\text{ N}$	evaluate
12. $\mathbf{B} = (0.628, -8.98)\text{ N}$	combine components

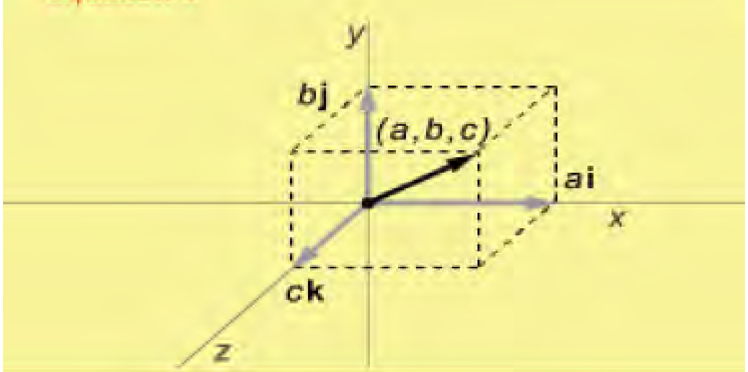
Now we add the *x* components of all three vectors and set the sum equal to zero. This lets us solve for the *x* component of vector **C**.

Step	Reason
13. $\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$	vector sum is zero
14. $A_x + B_x + C_x = 0$	sum of the x components is zero
15. $3.50 + 0.628 + C_x = 0$	enter values
16. $C_x = -4.13$ N	solve for C_x

Similarly, we find the y component of $\mathbf{C}$.

Step	Reason
17. $A_y + B_y + C_y = 0$	sum of the y components is zero
18. $5.00 + -8.98 + C_y = 0$	enter values
19. $C_y = 3.98$ N	solve for C_y

equation 1

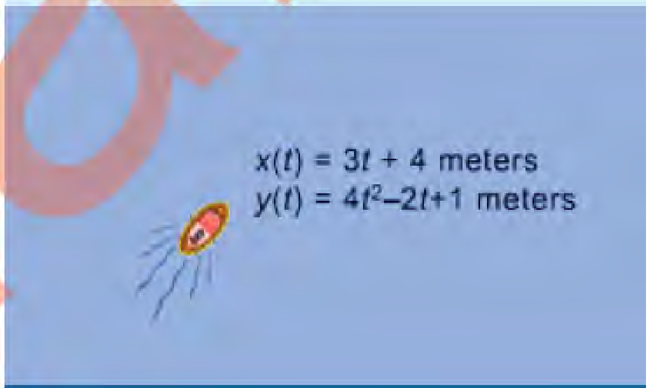


Unit vectors

$$(a, b, c) = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$$

a, b, c = vector components
$\mathbf{i}, \mathbf{j}, \mathbf{k}$ = unit vectors

example 1

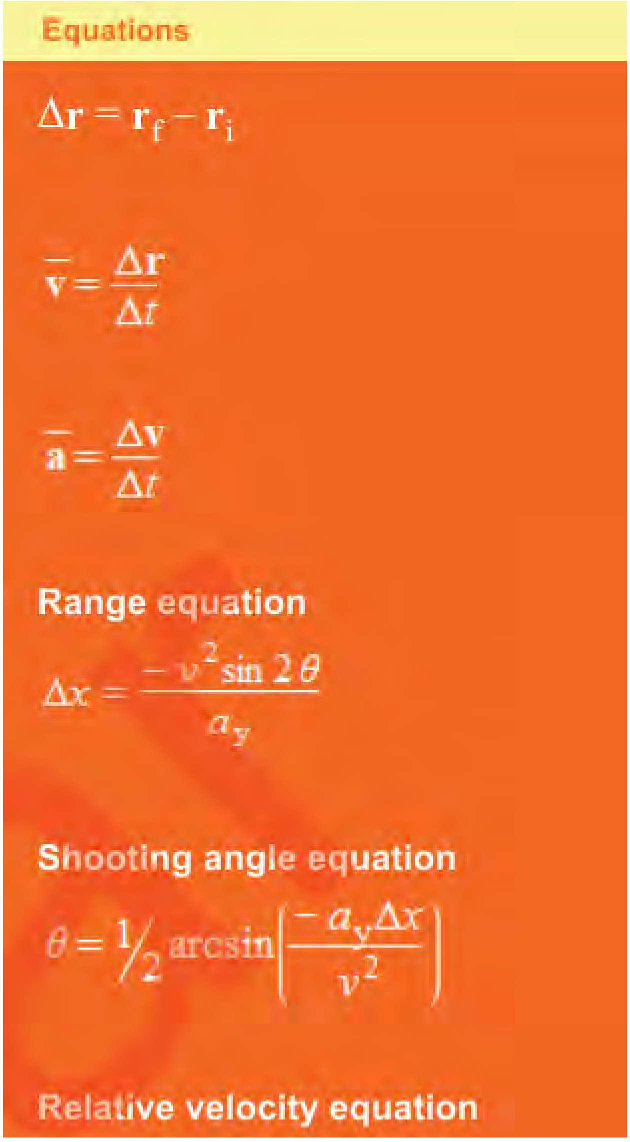


The *x* and *y* components of the boat's displacement are defined by the two equations shown. Express the displacement at 3.0 seconds as a vector r in unit vector notation.

$x(3.0) = 3(3.0) + 4 = 13$ m

$y(3.0) = 4(3.0)^2 - 2(3.0) + 1 = 31$ m

$\mathbf{r} = (13\mathbf{i} + 31\mathbf{j})$ m



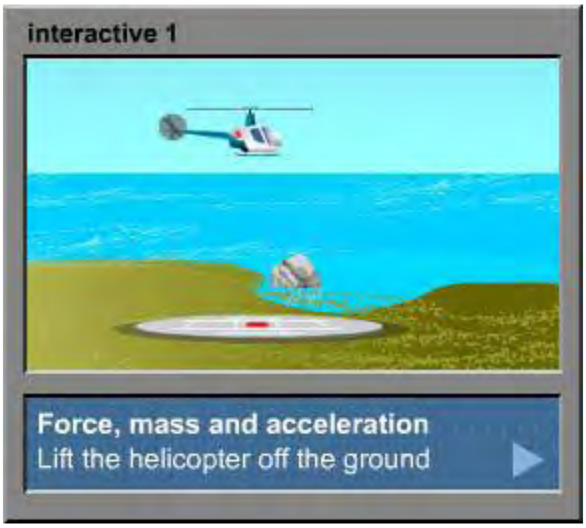
Introduction

Objects can speed up, slow down, and change direction while they move. In short, they accelerate. A famous scientist, Sir Isaac Newton, wondered how and why this occurs. Theories about acceleration existed, but Newton did not find them very convincing. His skepticism led him to some of the most important discoveries in physics. Before Newton, people who studied motion noted that the objects they observed on Earth always slowed down. According to their theories, objects possessed an internal property that caused this acceleration. This belief led them to theorize that a force was required to keep things moving.

This idea seems like common sense. Moving objects do seem to slow down on their own: a car coasts to a stop, a yo-yo stops spinning, a soccer ball rolls to a halt. Newton, however, rejected this belief, instead suggesting the opposite: The nature of objects is to continue moving unless some force acts on them. For instance, Newton would say that a soccer ball stops rolling because of forces like friction and air resistance, not because of some property of the soccer ball. He would say that if these forces were **not** present, the ball would roll and roll and roll. A force (a kick) is required to start the ball's motion, and a force such as the frictional force of the grass is required to stop its motion. Newton proposed several fundamental principles that govern forces and motion. Nearly 300 years later, his insights remain the foundation for the study of forces and much of motion. This chapter stands as a testament to a brilliant scientist. At the right, you can use a simulation to experience one of Newton's fundamental principles: his law relating a net force, mass and acceleration. In the simulation, you can attempt some of the basic tasks required of a helicopter pilot. To do so, you control the **net** force upward on the helicopter. When the helicopter is in the air, the net force equals the lift force minus its weight. (The lift force is caused by the interaction of the spinning blades with the air, and is used to propel the helicopter upward.) The net force, like all forces, is measured in newtons (N). When the helicopter is in the air, you can set the net force to positive, negative, or zero values. The net force is negative when the helicopter's lift force is less than its weight. When the helicopter is on the ground, there cannot be a negative net force because the ground opposes the downward force of the helicopter's weight and does not allow the helicopter to sink below the Earth's surface. The simulation starts with the helicopter on the ground and a net force of 0 N. To increase the net force on the helicopter, press the up arrow key (□) on your keyboard; to decrease it, press the down arrow key (□). This net force will continue to be applied until you change it. To start, apply a positive net force to cause the helicopter to rise off the ground. Next, attempt to have the helicopter reach a constant vertical velocity. For an optional challenge, have it hover at a constant height of 15 meters, and finally, attempt to land (not crash) the helicopter. Once in the air, you may find that controlling the craft is a little trickier than you anticipated □ it may act a little skittish. Welcome to (a) the challenge of flying a helicopter and (b) Newton's world. Here are a few hints: Start slowly! Initially, just use small net forces. You can look at the acceleration gauge to see in which direction you are accelerating. Try to keep your acceleration initially between plus or minus 0.25 m/s2. This simulation is designed to help you experiment with the relationship between force and acceleration. If you find that achieving a constant velocity or otherwise controlling the helicopter is challenging □ read on! You will gain insights as you do.

1 – Force *Force:* Loosely defined as "pushing" or "pulling."

Your everyday conception of force as pushing or pulling provides a good starting point for explaining what a force is. There are many types of forces. Your initial thoughts may be of forces that require direct contact: pushing a box, hitting a ball, pulling a wagon, and so on. Some forces, however, can act without direct contact. For example, the gravitational force of the Earth pulls on the Moon even though hundreds of thousands of kilometers separate the two bodies. The gravitational force of the Moon, in turn, pulls on the Earth.



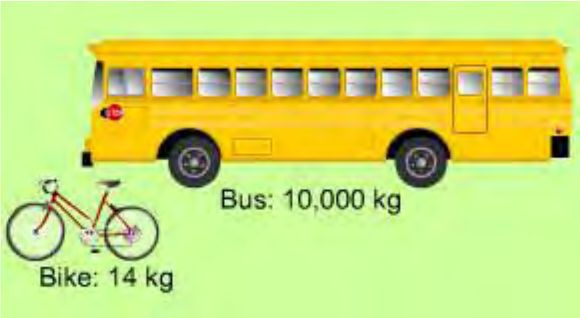
observe no net force acting on the cup. However, the nature of observations made in an accelerating reference frame is a topic far removed from this chapter's focus, and this marks the end of our discussion of reference frames in this chapter.

3 – Mass *Mass:* A property of an object that determines how much it will resist a change in velocity.

Newton's second law summarizes the relationship of force, mass and acceleration.

Mass is crucial to understanding the second law because an object's mass determines how much it resists a change in velocity. More massive objects require more net force to accelerate than less massive objects. An object's resistance to a change in velocity is called its *inertial mass.* It requires more force to accelerate the bus on the right at, say, five m/s2 than the much less massive bicycle.

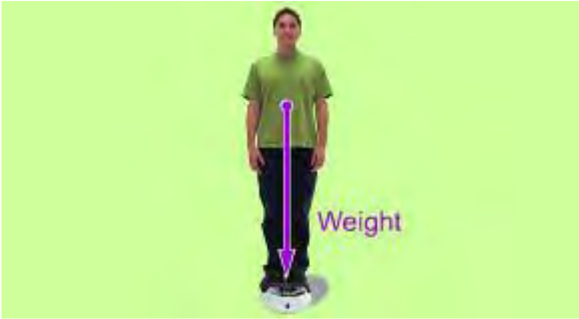
A common error is to confuse mass and weight. Weight is a force caused by gravity and is measured in newtons. Mass is an object's resistance to change in velocity and is measured in kilograms. An object's weight can vary: Its weight is greater on Jupiter's surface than on Earth's, since Jupiter's surface gravity is stronger than Earth's. In contrast, the object's mass does not change as it moves from planet to planet. The kilogram (kg) is the SI unit of mass.

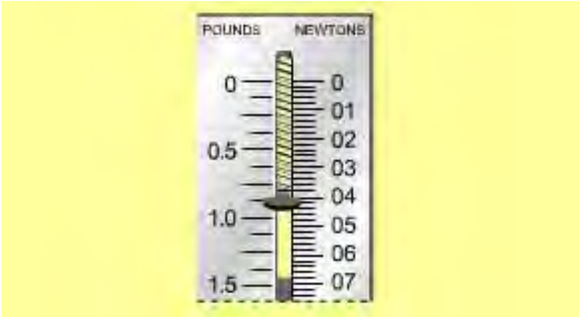


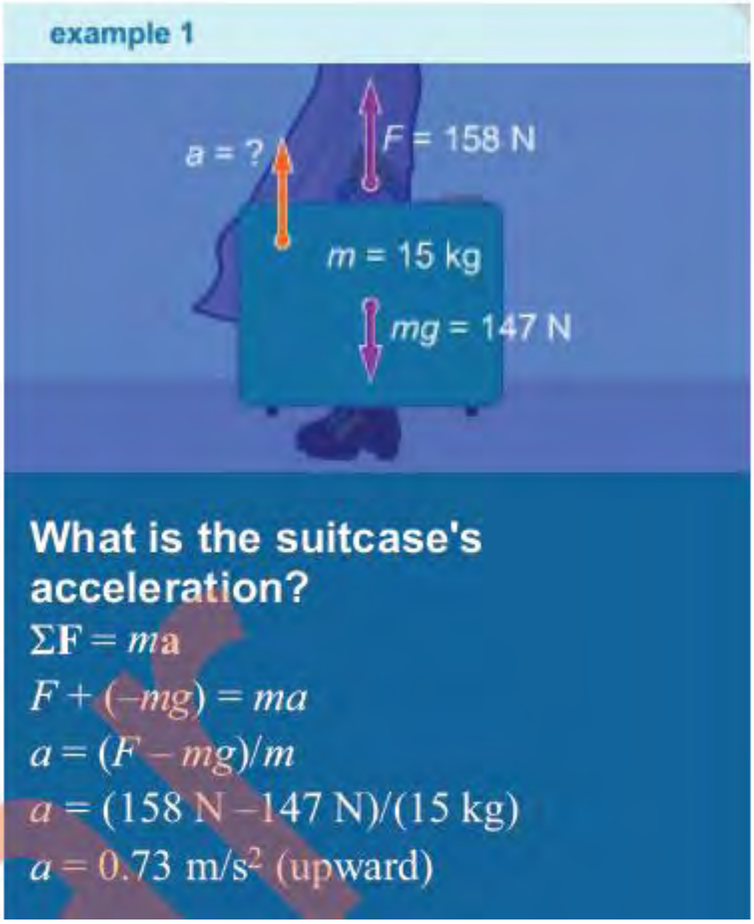
Gravitational force: weight *Weight:* The force of gravity on an object.

We all experience weight, the force of gravity. On Earth, by far the largest component of the gravitational force we experience comes from our own planet. To give you a sense of proportion, the Earth exerts 1600 times more gravitational force on you than does the Sun. As a practical matter, an object's weight on Earth is defined as the gravitational force the Earth exerts on it. Weight is a force; it has both magnitude and direction. At the Earth' surface, the direction of the force is toward the center of the Earth. The magnitude of weight equals the product of an object's mass and the rate of free fall acceleration due to gravity. On Earth, the rate of acceleration g due to gravity is 9.80 m/s2. The rate of free fall acceleration depends on a planet's mass and radius, so it varies from planet to planet. On Jupiter, for instance, gravity exerts more force than on Earth, which makes for a greater value for free fall acceleration. This means you would weigh more on Jupiter's surface than on Earth's.

Scales, such as the one shown in Concept 1, are used to measure the magnitude of weight. The force of Earth's gravity pulls Kevin down and compresses a spring. This scale is calibrated to display the amount of weight in both newtons and pounds, as shown in Equation 1. Forces like weight are measured in pounds in the British system. One newton equals about 0.225 pounds. A quick word of caution: In everyday conversation, people speak of someone who "weighs 100 kilograms," but kilograms are units for mass, not weight. Weight, like any force, is measured in newtons. A person with a mass of 100 kg weighs 980 newtons.


Weight


POUNDS
NEWTONS
0
0.5
1.0
1.5
0
01
02
03
04
05
06
07


example 1
a = ?
F = 158 N
m = 15 kg
mg = 147 N
What is the suitcase's acceleration?
ΣF = ma
F + (–mg) = ma
a = (F – mg)/m
a = (158 N – 147 N)/(15 kg)
a = 0.73 m/s2 (upward)

Above you see “Rocket Guy,” a superhero who wears a jet pack. The jet pack provides an upward force on him, while Rocket Guy’s weight points downward.

Variables

All the forces on Rocket Guy are directed along the y axis.

thrust	$F_T = 1250\ \mathrm{N}$
weight	$-mg = -905\ \mathrm{N}$
mass	m
acceleration	a

What is the strategy?

1. Determine the net force on Rocket Guy.
2. Determine Rocket Guy’s mass.
3. Use Newton’s second law to find his acceleration.

Physics principles and equations

Newton’s second law

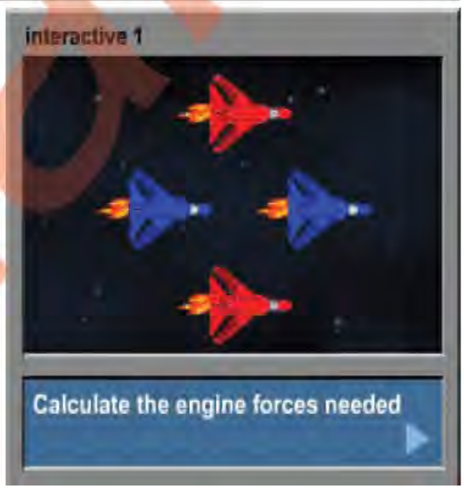
$$\Sigma \mathbf{F} = m\mathbf{a}$$

5.9 - Interactive problem: flying in formation

The simulation on the right will give you some practice with Newton’s second law. Initially, all the space ships have the same velocity. Their pilots want all the ships to accelerate at 5.15 m/s^2. The red ships have a mass of 1.27×10^4 kg, and the blue ships, a mass of 1.47×10^4 kg. You need to set the amount of force supplied by the ships’ engines so that they accelerate equally. The masses of the ships do not change significantly as they burn fuel.

Apply Newton’s second law to calculate the engine forces needed. The simulation uses scientific notation; you need to enter three-digit leading values. Enter your values and press GO to start the simulation. If all the ships accelerate at 5.15 m/s^2, you have succeeded. Press RESET to try again.

If you have difficulty solving this problem, review Newton’s second law.



concept 1

Newton's third law

Forces come in pairs

Equal in strength, opposite in direction

The forces act on different objects

equation 1

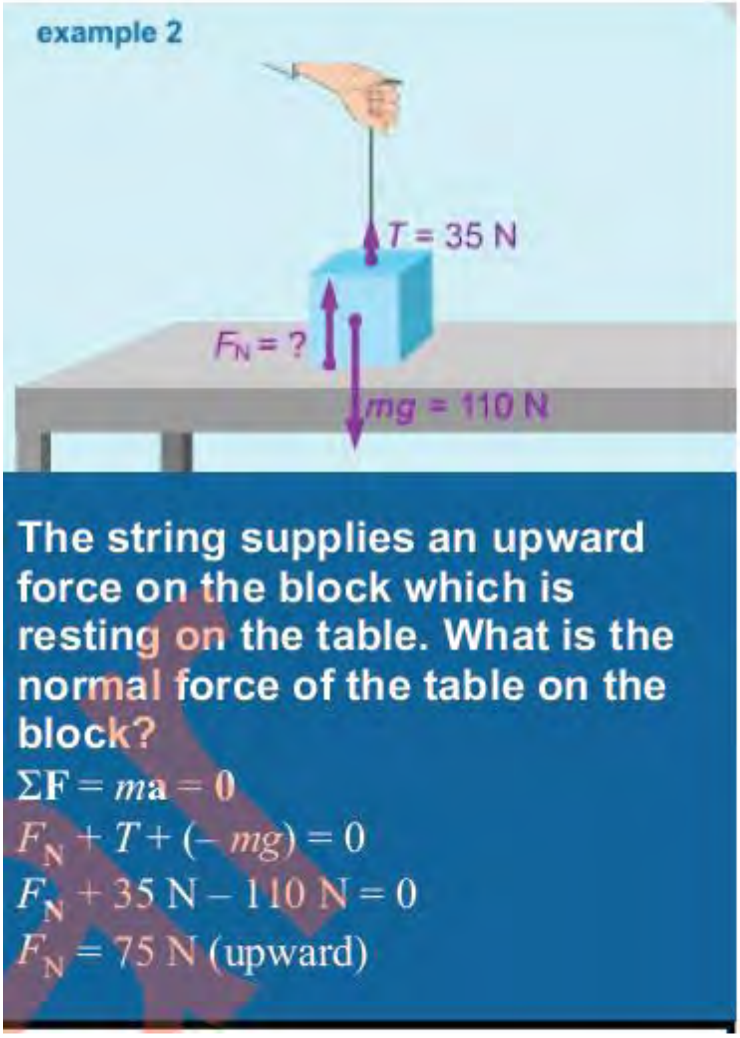
Newton's third law

$$\mathbf{F}_{ab} = -\mathbf{F}_{ba}$$

Newton's third law: "To any action there is always an opposite and equal reaction; in other words, the actions of two bodies upon each other are always equal and always opposite in direction."

Newton's third law states that forces come in pairs and that those forces are equal in magnitude and opposite in direction. When one object exerts a force on another, the second object exerts a force equal in magnitude but opposite in direction on the first.

For instance, if you push a button, it pushes back on you with the same amount of force. When someone leans on a wall, it pushes back, as shown in the illustration above.



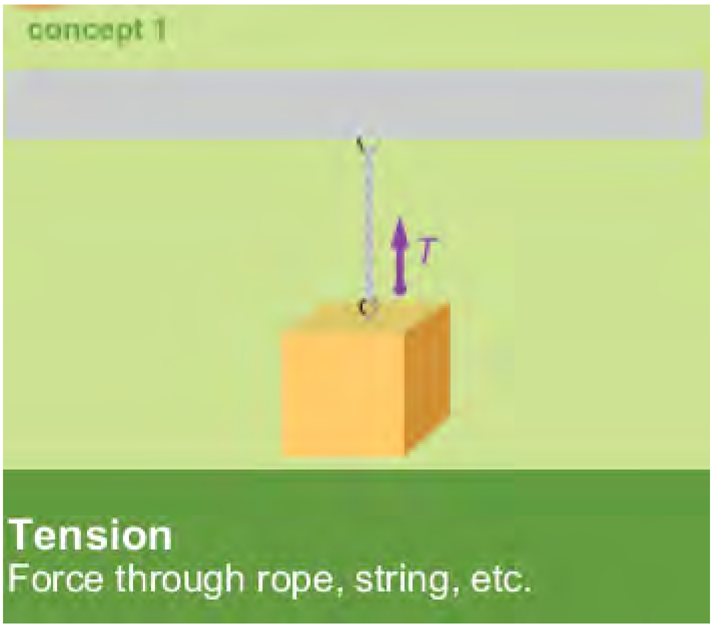
Tension: Force exerted by a string, cord, twine, rope, chain, cable, etc.

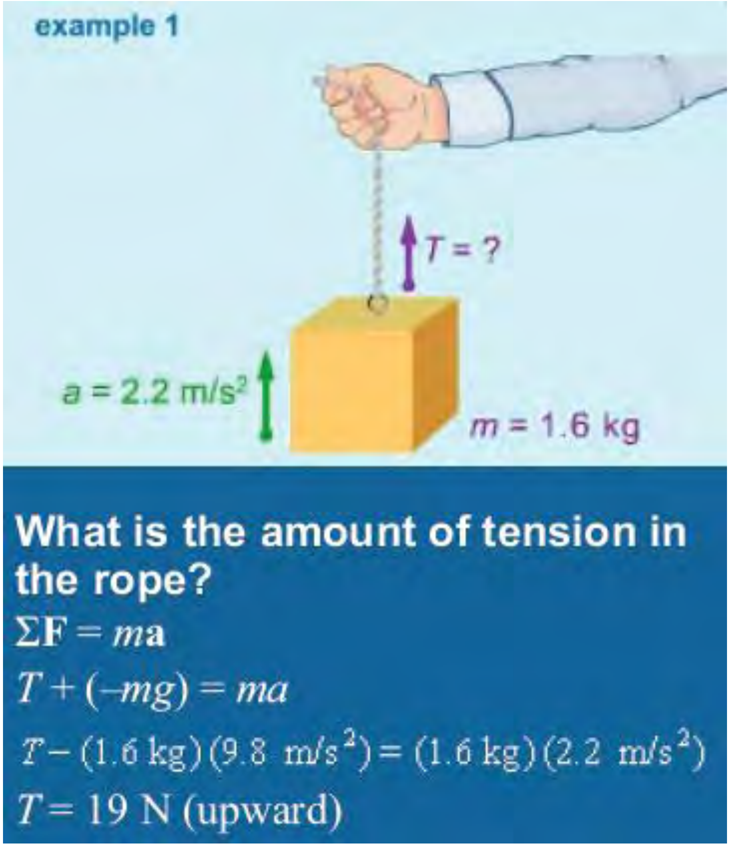
In physics textbooks, tension means the pulling force conveyed by a string, rope, chain, tow-bar, or other form of connection. In this section, we will use a rope to illustrate the concept of tension.

The rope in Concept 1 is shown exerting a force on the block; that force is called tension. This definition differs slightly from the everyday use of the word tension, which often refers to forces within a material or object − or a human brain before exams.

In physics problems, two assumptions are usually made about the nature of tension. First, the force is transmitted unchanged by the rope. The rope does not stretch or otherwise diminish the force. Second, the rope is treated as having no mass (it is massless). This means that when calculating the acceleration of a system, the mass of the rope can be ignored.

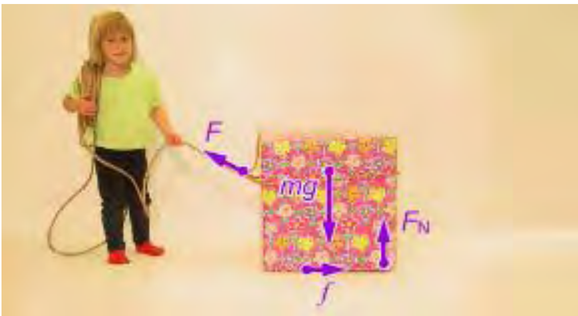
Example 1 shows how tension forces can be calculated using Newton's second law. There are two forces acting on the block: its weight and the tension. The vector sum of those forces, the net force, equals the product of its mass and acceleration. Since the mass and acceleration are stated, the problem solution shows how the tension can be determined.



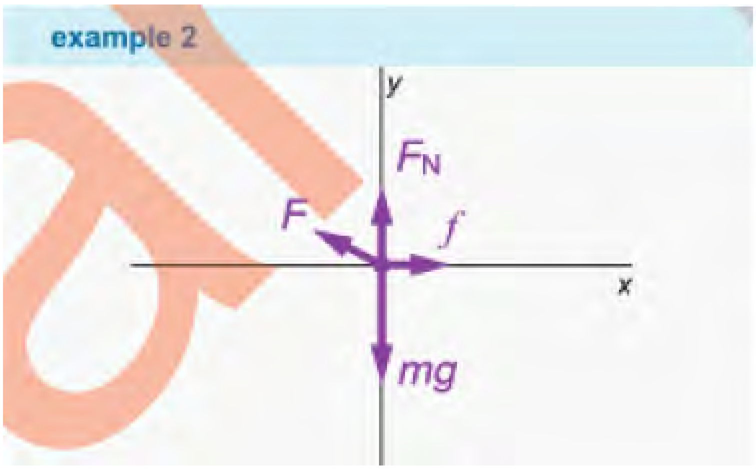


Newton's second and third laws

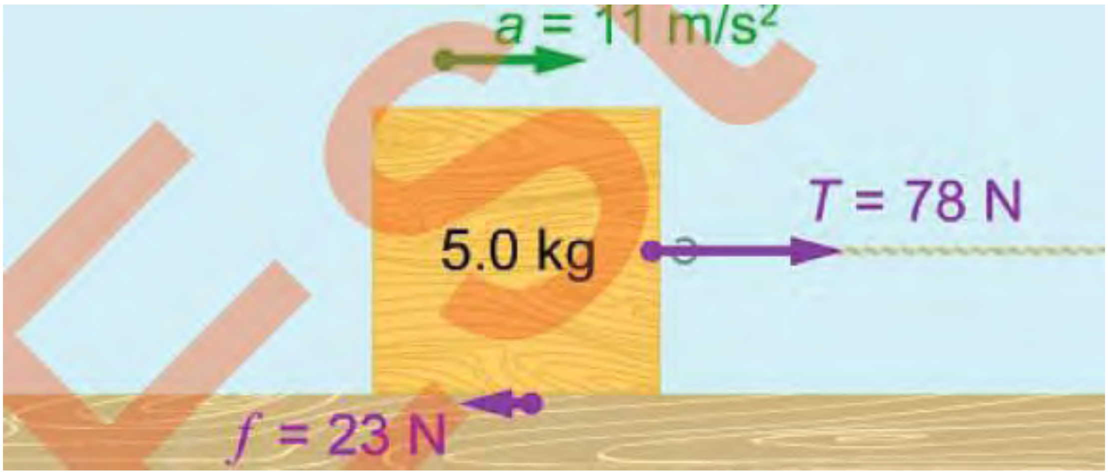
It might seem that Newton's third law could lead to the conclusion that forces do **not** cause acceleration, because for every force there is an equal but opposite force. If for every force there is an equal but opposite force, how can there be a net non-zero force? The answer lies in the fact that the forces do not act on the same object. The pair of forces in an action-reaction pair acts on **different** objects. In this section, we illustrate this often confusing concept with an example. balance as well. By considering the forces acting in both the horizontal and vertical directions, the tensions of the ropes can be determined. In Example 1, one of the forces shown is friction, *f*. Friction acts to oppose motion when two objects are in contact.



Draw a free-body diagram of the forces on the box.

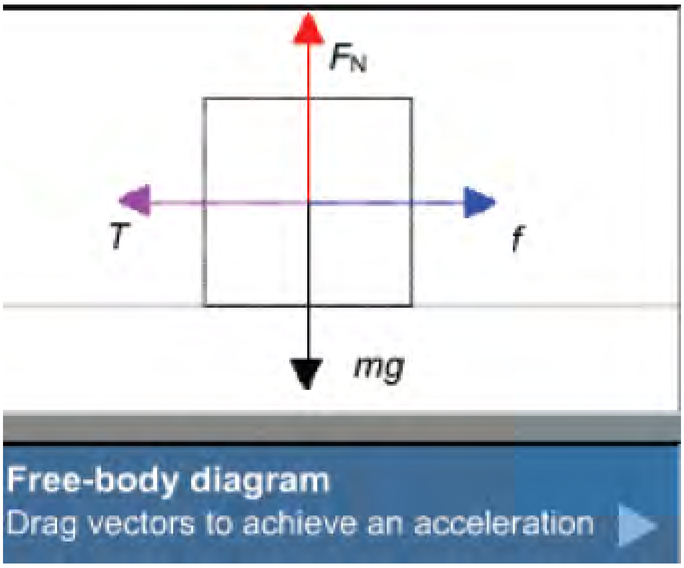


Free-body diagram of forces on box



A rope pulls the block against friction. Draw a free-body diagram. The block accelerates at 11 m/s^2 if the diagram is correct.

In this section, you practice drawing a free-body diagram. Above, you see the situation: A block is being pulled horizontally by a rope. It accelerates to the right at 11 m/s^2. In the simulation on the right, the force vectors on the block are drawn, but each one points in the wrong direction, has the wrong magnitude, or both. We ignore the force of air resistance in this simulation.



Your job is to fix the force vectors. You do this by clicking on the heads of the vectors and dragging them to point in the correct direction. (To simplify your work, they "snap" to vertical and horizontal orientations, but you do need to drag them close before they will snap.) You change both their lengths (which determine their magnitudes) and their directions with the mouse.

The mass of the block is 5.0 kg. The tension force *T* is 78 N and the force of friction *f* is 23 N. The friction force acts opposite to the direction of the motion. Calculate the magnitudes of the weight *mg* and the normal force *F*N to the nearest newton, and then drag the heads of the vectors to the correct positions, or click on the up and down arrow buttons, and press GO. If you are correct, the block will accelerate to the right at 11 m/s2. If not, the block will move based on the net force as determined by your vectors as well as its mass. Press RESET to try again. There is more than one way to arrange the vectors to create the same acceleration, but there is only one arrangement that agrees with all the information given. If you have difficulty solving this problem, review the sections on weight and normal force, and the section on free-body diagrams

Step-by-step solution

As noted, we use the convention that forces to the right are positive and those to the left are negative. A more rigorous approach would be to calculate the vector components of these forces using the cosine of 0° for the frictional force and the cosine of 180° for the pushing force. The result would be x components of 18.0 N and □34.0 N (the same conclusion we reached via inspection and convention). Many instructors prefer this approach. It does not change the answer to the problem, but the component method is more rigorous, and is required to solve more difficult problems.

Step	Reason
1. $\Sigma \mathbf{F} = \mathbf{F}_{push} + \mathbf{F}_{friction}$	net horizontal force
2. $\Sigma F = -34.0\ \text{N} + 18.0\ \text{N}$ $\Sigma F = -16.0\ \text{N}$	enter values and add
3. $\Sigma F = ma$	Newton's second law
4. $a = \Sigma F/m$	solve for a
5. $a = (-16.0\ \text{N}) / (15.0\ \text{kg})$	enter values
6. $a = -1.07\ \text{m/s}^2$	division

The helicopter on the right is being used as a scale, making it one of the more expensive scales in the world, we suspect. This simulation includes three crates; each has a slightly different mass. Your assignment is to find the crate with a mass of 661 kg. Do this by lifting each crate with the helicopter and noting the acceleration. The helicopter lifts each crate with a force of 10,748 N via the tension in the cable. The resulting acceleration of each crate will let you calculate its mass.

Click on the graphic to start the simulation. To determine the answer, drag the helicopter to each of the three crates and press GO to make the helicopter lift the crate. Record the acceleration of each crate and use the acceleration to calculate the mass. When you have found the crate with a mass of 661 kg, select it by clicking on it. The simulation will tell you whether you clicked on the correct one.

If you cannot solve the problem, review Newton's second law and the section on weight.

Friction: A force that resists the motion of one object sliding past another.

Friction between the buffalo's back and the tree scratches an itch.

If you push a cardboard box along a wooden floor, you have to push to overcome the force of friction. This force makes it harder for you to slide the box. The force of friction opposes any force that can cause one object to slide past another. There are two types of friction: static and kinetic. These forces are discussed in more depth in other sections. In this section, we discuss some general properties of friction

The amount of friction depends on the materials in contact. For example, the box would slide more easily over ice than wood. Friction is also proportional to the normal force. For a box on the floor, the greater its weight, the greater the normal force, which increases the force of friction. Humans expend many resources to combat friction. Motor oil, Teflon™, WD-40™, Tri-Flo™ and many other products are designed to reduce this force. However, friction can be very useful. Without it, a nail would slip out of a board, the tires of a car would not be able to “grip” the road, and you would not be able to walk. Friction exists even between seemingly smooth surfaces. Although a surface may appear smooth, when magnified sufficiently, any surface will look bumpy or rough, as the illustration in Concept 2 on the right shows. The magnified picture of the “smooth” crystal reveals its microscopic “rough” texture. Friction is a force caused by the interaction of molecules in two surfaces. maximum amount of static friction is constant. Why? With the greater contact area, the normal and frictional forces per unit area diminish proportionally.

Static friction

$$f_{s,max} = \mu_s F_N$$

$f_{s,max}$ = maximum static friction
μ_s = coefficient of static friction
F_N = normal force

equation 2

Coefficient of static friction

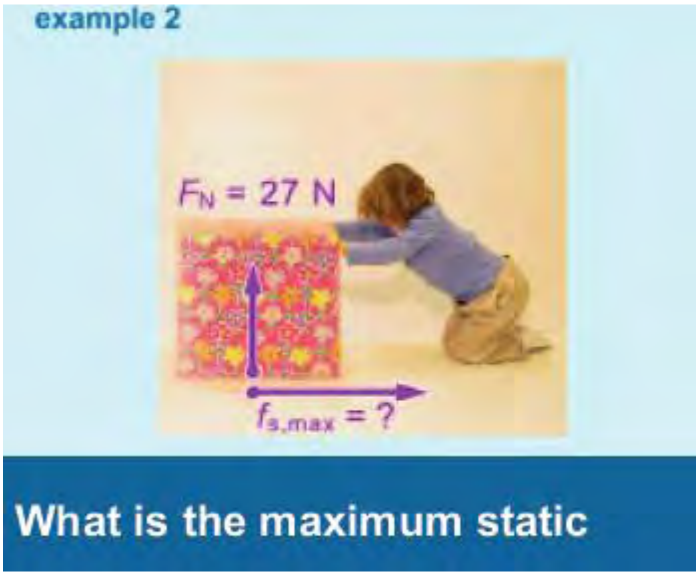
Tires on dry pavement	0.90
Tires on wet pavement	0.42
Glass on glass	0.94
Steel on steel	0.78
Oak on oak	0.54
Waxed ski on dry snow	0.04
Teflon™ on Teflon™	0.04

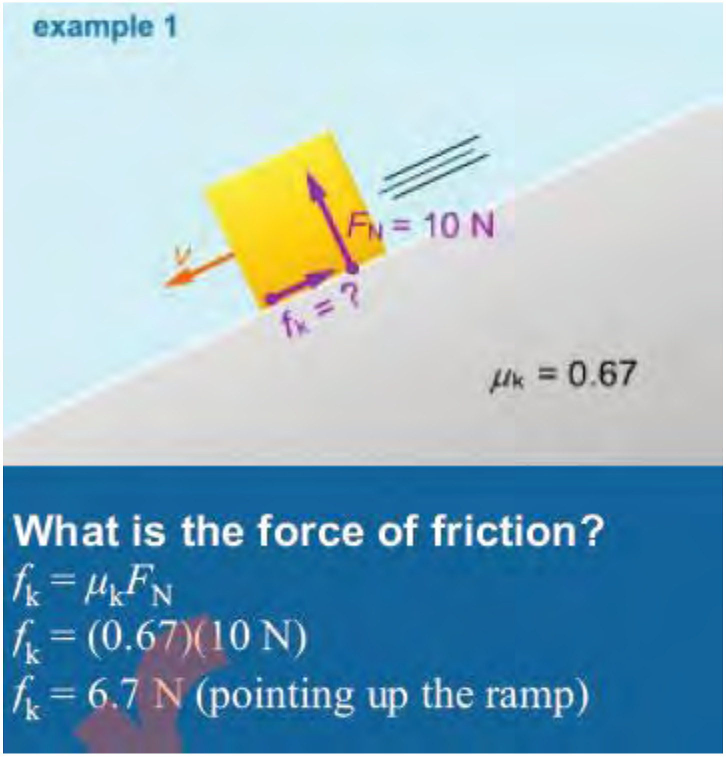
Coefficients of static friction

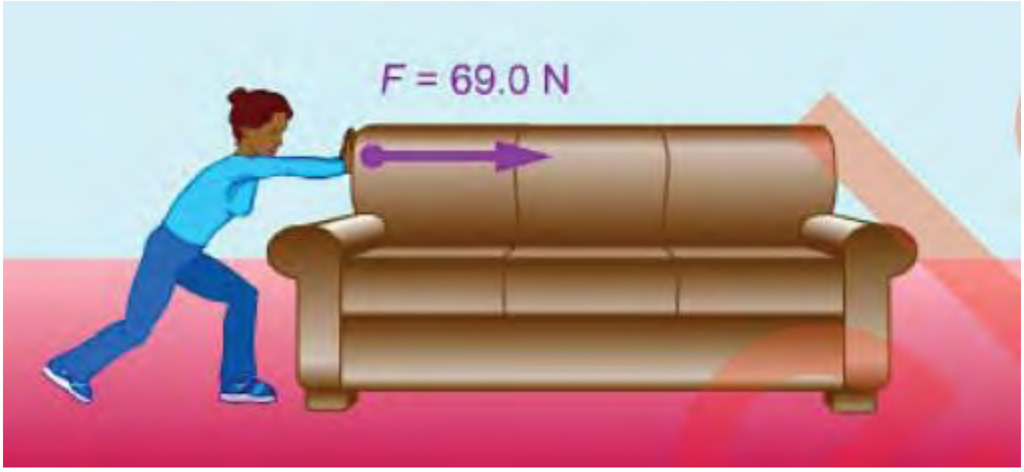
example 1

Anna is pushing but the box does not move. What is the force of static friction?

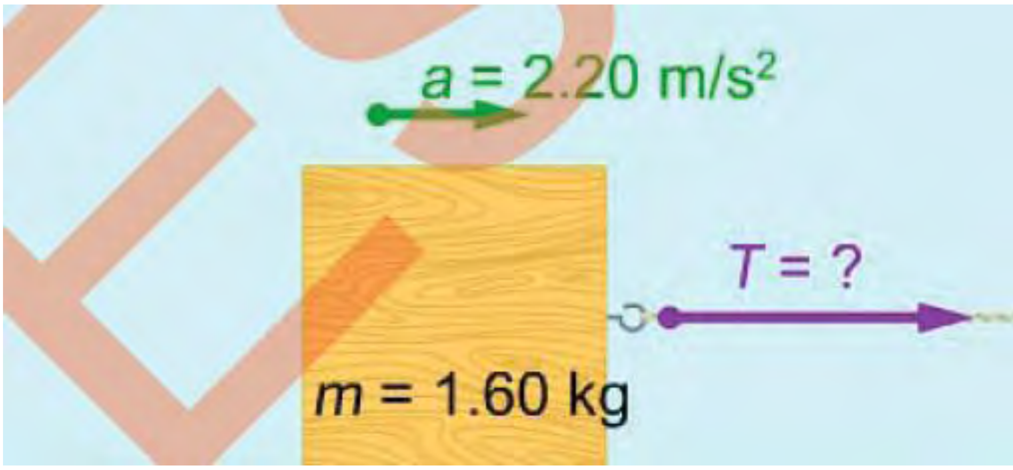
f_s = 7 N to the right


example 2
F_N = 27 N
f_s,max = ?
What is the maximum static


example 1
v
F_N = 10 N
f_k = ?
μ_k = 0.67
What is the force of friction?
$f_k = \mu_k F_N$
$f_k = (0.67)(10\text{ N})$
$f_k = 6.7\text{ N}$ (pointing up the ramp)



While rearranging your living room, you push your couch across the floor at a constant speed with a horizontal force of 69.0 N. You are using specia pads on the couch legs that help it slide easier. If the couch has a mass of 59.5 kg, what is the coefficient of kinetic friction between the pads and the floor?

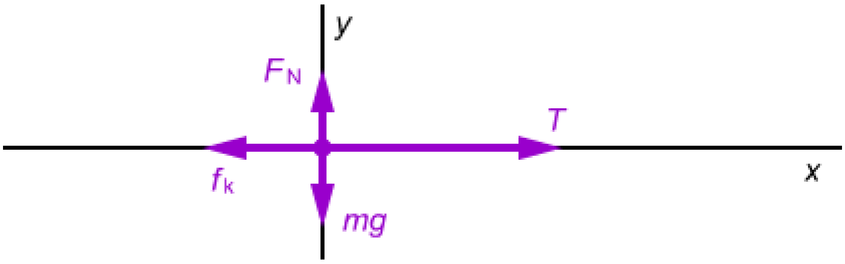


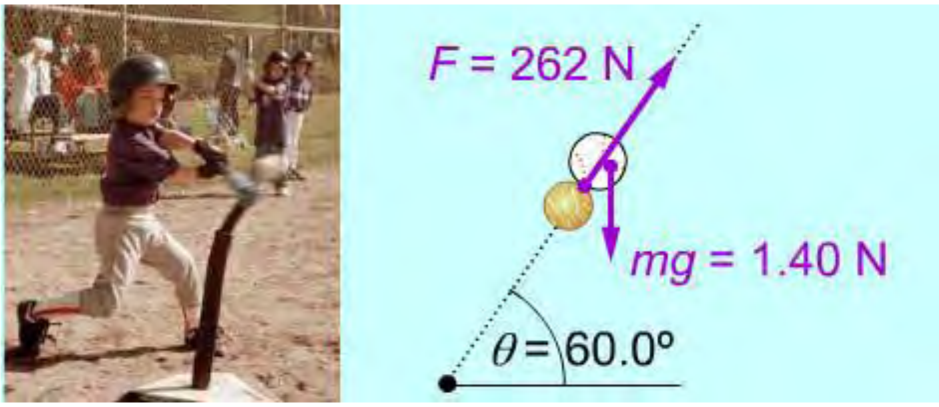
The coefficient of kinetic friction is 0.200. What is the magnitude of the tension force in the rope?

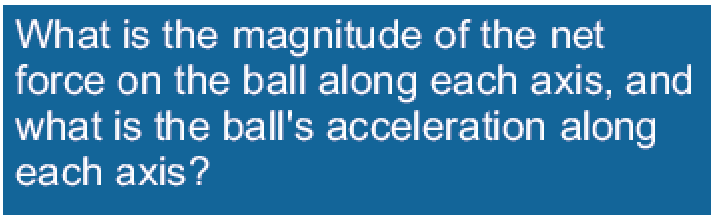
Above, you see a block accelerating to the right due to the tension force applied by a rope. What is the magnitude of tension the rope applies to the block?

Starting this type of problem with a free-body diagram usually proves helpful.

Draw a free-body diagram

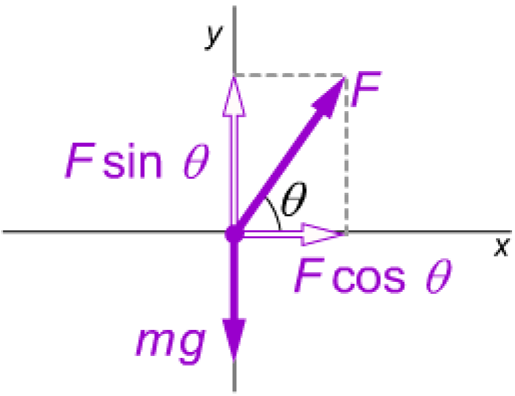






Above, you see a bat hitting a ball at an angle. You are asked to find the net force and the acceleration of the ball along the x and y axes.

Draw a free-body diagram



The forces on the ball are its weight down and the force of the bat at the angle to the *x* axis.

Variables

	x component	*y* component
weight	0	$mg \sin 270° = -1.40$ N
force	$F \cos \theta$	$F \sin \theta$
acceleration	a_x	a_y
force	$F = 262$ N	
angle	$\theta = 60.0°$	
mass	$m = mg/g = (1.40\text{ N}) / (9.80\text{ m/s}^2) = 0.143$ kg	

What is the strategy?

1. Draw a free-body diagram.
2. Use trigonometry to calculate the net force on the ball along each axis.
3. Use Newton's second law to find the acceleration of the ball along each axis. The mass of the ball is not given, but you can determine it because you are told its weight. We do this in the variables table.

Physics principles and equations

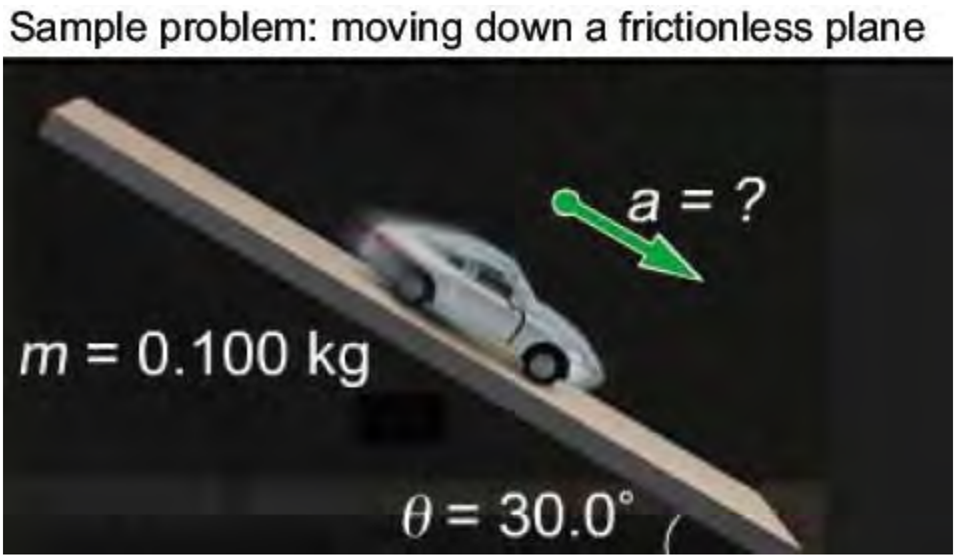
Newton's second law

$$\Sigma\mathbf{F} = m\mathbf{a}$$

Step-by-step solution

We begin by calculating the net force along the *x* axis.

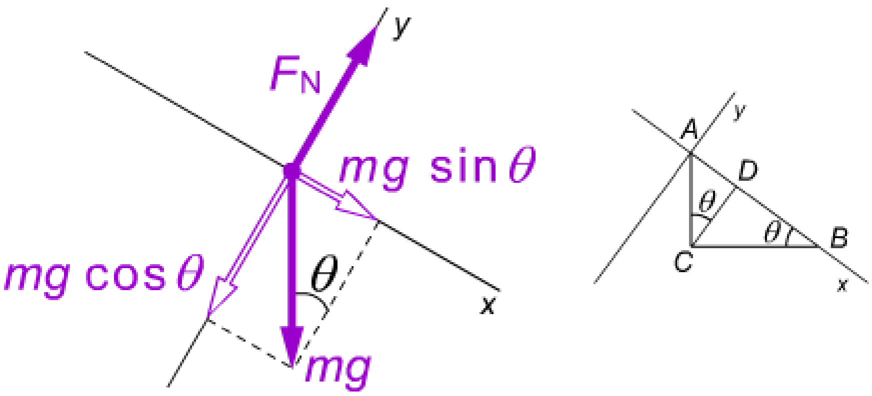
Step	Reason
1. $\Sigma F_x = F\cos\theta$	net force along x axis
2. $\Sigma F_x = (262\text{ N})(\cos 60.0°)$	x component of force
3. $\Sigma F_x = 131\text{ N}$	evaluate



What is the car's acceleration?

Above, you see a toy car going down an inclined plane. The diagram shows the mass of the car and the angle the plane makes with the horizontal. You are asked to calculate the car's acceleration. In this problem, ignore any friction or air resistance, as well as any energy consumed by the rotation of the wheels.

Draw a free-body diagram



By rotating the axes so that the x axis is parallel to the car's motion down the ramp, we make the forces along the y axis sum to zero. (These two forces are the y component of the car's weight and the normal force from the ramp.) Rotating the axes means there is a net force only along the x axis, and this reduces the steps required to solve the problem. It may be a little difficult to see why □, the angle that the plane makes with the horizontal, is the same as the angle □ in the free-body diagram. The drawing to the right of the free-body diagram uses two similar right triangles to show why this is true. The triangle ABC has one leg (AC) that is the weight vector, and its hypotenuse (AB) lies along the x axis. The hypotenuse of the smaller triangle ACD is the weight vector. These are both right triangles and share a common angle at A, so they are similar. It is often useful to check this angle with the situation shown. At a 30° angle, the y component of the weight is larger than the x component (the cosine of 30° is greater than the sine of 30°). Looking at the picture above, this is what you would expect. The component of the weight down the plane is less than the component on the plane. It may help to push it to the extreme: What would you expect at a 0° angle? At 90°?

Variables

With the axes rotated and □ as shown, the x component of the weight is computed using the sine, and the y component with the cosine. (Without the rotation, the x component would be calculated with the cosine, and the y component with the sine.)

	x component	y component
weight	$mg \sin \theta$	$-mg \cos \theta$
normal force	0 N	F_N
acceleration	a	0 m/s^2
mass	$m = 0.100$ kg	
angle	$\theta = 30.0°$	

What is the strategy?

1. Draw a free-body diagram, rotating the axes so the x axis is parallel to the motion of the car.

2. Use trigonometry to calculate the net force on the car.

3. Use Newton's second law to determine the acceleration of the car.

Physics principles and equations

Newton's second law

□**F** = *m***a**

What is the strategy?

1. Draw a free-body diagram.
2. Calculate the net force on the ball along each axis by finding the components of the two forces using trigonometry.
3. Use Newton's second law to find the acceleration of the ball along each axis.

Physics principles and equations

Newton's second law

□**F** = *m***a**

Step-by-step solution

We begin by calculating the net force along the *x* axis.

Step	Reason
1. $\Sigma F_x = F_1 \cos\theta_1 + F_2 \cos\theta_2$	net force along *x* axis
2. $\Sigma F_x = 162\cos(171°)\text{N} + 215\cos(285°)\text{N}$	enter values
3. $\Sigma F_x = -104\text{ N}$	evaluate

Next we calculate the net force along the *y* axis.

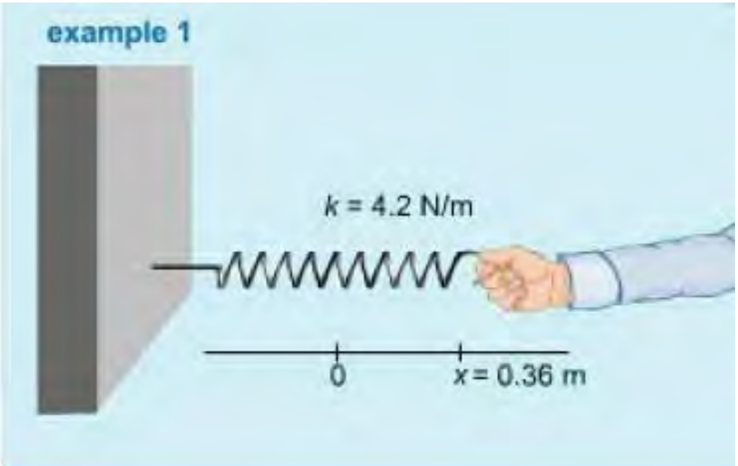
Step	Reason
4. $\Sigma F_y = F_1 \sin\theta_1 + F_2 \sin\theta_2$	net force along *y* axis
5. $\Sigma F_y = 162\sin(171°)\text{N} + 215\sin(285°)\text{N}$	enter values
6. $\Sigma F_y = -182\text{ N}$	evaluate

Using Newton's second law and the net force in the x dimension, calculated above, we find the acceleration in the x dim

Step	Reason
7. $\Sigma F_x = ma_x$	Newton's second law
8. $a_x = \Sigma F_x / m$	solve for a_x
9. $a_x = (-104\text{ N}) / (0.420\text{ kg})$	enter values
10. $a_x = -248\text{ m/s}^2$	division

And finally we calculate the acceleration in the *y* dimension.

Step	Reason
11. $\Sigma F_y = ma_y$	Newton's second law
12. $a_y = \Sigma F_y / m$	solve for a_y
13. $a_y = (-182\text{ N}) / (0.420\text{ kg})$	enter values
14. $a_y = -433\text{ m/s}^2$	division



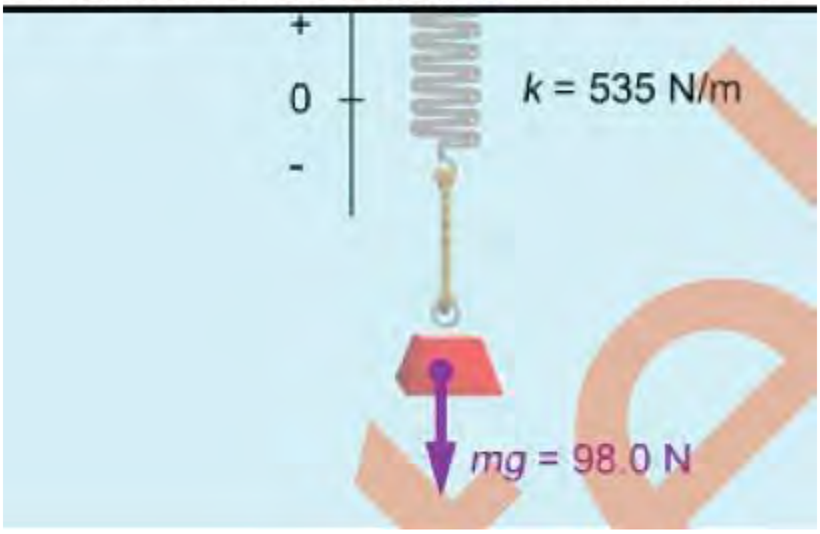
What is the force exerted by the spring?

$F_s = -kx$

$F_s = -(4.2\text{ N/m})(0.36\text{ m})$

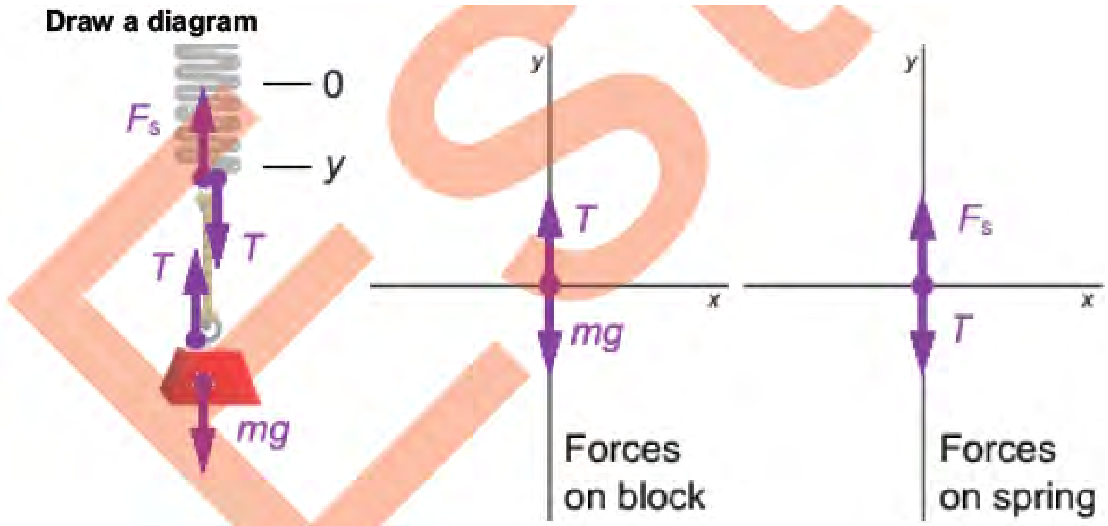
$F_s = -1.5\text{ N}$ (to the left)

Sample problem: spring force and tension



What is the amount of tension in the rope? What is the position of the end of the spring away from its rest position?

You see a block hanging from a rope attached to a spring. The block is stationary. You are asked to determine the tension in the rope and the position of the end of the spring relative to its rest point



Above on the left, we draw the forces: the weight of the block, the tension forces in the rope, and the spring force. The tension forces are equal in magnitude, so we use the same variable T for each of them. Then we draw two free-body diagrams, one for the block and one for the lower end of the spring.

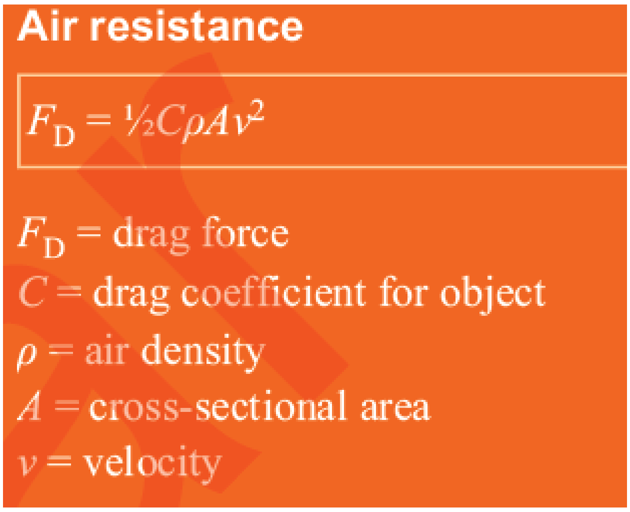
Variables

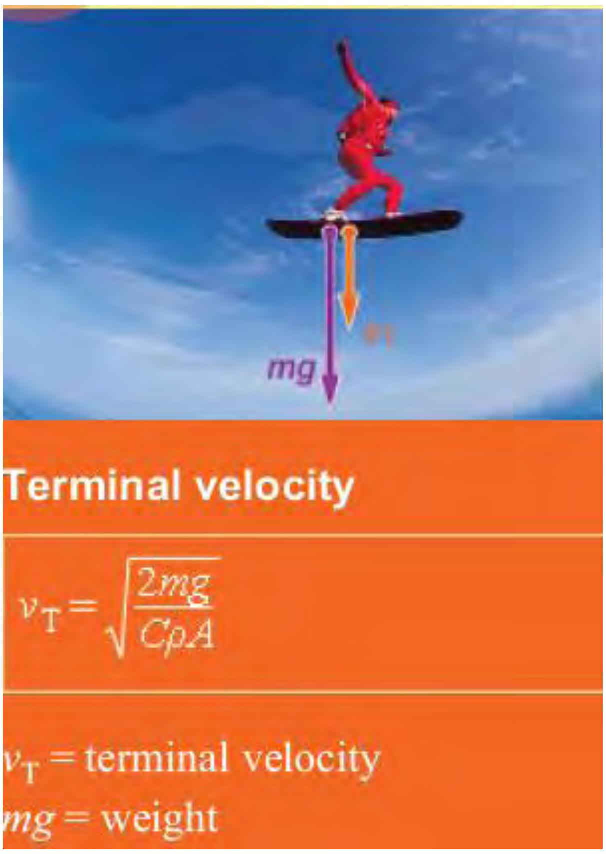
weight	$-mg = -98.0$ N
tension	T
spring force	F_s
spring constant	$k = 535$ N/m
displacement	y

What is the strategy?

1. Draw a free-body diagram.
2. Apply Newton's second law to calculate the tension force in the rope.
3. Then apply Newton's second law with Hooke's law to find the position of the end of the spring.

Research has actually determined that cats reach terminal velocity after falling six stories. In fact, they tend to slow down after six stories. Here's why this occurs: The cat achieves terminal velocity and then relaxes a little, which expands its cross sectional area and increases its drag force. As a result, it slows down. One has to admire the cat for relaxing in such a precarious situation (or perhaps doubt its intelligence). If you think this may be an urban legend, consult the *Journal of the American Veterinary Association*, volume 191, page 1399.





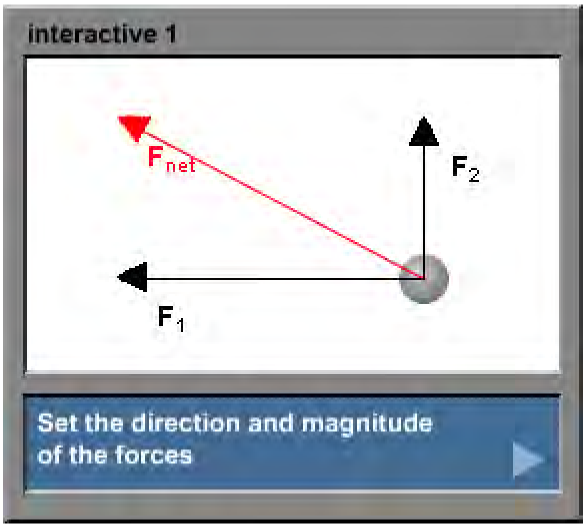
Introduction

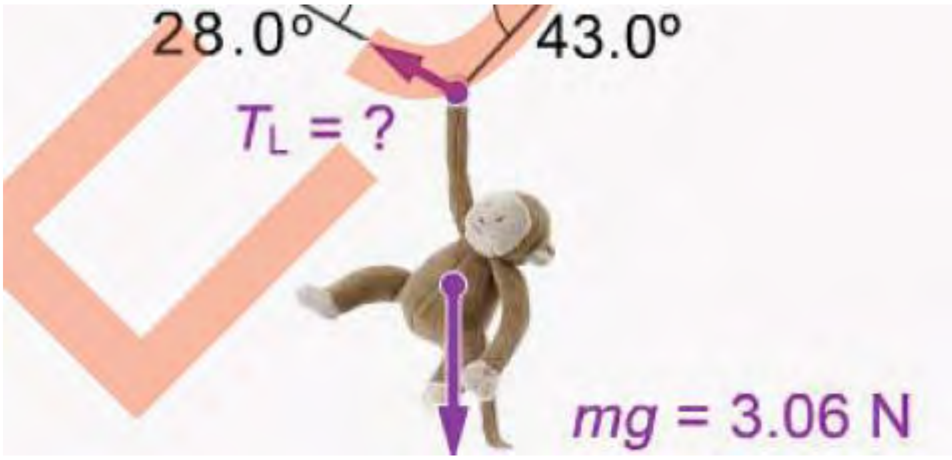
Now, you will get some additional practice applying Newton's laws. More specifically, you will use them in situations where multiple forces are acting on a single object. If the application of multiple forces results in a net force acting on an object, it accelerates. On the other hand, if the forces acting on it sum to zero in every dimension, the result is equilibrium. The object does not accelerate; it either maintains a constant velocity, or remains stationary. (Forces can also cause an object to rotate, but rotational motion is a later topic in mechanics.) Equilibrium is an important topic in engineering. The school buildings you study in, the bridges you travel across all such structures require careful design to ensure that they remain in equilibrium. The simulation on the right will help you develop an understanding for how forces in different directions combine when applied to an object. The 5.0 kg ball has two forces acting on it, **F**1 and **F**2. They act on it as long as the ball is on the screen. You control the direction and magnitude of each force. In the simulation, you set a force vector's direction and magnitude by dragging its arrowhead; You will notice the angles are restricted to multiples of 90°. You can also adjust the magnitude of each vector with a controller in the control panel. The net force is shown in the simulation; it is the vector sum of **F**1 and **F**2.

You can check the box “Display vectors head to tail” if you would like to see them graphically combined in that fashion. Press GO to start the simulation and set the ball moving in response to the forces on it. Here are some challenges for you. First, set the forces so that the ball does not move at all when you press GO. The individual forces must be at least 10 newtons, so setting them both to zero is not an option! Next, hit each of the three animated targets. The center of one is directly to the right of the ball and the center of another is at a 45° angle above the horizontal from the ball. Set the individual vectors and press GO to hit the center of each target in turn. The target to the left is at a 150° angle. It is the “extra credit” target. Determining the correct ratio of vectors will require a little thought. We allow for rounding with this target; if you set one of the vectors to 10 N, you can solve the problem by setting the other one to the appropriate closest integer value.

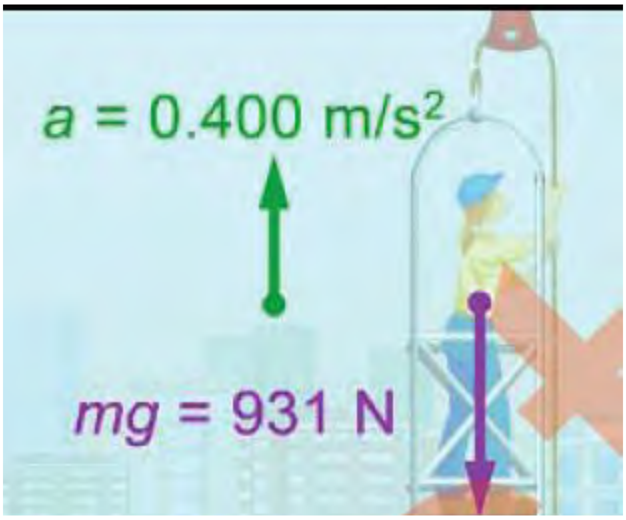
Sample problem: a mass on ropes

Since it is stationary, the monkey is not accelerating, which means no net force is acting on it. This section shows you a useful technique for solving problems that involve multiple forces acting on a single object. To calculate the overall force on an object like the rope, the x and y components of each force need to be determined. Since there is no acceleration in any direction, there is no net force along any dimension. Two equations can be developed: The sum of the x components equals zero, and the sum of the y components equals zero. We will use a consistent approach to solving multi-force problems. First we draw a free-body diagram to help us identify the variables and the force components. Then we state the variables and their values when they are known. Next, we use Newton’s second law, relating the net force to the acceleration and the mass of objects in the problem. Finally, we perform the algebraic and mathematical steps needed to solve the problem.





The monkey hangs without moving in the configuration you see here. What is the magnitude of the tension in the left rope?



What is the magnitude of the tension in the rope above the window washer's hands?

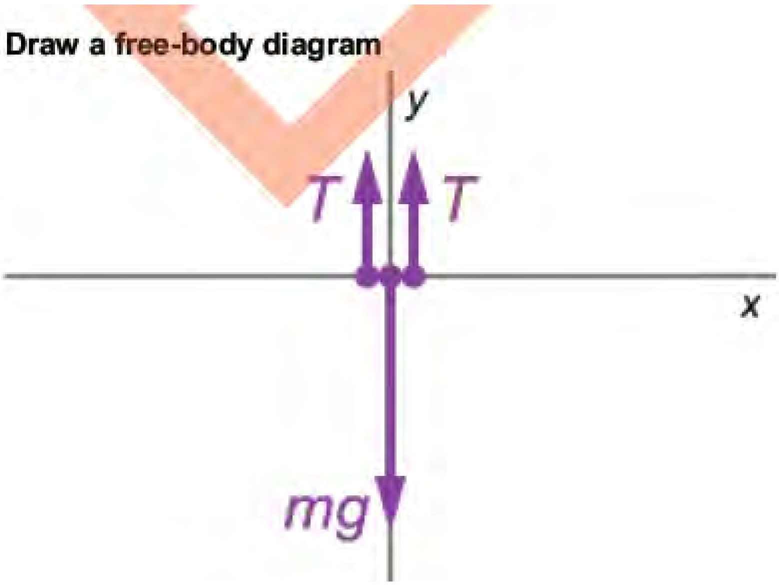
If you have ever worked in a skyscraper, you have probably seen window washers raising and lowering themselves and their scaffolding. Let's say a particularly energetic window washer is accelerating herself upward, as shown above. You are asked to find the amount of tension in one of the two ropes.

Ropes and pulleys are fairly common in mechanics problems. We will always assume that tension is transmitted with a change in direction but no change in magnitude in a rope that goes around a pulley. You may often be told to draw these conclusions when a problem states that the pulley is assumed to be frictionless and massless, or the rope is massless and does not stretch.

The drawing above shows the weight of the combination of the scaffolding and the window washer, which is 931 N. To apply Newton's second law, we need to compute the mass of the system (window washer plus scaffolding) from its weight.

Draw a free-body diagram

Draw a free-body diagram



Variables

weight	$-mg = -931$ N
tension in left rope	T
tension in right rope	T
acceleration	$a = 0.400$ m/s^2
mass	$m = 931$ N / 9.80 m/s^2 = 95.0 kg

Variables

Block A on table	x component	y component
tension	T	0 N
weight	0 N	$-m_A g$
normal force	0 N	F_N
acceleration	a	0 m/s
mass	$m_A = 4.20$ kg	
Falling block B		
tension	0 N	T
weight	0 N	$-m_B g$
acceleration	0 m/s	$-a$
mass	$m_B = 5.70$ kg	

What is the strategy?

1. Draw a free-body diagram for each block.

2. Calculate the net force on each block. Block A moves only in the horizontal direction, so we can ignore the vertical forces on it. Block B moves only vertically, and there are no horizontal forces on it.

3. Use Newton's second law for each block to find two expressions for the tension force in the rope, and set the expressions equal to find the acceleration.

Physics principles and equations

Newton's second law relates the net force and acceleration. Block A moves only horizontally, so we will consider only the *x* direction for it; similarly, we consider only the *y* direction for block B.

□**F** = *m***a**

Step-by-step solution

We start by considering block A, and use Newton's second law to find an equation that gives an expression for the tension force in the rope.

Step	Reason
1. $\Sigma \mathbf{F}_x = m_A \mathbf{a}_x$	Newton's second law applied to A
2. $T = m_A a$	tension is net force on A

Then we apply Newton's second law to block B to find another expression for the tension. Since block B falls, we assign its acceleration a negative value.

Step	Reason
3. $\Sigma \mathbf{F}_y = m_B \mathbf{a}_y$	Newton's second law applied to B
4. $T + (-m_B g) = m_B(-a)$	net force is sum of tension and weight
5. $T = m_B g - m_B a$	solve for T

We set the two expressions for the tension equal. The rest is algebra.

Step	Reason
6. $m_A a = m_B g - m_B a$	set tensions equal, from 2 and 5
7. $a = (m_B g)/(m_A + m_B)$	solve for a
8. $a = \dfrac{(5.70\ \text{kg})(9.80\ \text{m/s}^2)}{4.20\ \text{kg} + 5.70\ \text{kg}}$	enter values
9. $a = 5.64\ \text{m/s}^2$	evaluate

The steps above determine the magnitude of the acceleration. Since the question asked for the acceleration of the block on the table, the full answer is 5.64 m/s2 to the right.

Variables

	x component	y component
weight	$-mg \sin \theta$	$-mg \cos \theta$
thrust	F_T	0 N
lift	0 N	F_L
air resistance	F_R	0 N
weight	$mg = 2.60 \times 10^5$ N	
thrust	$F_T = 3.00 \times 10^5$ N	
flight angle	$\theta = 60.0°$	

What is the strategy?

1. Draw a free-body diagram of the forces on the plane, rotating the axes so three of the forces lie along an axis.

2. Calculate the net force on the plane along the axis of the lift force, and solve for the lift force.

Physics principles and equations

Newton's second law

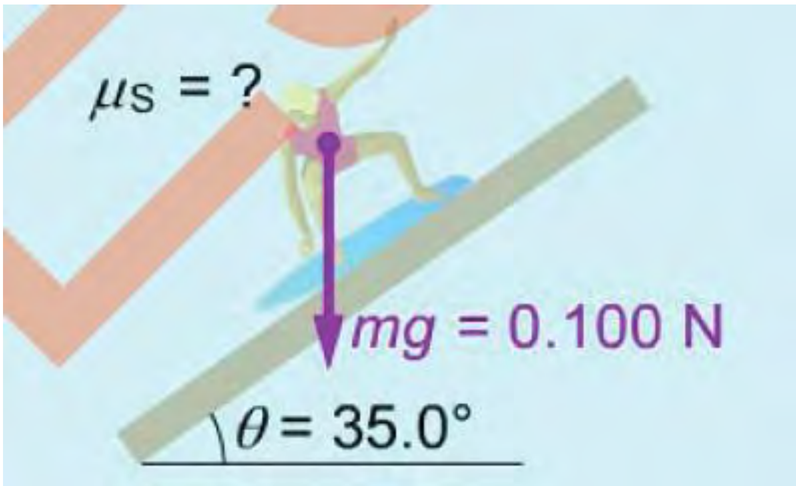
□**F** = *m***a**

When the acceleration is zero, the forces along every dimension must sum to zero.

Step-by-step solution

The lift force acts in the y dimension. In the y column of the variables table, all the values are known except the lift force. So, we need only apply equilibrium in the y dimension to solve this problem.

Step	Reason
1. $\Sigma F_y = 0$	no acceleration in y dimension
2. $F_L + (-mg \cos \theta) = 0$	lift and y component of weight
3. $F_L = (2.60 \times 10^5 \text{ N}) \cos 60.0°$	rearrange and enter values
4. $F_L = 1.30 \times 10^5$ N	evaluate



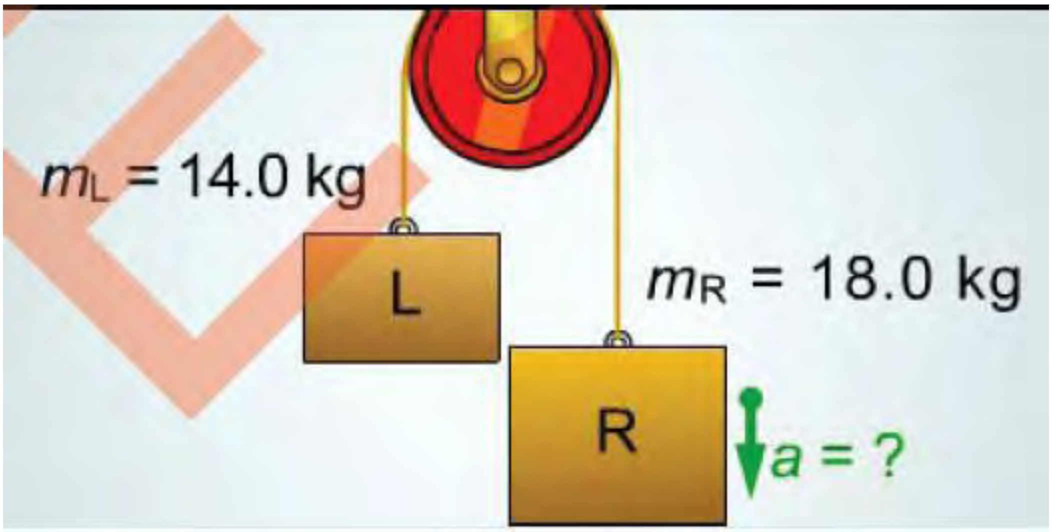
The Surfer Bob toy is just about to slide. What is the coefficient of static friction?

A classic physics lab exercise asks you to use a block on a plane to calculate the coefficient of static friction for two materials. You see that configuration shown above, although instead of a block, we are using Surfer Bob. You are given Bob's weight and the angle the plane makes with the horizontal just before Bob begins to slide. From this information, you are asked to calculate the coefficient of static friction. Since Bob is not accelerating, no net force is acting on him.

You may have performed a lab experiment like this at some point during your studies. You incline a plane until the force of gravity just overcomes friction and causes a block on the plane to slide. You then incline it a little less until the plane is at the angle at which the force of static friction balances the force of gravity down the plane. At this point, the static friction is at its maximum and you can calculate the coefficient. As you see below, we can solve this problem in fewer steps by rotating the axes. Two of the forces are acting along the inclined plane. By rotating the axes so the x axis is parallel to the plane, we can reduce the amount of trigonometry required. The rotation means that two of the forces act solely along an axis. If we did not do the rotation, each force we analyzed would have to be decomposed into its x and y components with the use of sines and cosines in order for us to solve the problem. If you do not like this axis rotation "trick," then you can always solve the problem by keeping the axes horizontal and vertical and using components. Now we substitute the expression for the normal force from step 6 into the equation of step 3 and find the coefficient of static friction.

Step	Reason
7. $\mu_s = (mg \sin \theta) / (mg \cos \theta)$	substitute step 6 into step 3
8. $\mu_s = \sin \theta / \cos \theta$	simplify
9. $\mu_s = \tan \theta$	trigonometric identity
10. $\mu_s = \tan 35.0°$	enter value
11. $\mu_s = 0.700$	evaluate

Even though we were given the weight of the dude, it turns out the coefficient of static friction does not depend on weight, but solely on the angle of the plane.



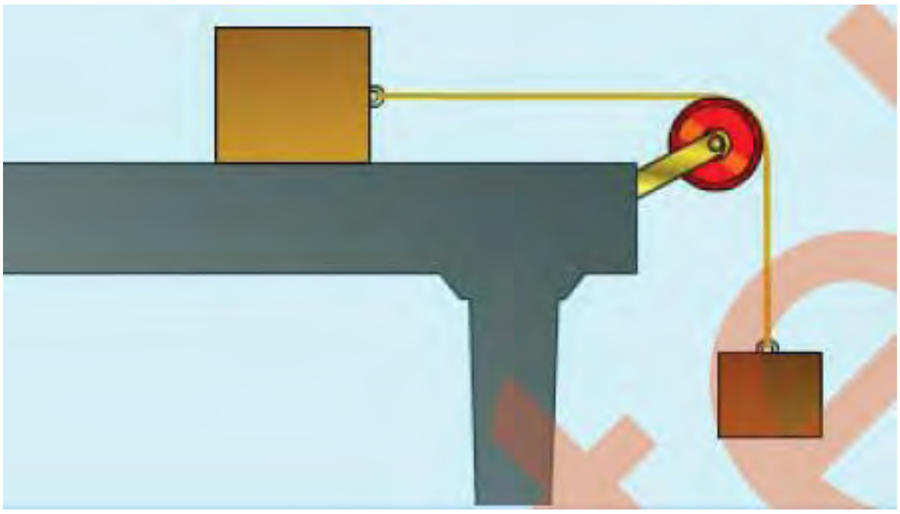
What is the magnitude of acceleration of the blocks?

The illustration above shows two blocks connected by a rope passing over a pulley. Because the blocks partially counterbalance each other, the force required to lift either of them is less than its weight. This type of system is called an *Atwood machine.* An application can be found in elevators, where a massive block partly counterbalances the weight of the elevator car to reduce the force required from the motor that lifts the car. In this sample problem, you are asked to find the rate at which the blocks accelerate. As usual, the rope and pulley are massless, the rope does not stretch, and the pulley has no friction. The rope exerts an equal tension force on both blocks. The blocks' accelerations are equal in magnitude but opposite in direction.

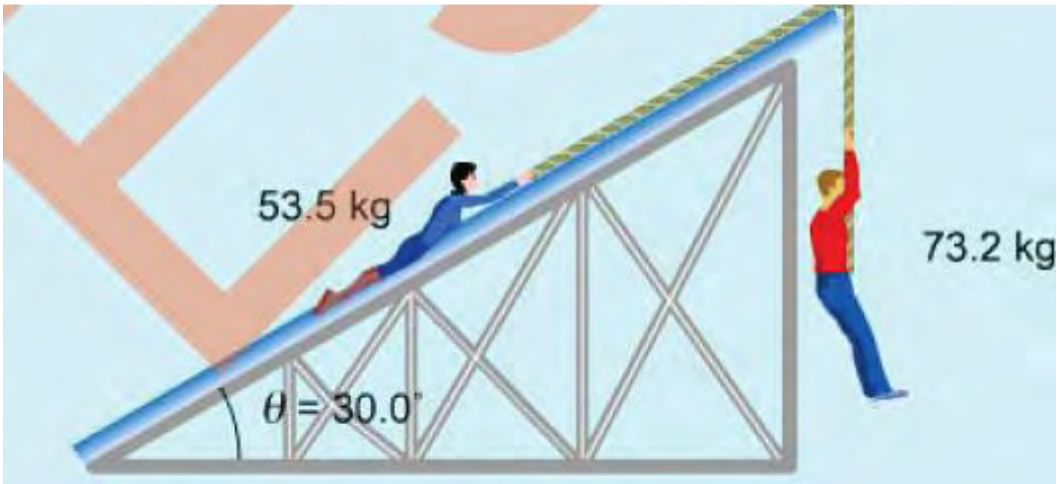
We set the two expressions for tension equal and solve for the acceleration.

Step	Reason
7. $m_L g + m_L a = m_R g - m_R a$	set tensions equal, from steps 3 and 6
8. $a = \frac{(m_R - m_L)(g)}{m_R + m_L}$	solve for a
9. $a = \frac{(18.0 - 14.0\ \text{kg})(9.80\ \text{m/s}^2)}{18.0\ \text{kg} + 14.0\ \text{kg}}$	enter values
10. $a = 1.23\ \text{m/s}^2$	arithmetic

Equation 8 can be profitably analyzed by considering a couple of special cases. If mL equals zero, equation 8 states that the acceleration equals g. This makes sense: The block on the right would be in free fall, since no force would be opposing the force of its weight. Also, if mR = mL, the acceleration would be zero, since there would be no net force. If we let mR go to infinity, equation 8 states that the acceleration equals g. This too makes sense. If mR is very much bigger than mL, then mL will hardly slow down mR as it falls in free fall.

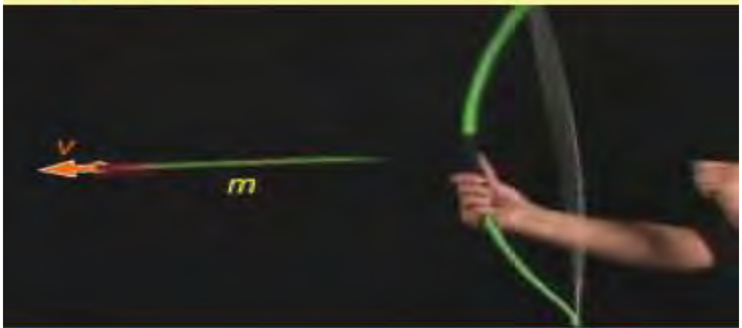


A 15.8 kg block sits on a frictionless horizontal table. The block is attached to a horizontal string that goes over a pulley and is connected to another block that hangs freely. The string is massless and does not stretch. The acceleration of the block on the table is 3.89 m/s^2. What is the mass of the hanging block?



Frances, a 53.5 kg woman, slides on a frictionless, icy ramp that is incline at 30.0° to the horizontal. An unstretchable rope connects her to Andre, who has a mass of 73.2 kg and is accelerating at the same rate, but parallel to the vertical wall of the ramp. What is the magnitude of their acceleration?

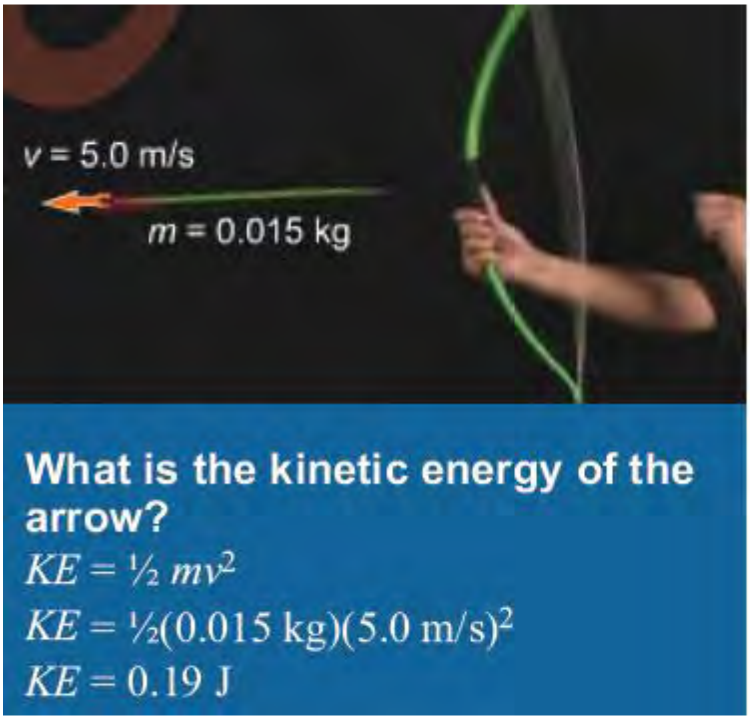
equation 1



Kinetic energy

$$KE = \frac{1}{2} mv^2$$

KE = kinetic energy
m = mass
v = speed
Unit: joule (J)



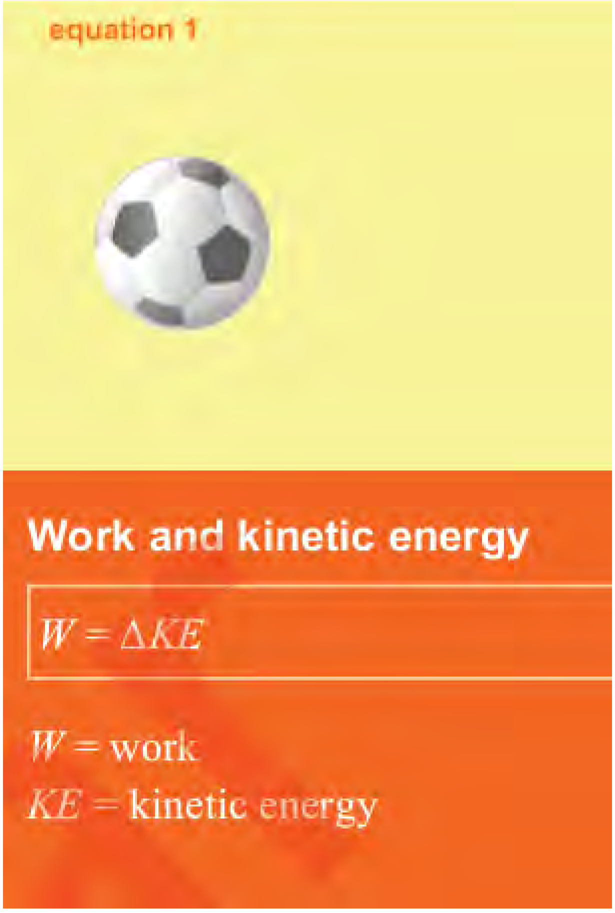
Work-kinetic energy theorem: The net work done on a particle equals its change in kinetic energy.

Consider the foot kicking the soccer ball in Concept 1. We want to relate the work done by the force exerted by the foot on the ball to the ball's change in kinetic energy. To focus solely on the work done by the foot, we ignore other forces acting on the ball, such as friction.

Initially, the ball is stationary. It has zero kinetic energy because it has zero speed. The foot applies a force to the ball as it moves through a short displacement. This force accelerates the ball. The ball now has a speed greater than zero, which means it has kinetic energy. The work-kinetic energy theorem states that the work done by the foot on the ball equals the change in the ball's kinetic energy. In this example, the work is positive (the force is in the direction of the displacement) so the work increases the kinetic energy of the ball. As shown in Concept 2, a goalie catches a ball kicked directly at her. The goalie's hands apply a force to the ball, slowing it. The force on the ball is opposite the ball's displacement, which means the work is negative. The negative work done on the ball slows and then stops it, reducing its kinetic energy to zero. Again, the work equals the change in energy; in this case, negative work on the ball decreases its energy. In the scenarios described here, the ball is the object to which a force is applied. But you can also think of the soccer ball doing work. The ball applies a force on the goalie, causing the goalie's hands to move backward. The ball does positive work on the goalie because the force it applies is in the direction of the displacement of the goalie's hands.

Derivation: work-kinetic energy theorem

In this section, we show that the net work done on an object and its change in kinetic energy are equal by using the definition of work and Newton's second law. We will again use the illustration of a soccer ball being kicked and model the ball as a particle. The ball starts at rest and we assume the force applied by the foot equals the net force on the ball, and that the ball moves without rotating.



Variables

Variables

work	W
force	F
displacement of object	Δx
mass of object	m
acceleration of object	a
initial speed of object	v_i
final speed of object	v
kinetic energy	KE

Strategy

1. Start with the definition of work.

2. Use Newton's second law to replace the net force in the definition of work by mass times the acceleration.

3. Use a motion equation from the study of kinematics to replace acceleration times displacement with one-half the speed squared.

Physics principles and equations

We will use the definition of work for when the force is in the direction of displacement.

$W = F\Delta x$

Newton's second law

$F = ma$

Linear motion equation

$v2 = vi$

$2 + 2a\Delta x$

The definition of kinetic energy

$KE = ½mv2$

Step-by-step derivation

State the definition of work and use Newton's second law to substitute *ma* for force.

Step	Reason
1. $W = F\Delta x$	definition of work
2. $W = ma\Delta x$	Newton's second law

We need to replace the acceleration and displacement terms with speed squared to end up with the definition of kinetic energy. We use a motion equation to make this substitution.

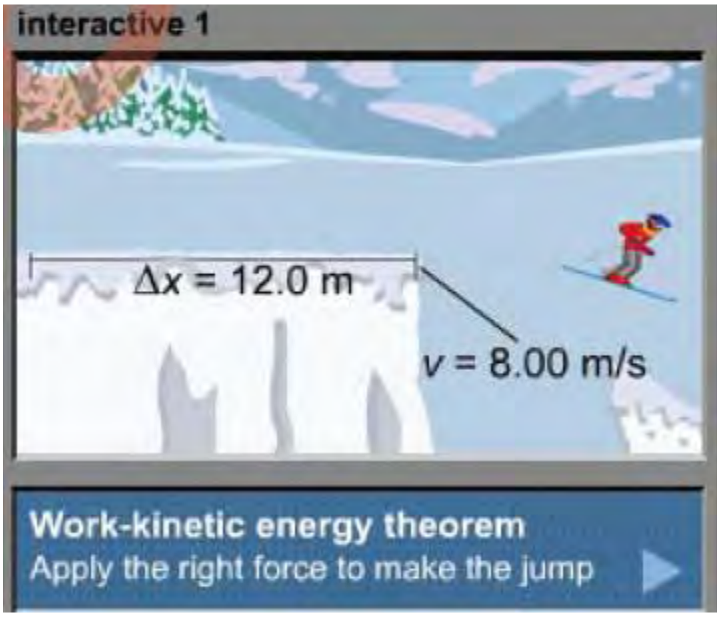
Step	Reason
3. $v^2 = v_i^2 + 2a\Delta x$	motion equation
4. $a\Delta x = ½\, v^2$	set $v_i = 0$ and rearrange
5. $W = ½\, mv^2$	substitute equation 4 into equation 2
6. $W = KE$	definition of kinetic energy
7. $W = \Delta KE$	no initial kinetic energy

Use the work-kinetic energy theorem to find the work done on the sled. Then, use the definition of work to determine how much force was exerted on the sled.

Step	Reason
6. $W = \Delta KE$	work-kinetic energy theorem
7. $W = (F \cos \theta)\Delta x$	definition of work
8. $(F \cos \theta)\Delta x = \Delta KE$	set two work equations equal
9. $F \cos \theta = \Delta KE/\Delta x$	rearrange
10. $F = \Delta KE/\Delta x$	force in direction of displacement
11. $F = 11{,}800\text{ J}/50.0\text{ m}$	enter values
12. $F = 236\text{ N}$	solve

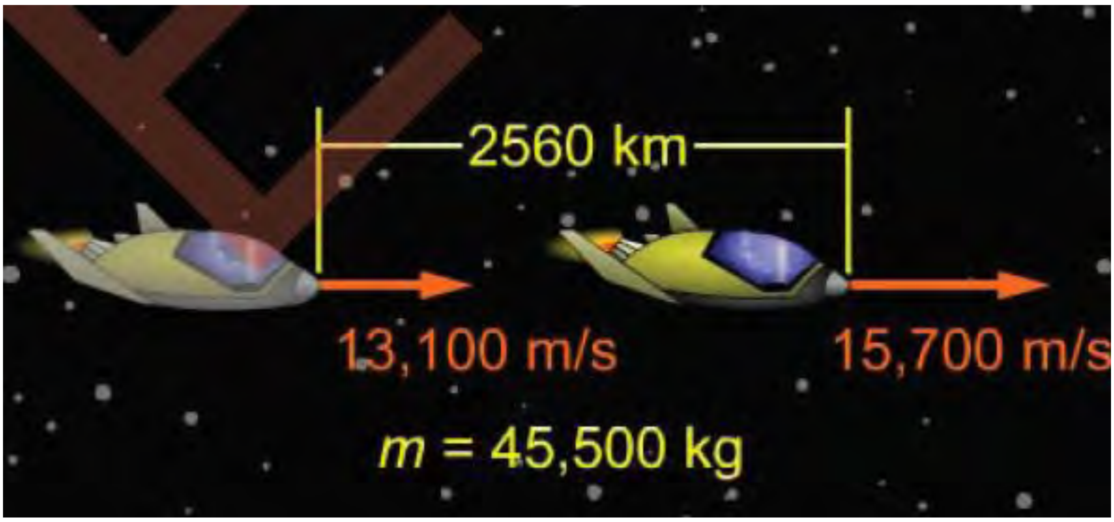
Interactive problem: work-kinetic energy theorem

In this simulation, you are a skier and your challenge is to do the correct amount of work to build up enough energy to soar over the canyon and land near the lip of the slope on the right. You, a 50.0 kg skier, have a flat 12.0 meter long runway leading up to the lip of the canyon. In that stretch, you must apply a force such that at the end of the straightaway, you are traveling with a speed of 8.00 m/s. Any slower, and your jump will fall short. Any faster, and you will overshoot. How much force must you apply, in newtons, over the 12.0 meter flat stretch? Ignore other forces like friction and air resistance



Enter the force, to the nearest newton, in the entry box and press GO to check your result.

If you have trouble with this problem, review the section on the work-kinetic energy theorem. (If you want to, you can check your answer using a linear motion equation and Newton's second law.)



A 45,500 kg spaceship is far from any significant source of gravity. It accelerates at a constant rate from 13,100 m/s to 15,700 m/s over a distance of 2560 km. What is the magnitude of the force on the ship due to the action of its engines? Use equations involving work and energy to solve the problem, and assume that the mass is constant.

example 1

Applying a force of 2.0×10⁵ N, the tugboat moves the log boom 1.0 kilometer in 15 minutes. What is the tugboat's average power?

$W = F\Delta x$

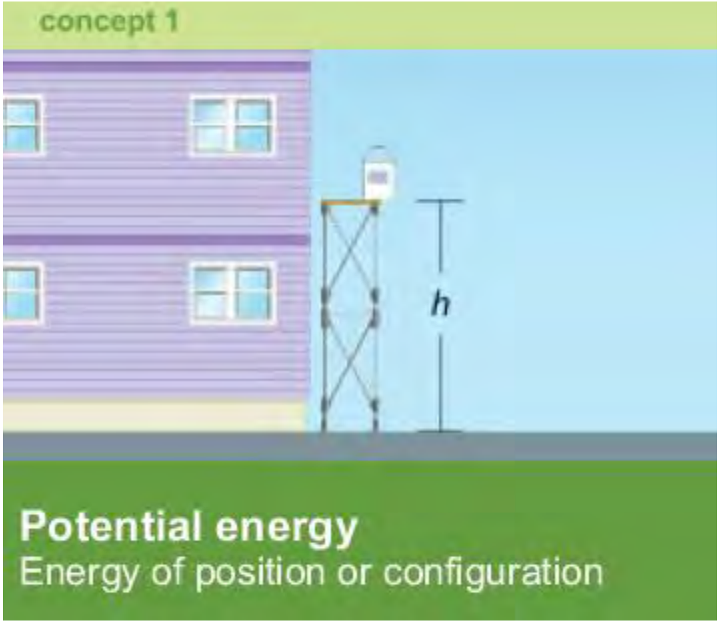
$W = F\Delta x = (2.0\times10^5\ \text{N})(1.0\times10^3\ \text{m})$

$W = 2.0\times10^8\ \text{J}$

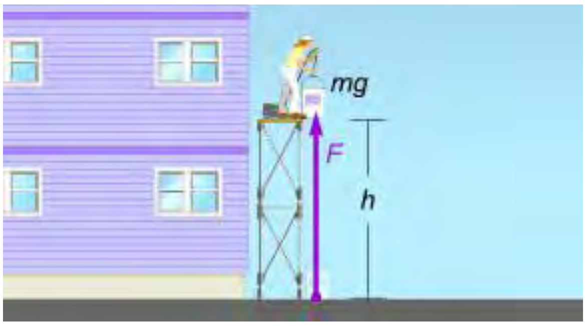
$\bar{P} = \frac{W}{\Delta t} = \frac{2.0\times10^8\ \text{J}}{9.0\times10^2\ \text{s}}$

$\bar{P} = 2.2\times10^5\ \text{W}$

Potential energy: Energy related to the positions of and forces between the objects that make up a system.



Although the paint bucket in Concept 1 is not moving, it makes up part of a system that has a form of energy called potential energy. In general, potential energy is the energy due to the configuration of objects that exert forces on one other. In this section, we focus on one form of potential energy, *gravitational potential energy*. The paint bucket and Earth make up a system that has this form of potential energy. A *system* is some "chunk" of the universe that you wish to study, such as the bucket and the Earth. You can imagine a boundary like a bubble surrounding the system, separating it from the rest of the universe. The particles within a system can interact with one another via internal forces or fields. Particles outside the system can interact with the system via external forces or fields. Gravitational potential energy is due to the gravitational force between the bucket and Earth. As the bucket is raised or lowered, its **change** in potential energy (ΔPE) equals the magnitude of its weight, mg, times its vertical displacement, Δh. (We follow the common convention of using Δh for change in height, instead of Δy.) The weight is the amount of force exerted on the bucket by the Earth (and vice versa). This formula is shown in Equation 1. A change in *PE* can be positive or negative. The magnitude of weight is a positive value, but change in height can be positive (when the bucket moves up) or negative (when it moves down). To define a system's *PE*, we must define a configuration at which the system has zero *PE*. Unlike kinetic energy, where zero *KE* has a natural value (when an object's speed is zero), the configuration with zero *PE* is defined by you, the physicist. In the diagrams to the right, it is convenient to say the system has zero *PE* when the bucket is on the Earth's surface. This convention means its *PE* equals its weight times its height above the ground, *mgh*. Only the bucket's distance above the Earth, *h*, matters here; if the bucket moves left or right, its *PE* does not change. In Example 1, we calculate the paint bucket's gravitational potential energy as it sits on the scaffolding, four meters above the ground. There are other types of potential energy. One you will frequently encounter is *elastic potential energy*, which is the energy stored in a compressed or stretched object such as a spring. As you may recall, this form of energy was present in the bow that was used to fire an arrow.



Change in gravitational potential energy

$$\Delta PE = mg\Delta h$$

PE = potential energy
mg = object's weight
Δh = vertical displacement

energy, as seen in Equation 2. Imagine that the painter drops the bucket from the scaffolding. Only the force of gravity does work on the bucket as it falls. The system has more potential energy when the bucket is at the top of the scaffolding than when it is at the bottom, so the work done by gravity has lowered the system's *PE*: the change in *PE* due to the work done by gravity is negative.

$$W = \Delta PE$$

W = work done against gravity
PE = potential energy of system

Work done by gravity

$$W = -\Delta PE$$

W = work done by gravity
PE = potential energy of system

Sample problem: potential energy and Niagara Falls