الجامعة التكنولوجية

قسم الهندسة الكيمياوية

المرحلة الاولى

مباديء الهندسة الكيمياوية

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Chapter 4

Temperature

- **Temperature** is a measure of the energy (mostly kinetic) of the molecules in a system. This definition tells us about the amount of energy.
- Other scientists prefer to say that **Temperature** is a property of the state of thermal

equilibrium of the system with respect to other systems because temperature tells us about the capability of a system to transfer energy (as heat).

☒ Four types of temperature:

Two based on a **relative scale**, **degrees Fahrenheit (°F)** and **Celsius (°C)**, and two based on an **absolute scale**, **degree Rankine (°R)** and **Kelvin (K)**.

☒ Temperature Conversion

$$\Delta°F = \Delta°R$$

$$\Delta°C = \Delta K$$

Also, the Δ°C is larger than the Δ°F

$$\frac{\Delta°C}{\Delta°F} = 1.8 \qquad \text{or} \qquad \Delta°C = 1.8\ \Delta°F$$

$$\frac{\Delta K}{\Delta°R} = 1.8 \qquad \text{or} \qquad \Delta K = 1.8\ \Delta°R$$

Also, because of the temperature difference between boiling water and ice (Celsius: 100°C – 0°C =100°C; Fahrenheit: 212°F – 32°F = 180°F), the following relationships hold:

$\Delta°C = 1.8000\ \Delta°F$ and $\Delta K = 1.8000\ \Delta°F$

The proper meaning of the symbols **°C, °F, K,** and **°R,** as either the temperature or the unit temperature difference, must be interpreted from the context of the equation or sentence being examined.

Suppose you have the relation:

$$T_{°F} = a + bT_{°C}$$

What are the units of **a** and **b**? The units of **a** must be **°F** for consistency. The correct units for **b** must involve the conversion factor **(1.8 Δ°F\Δ°C)**, the factor that converts the size of an interval on one temperature scale

$$T_{°F} = a_{°F} + \left(\underbrace{\frac{1.8\ \Delta°F}{\Delta°C}}_{b}\right) T_{°C}$$

Unfortunately, the units for b are usually ignored; just the value of b (1.8) is employed.

✶ The relations between **°C, °F, K,** and **°R** are:

$$T_{°R} = T_{°F}\left(\frac{1\ \Delta°R}{1\ \Delta°F}\right) + 460°R$$

Or $\boxed{\mathbf{T_{°R} = T_{°F} + 460}}$

$$T_K = T_{°C}\left(\frac{1\ \Delta K}{1\ \Delta°C}\right) + 273\ K$$

$$T_{°F} - 32°F = T_{°C}\left(\frac{1.8\ \Delta°F}{1\ \Delta°C}\right)$$

Or $\boxed{\mathbf{T_K = T_{°C} + 273}}$

$$T_{°C} = (T_{°F} - 32°F)\left(\frac{1\ \Delta°C}{1.8\ \Delta°F}\right)$$

Or $\boxed{\mathbf{T_{°F} = 1.8\ T_{°C} + 32}}$

Example 4.1

Convert 100 °C to (a) K, (b) °F, and (c) °R.

Solution

$$\text{(a)}\quad (100 + 273)°C\,\frac{1\ \Delta K}{1\ \Delta°C} = 373\ K$$

or with suppression of the Δ symbol,

$$(100 + 273)°C\,\frac{1\ K}{1°C} = 373\ K$$

$$\text{(b)}\quad (100°C)\frac{1.8\ \Delta°F}{1\ \Delta°C} + 32°F = 212°F$$

$$\text{(c)}\quad (212 + 460)°F\,\frac{1\ \Delta°R}{1\ \Delta°F} = 672°R$$

or

$$(373\ \text{K})\frac{1.8\ \Delta°\text{R}}{1\ \Delta\text{K}} = 672°\text{R}$$

Example 4.2

The heat capacity of sulfuric acid has the units J/(g mol)(°C), and is given by the relation

$$\text{Heat capacity} = 139.1 + 1.56 * 10^{-1}\ T$$

where T is expressed in °C. Modify the formula so that the resulting expression has the associated units of Btu/(lb mol) (°R) and T is in °R.

Solution

$T_{°F} = 1.8\ T_{°C} + 32 \Longrightarrow T_{°C} = (T_{°F} - 32)/1.8$

$T_{°R} = T_{°F} + 460 \Longrightarrow T_{°F} = T_{°R} - 460$

$\therefore\ T_{°C} = [T_{°R} - 460 - 32]/1.8$

$$\text{heat capacity} = \left\{139.1 + 1.56 \times 10^{-1}\overbrace{\left[(T_{°R} - 460 - 32)\frac{1}{1.8}\right]}^{T_{°C}}\right\} \times \frac{1\ \text{J}}{(\text{g mol})(°\text{C})}\underbrace{\left|\frac{1\ \text{Btu}}{1055\ \text{J}}\right|\frac{454\ \text{g mol}}{1\ \text{lb mol}}\left|\frac{1°\text{C}}{1.8°\text{R}}\right.}_{\text{conversion factors}} =$$

$$= 23.06 + 2.07 \times 10^{-2} T_{°R}$$

Note the suppression of the Δ symbol in the conversion between °C and °R.

Problems

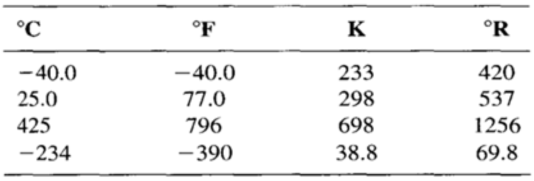
1. Complete the following table with the proper equivalent temperatures:

°C	°F	K	°R
–40	____	____	____
____	77.0	____	____
____	____	698	____
____	____	____	69.8

2. The heat capacity of sulfur is $C_p = 15.2 + 2.68T$, where C_p is in J/(g mol)(K) and T is in K. Convert this expression so that C_p is in cal/(g mol)(°F) with T in °F.

Answers:

1.

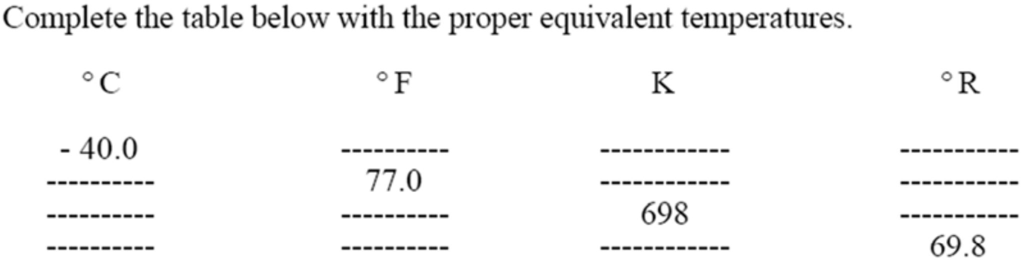


°C	°F	K	°R
−40.0	−40.0	233	420
25.0	77.0	298	537
425	796	698	1256
−234	−390	38.8	69.8

2. $C_p = 93.2 + 0.186\, T_{°F}$

Supplementary Problems (Chapter Four):

Problem 1



Complete the table below with the proper equivalent temperatures.

°C	°F	K	°R
- 40.0	----------	------------	-----------
----------	77.0	------------	-----------
----------	----------	698	-----------
----------	----------	------------	69.8

Solution

The conversion relations to use are:

$$\begin{array}{lclcl} ^\circ F & = & 1.8\ ^\circ C & + & 32 \\ K & = & ^\circ C & + & 273 \\ ^\circ R & = & ^\circ F & + & 460 \\ ^\circ R & = & 1.8\ K & & \end{array}$$

°C	°F	K	°R
- 40.0	**- 40.0**	**233**	**420**
25.0	77.0	**298**	**437**
425	**797**	698	**1257**
- 235	**-390**	**38.4**	69.8

Problem 2

The specific heat capacity of toluene is given by following equation

$$C_p = 20.869 + 5.293 \times 10^{-2}\, T$$

where C_p is in Btu/(LB mol) (° F) and T is in ° F

Express the equation in cal/(g mol) (K) with T in K.

Solution

First, conversion of the units for the overall equation is required.

$$C_p = \frac{[20.869 + 5.293 \times 10^{-2}\,(T_{^\circ F})] \quad \text{Btu}}{1\ (\text{lb mol})\ (^\circ F)} \left| \frac{252\ \text{cal}}{1\ \text{Btu}} \right| \frac{1\ \text{lb mol}}{454\ \text{g mol}} \left| \frac{1.8\ ^\circ F}{1\ K} \right.$$

$$= [\,20.869 + 5.293 \times 10^{-2}\,(T_{^\circ F})]\,\frac{\text{cal}}{(\text{g mol})\ (K)}$$

Note that the coefficients of the equation remain unchanged in the new units for this particular conversion. The T of the equation is still in °F, and must be converted to kelvin.

$$T_{^\circ F} = (T_K - 273)\ 1.8 + 32$$

$$C_p = 20.69 + 5.293 \times 10^{-2}\ [(T_K - 273)\ 1.8 + 32]$$

Simplifying $C_p = -3.447 + 9.527 \times 10^{-2}\ T_K$

Chapter 5

Pressure

5.1 Pressure and Its Units

Pressure is defined as "the normal (perpendicular) **force** per unit **area** (Figure 5.1). The pressure at the bottom of the static (nonmoving) column of mercury exerted on the sealing plate is

$$p = \frac{F}{A} = \rho g h + p_0 \qquad \ldots 5.1$$

Where p = pressure at the bottom of the column of the fluid, F = force, A = area, ρ = density of fluid g = acceleration of gravity, h = height of the fluid column, and p_0 = pressure at the top of the column of fluid

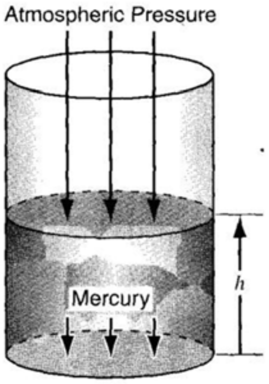


Figure 5.1 Pressure is the normal force per unit area. Arrows show the force exerted on the respective areas

For Example, suppose that the cylinder of fluid in Figure 5.1 is a column of mercury that has an area of 1 cm^2 and is 50 cm high. The density of the Hg is 13.55 g/cm^3. Thus, the force exerted by the mercury alone on the 1 cm^2 section of the bottom plate by the column of mercury is

$$F = \frac{13.55\ \text{g}}{\text{cm}^3}\left|\frac{980\ \text{cm}}{\text{s}^2}\right|\frac{50\ \text{cm}}{}\left|\frac{1\ \text{cm}^2}{}\right|\frac{1\ \text{kg}}{1000\ \text{g}}\left|\frac{1\ \text{m}}{100\ \text{cm}}\right|\frac{1(\text{N})(\text{s}^2)}{1(\text{kg})(\text{m})}$$

$$= 6.64\ \text{N}$$

P=F/A …. F= P* A = ρ g h*A

The pressure on the section of the plate covered by the mercury is the force per unit area of the mercury plus the pressure of the atmosphere

$$p = \frac{6.64\ \text{N}}{1\ \text{cm}^2}\left|\left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^2\right|\frac{(1\ \text{m}^2)(1\ \text{Pa})}{(1\ \text{N})}\left|\frac{1\ \text{kPa}}{1000\ \text{Pa.}}\right. + p_0 = 66.4\ \text{kPa} + p_0$$

If we had started with units in the AE system, the pressure would be computed as [the density of mercury is 845.5 lb_m/ft^3]

$$p = \frac{845.5\ \text{lb}_\text{m}}{1\ \text{ft}^3}\bigg|\frac{32.2\ \text{ft}}{\text{s}^2}\bigg|\frac{50\ \text{cm}}{}\bigg|\frac{1\ \text{in.}}{2.54\ \text{cm}}\bigg|\frac{1\text{ft}}{12\ \text{in.}}\bigg|\frac{(\text{s})^2(\text{lb}_\text{f})}{32.174(\text{ft})(\text{lb}_\text{m})} + p_0$$

$$= 1388\frac{\text{lb}_\text{f}}{\text{ft}^2} + p_0$$

5.2 Measurement of Pressure

Pressure, like temperature, can be expressed using either an **absolute** or a **relative scale**.

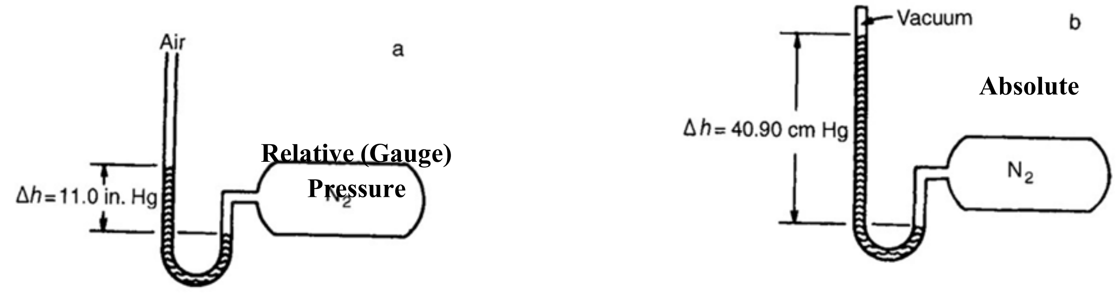


Figure 5.2 (a) **Open-end manometer** showing a pressure above atmospheric pressure. (b) **Manometer** measuring an **absolute pressure**.

The relationship between **relative** and **absolute pressure** is given by the following expression:

Gauge Pressure + Barometer Pressure (atmospheric)= Absolute Pressure …5.2

$$P_{\text{vacuum}} = p_{\text{atmospheric}} - p_{\text{absolute}}$$

☒ The **standard atmosphere** is defined as the pressure (in a standard gravitational field) equivalent to 1 atm or 760 mm Hg at 0°C or other equivalent.

The **standard atmosphere** is equal to

- 1.00 atmospheres (atm)
- 33.91 feet of water (ft H_2O)
- 14.7 pounds (force) per square inch absolute (psia)

- 29.92 inches of mercury (in. Hg)
- 760.0 millimeters of mercury (mm Hg)
- $1.013 * 10^5$ pascal (Pa) or newtons per square meter (N/m^2); or 101.3 kPa

For Example, convert 35 psia to inches of mercury and kPa.

$$\frac{35\ \text{psia}}{}\left|\frac{29.92\ \text{in. Hg}}{14.7\ \text{psia}}\right. = 71.24\ \text{in Hg}$$

And,

$$\frac{35\ \text{psia}}{}\left|\frac{101.3\ \text{kPa}}{14.7\ \text{psia}}\right. = 241\ \text{kPa}$$

For Example, What is the equivalent **pressure** to **1 kg/cm^2** (i.e., **kg$_f$/cm^2**) in pascal (g = 9.8 m/s^2)
[1 kg/cm^2] * [9.8 m/s^2] * [(100 cm/1 m)^2] = 9.8 * 10^4 N/m^2 (or Pa)

Example 5.1

What is the equivalent pressure to 60 Gpa (gigapascal) in

(a) atmospheres (b) psia (c) inches of Hg (d) mm of Hg

Solution

Basis: 60 GPa

$$\text{(a)}\quad \frac{60\ \text{GPa}}{}\left|\frac{10^6\ \text{kPa}}{1\ \text{GPa}}\right|\frac{1\ \text{atm}}{101.3\ \text{kPa}} = 0.59\times 10^6\ \text{atm}$$

$$\text{(b)}\quad \frac{60\ \text{GPa}}{}\left|\frac{10^6\ \text{kPa}}{1\ \text{GPa}}\right|\frac{14.696\ \text{psia}}{101.3\ \text{kPa}} = 8.70\times 10^6\ \text{psia}$$

$$\text{(c)}\quad \frac{60\ \text{GPa}}{}\left|\frac{10^6\ \text{kPa}}{1\ \text{GPa}}\right|\frac{29.92\ \text{in. Hg}}{101.3\ \text{kPa}} = 1.77\times 10^7\ \text{in. Hg}$$

$$\text{(d)}\quad \frac{60\ \text{GPa}}{}\left|\frac{10^6\ \text{kPa}}{1\ \text{GPa}}\right|\frac{760\ \text{mm Hg}}{101.3\ \text{kPa}} = 4.50\times 10^8\ \text{mm Hg}$$

Example 5.2

The pressure gauge on a tank of CO_2 used to fill soda-water bottles reads 51.0 psi. At the same time the barometer reads 28.0 in. Hg. What is the absolute pressure in the tank in psia? See Figure E5.2.

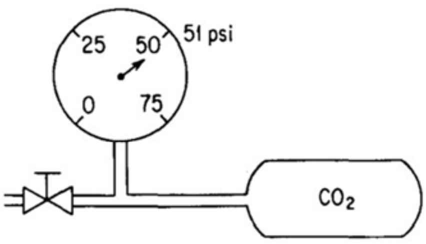


Figure E5.2

Solution

$$\text{Atmospheric pressure} = \frac{28.0\ \text{in. Hg}}{}\left|\frac{14.7\ \text{psia}}{29.92\ \text{in Hg}}\right. = 13.76\ \text{psia}$$

The absolute pressure in the tank is

51.0 psia + 13.76 psia = 64.8 psia

Example 5.3

Small animals such as mice can live (although not comfortably) at reduced air pressures down to 20 kPa absolute. In a test, a mercury manometer attached to a tank, as shown in Figure E5.3, reads 64.5 cm Hg and the barometer reads 100 kPa. Will the mice survive?

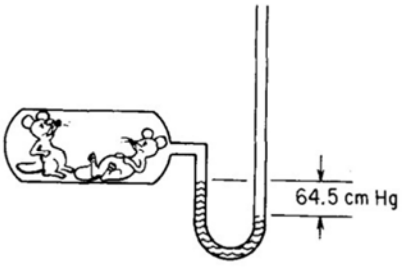


Figure E5.3

Solution

You are expected to realize from the figure that the tank is **below atmospheric pressure** because the **left leg** of the **manometer is higher** than the **right leg**, which is open to the atmosphere. Consequently, to get the **absolute pressure** you **subtract** the **64.5 cm Hg** from the **barometer reading**.

The **absolute pressure** in the tank is

$$100\ \text{kPa} - \frac{64.5\ \text{cm Hg}}{}\left|\frac{101.3\ \text{kPa}}{76.0\ \text{cm Hg}}\right. = 100 - 86 = 14\ \text{kPa absolute}$$

The mice probably will **not survive**.

5.3 Differential Pressure Measurements

When the columns of fluids are at equilibrium (see Figure 5.3), the relationship among ρ_1, ρ_2, ρ_3, and the heights of the various columns of fluid is as follows:

$$\mathbf{P_1 + \rho_1 d_1 g = P_2 + \rho_2 d_2 g + \rho_3 d_3 g} \qquad \ldots 5.3$$

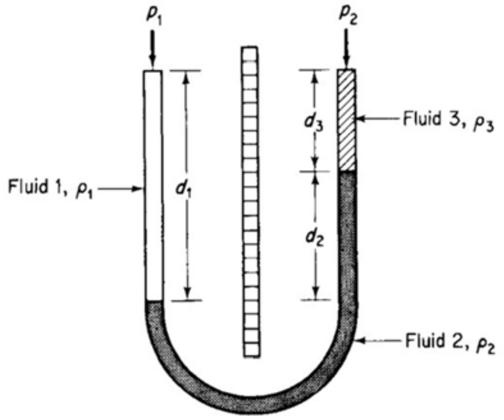


Figure 5.3 Manometer with three fluids.

Note

If fluids 1 and 3 are **gases**, and fluid 2 is **mercury**, the density of the **gas** is **so much less** than that of **mercury** that you can **ignore** the term involving the gas in Equation (5.3) for practical applications.

- ✶ Can you show for the case in which $\rho_1 = \rho_3 = \rho$ that the manometer expression reduces to the differential manometer equation:

$$\mathbf{P_1 - P_2 = (\rho_2 - \rho)\, g\, d_2} \qquad \dots 5.4$$

Example 5.4

In measuring the flow of fluid in a pipeline as shown in Figure E5.4, a differential manometer was used to determine the pressure difference across the orifice plate. The flow rate was to be calibrated with the observed pressure drop (difference). Calculate the **pressure drop $p_1 - p_2$** in pascals for the manometer reading in Figure E5.4.

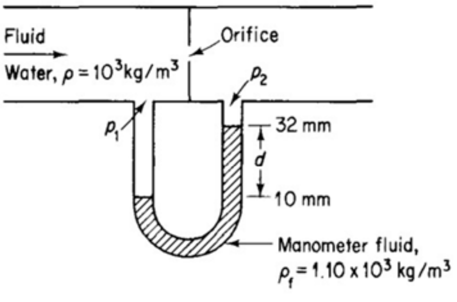


Figure E5.4

Solution

In this problem you **cannot ignore the water density** above the manometer fluid.

$$p_1 - p_2 = (\rho_f - \rho)gd$$

$$= \frac{(1.10 - 1.00)10^3\,\text{kg}}{\text{m}^3}\left|\frac{9.807\,\text{m}}{\text{s}^2}\right|\frac{(22)(10^{-3})\text{m}}{}\left|\frac{1(\text{N})(\text{s}^2)}{(\text{kg})(\text{m})}\right|\frac{1(\text{Pa})(\text{m}^2)}{1(\text{N})}$$

$$= 21.6\ \text{Pa}$$

Example 5.5

Air is flowing through a duct **under** a draft of 4.0 cm H_2O. The barometer indicates that the atmospheric pressure is 730 mm Hg. What is the absolute pressure of the air in inches of mercury? See Figure E5.5

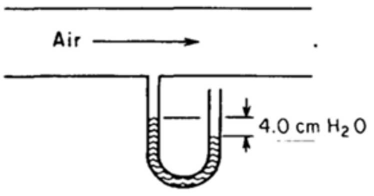


Figure E5.5

Solution

In this problem you **can ignore the gas density** above the manometer fluid and the air above the open end of the manometer.

$$\text{Atmospheric pressure} = \frac{730\ \text{mm Hg}}{}\bigg|\frac{29.92\ \text{in. Hg}}{760\ \text{mm Hg}} = 28.7\ \text{in. Hg}$$

Next, convert 4.0 cm H_2O to in. Hg:

$$\frac{4.0\ \text{cm}\ H_2O}{}\bigg|\frac{1\ \text{in.}}{2.54\ \text{cm}}\bigg|\frac{1\ \text{ft}}{12\ \text{in.}}\bigg|\frac{29.92\ \text{in. Hg}}{33.91\ \text{ft}\ H_2O} = 0.12\ \text{in. Hg}$$

Since the reading is 4.0 cm H_2O draft (**under atmospheric**), the absolute reading in uniform units is

$$28.7\ \text{in. Hg} - 0.12\ \text{in. Hg} = 28.6\ \text{in. Hg absolute}$$

<u>Questions</u>

1. Figure SAT5.1Q2 shows four closed containers completely filled with water. Order the containers from the one exerting the highest pressure to the lowest on their respective base.

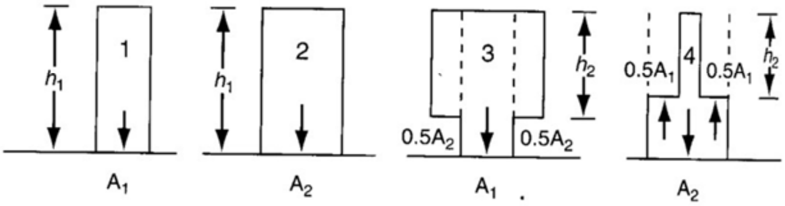


Figure SAT5.1Q2

2. Answer the following questions true or false:
    a. Atmospheric pressure is the pressure of the air surrounding us and changes from day to day
    b. The standard atmosphere is a constant reference atmosphere equal to 1.000 atm or the equivalent pressure in other units.
    c. Absolute pressure is measured relative to a vacuum.
    d. Gauge pressure is measured upward relative to atmospheric pressure.
    e. Vacuum and draft pressures are measured downward from atmospheric pressure.

f. You can convert from one type of pressure measurement to another using the standard atmosphere.
g. A manometer measures the pressure difference in terms of the height of fluid (s) in the manometer tube.

3. What is the equation to convert gauge pressure to absolute pressure?
4. What are the values and units of the standard atmosphere for six different methods of expressing pressure?
5. What is the equation to convert vacuum pressure to absolute pressure?

Answers:

1. 3 is the highest pressure; next are 1 and 2, which are the same; and 4 is last. The decisions are made by dividing the weight of water by the base area.
2. All are true
3. Gauge pressure + barometric pressure = absolute pressure
4. See lectures
5. Barometric pressure - vacuum pressure = absolute pressure

Problems

1. Convert a pressure of 800 mm Hg to the following units:
   a. psia b. kPa c. atm d. ft H_2O
2. Your textbook lists five types of pressures: atmospheric pressure, barometric pressure, gauge pressure, absolute pressure, and vacuum pressure.
   a. What kind of pressure is measured by the device in Figure SAT5.2P2A?

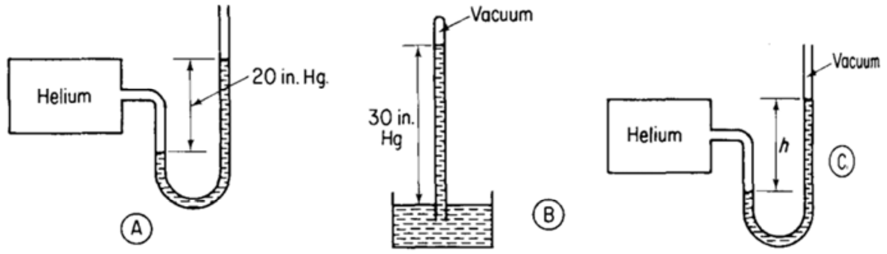


Figure SAT5.2P2A

   b. What kind of pressure is measured by the device in Figure SAT5.2P2B?
   c. What would be the reading in Figure SAT5.2P2C assuming that the pressure and temperature inside and outside the helium tank are the same as in parts (a) and (b)?
3. An evaporator shows a reading of 40 kPa vacuum. What is the absolute pressure in the evaporator in kPa?
4. A U-tube manometer filled with mercury is connected between two points in a pipeline. If the manometer reading is 26 mm of Hg, calculate the pressure difference in kPa between the

points when (a) water is flowing through the pipeline, and (b) also when air at atmospheric pressure and 20°C with a density of 1.20 kg/m^3 is flowing in the pipeline.

5. A Bourdon gauge and a mercury manometer are connected to a tank of gas, as shown in Figure SAT5.3P2. If the reading on the pressure gauge is 85 kPa, what is h in centimeters of Hg?

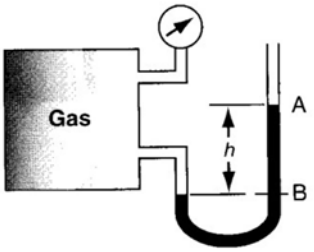


Figure SAT5.3P2

Answers:

1. (a) 15.5; (b) 106.6; (c) 1.052; (d) 35.6
2. (A) Gauge pressure; (B) barometric pressure, absolute pressure; (C) 50 in. Hg
3. In the absence of a barometric pressure value, assume 101.3 kPa. The absolute pressure is 61.3 kPa.
4. The Hg is static. (a) 3.21 kPa; (b) 3.47 kPa
5. 63.8 cm Hg

Supplementary Problems (Chapter Five):

Problem 1

A solvent storage tank, 15.0 m high contains liquid styrene (sp. gr. 0.909). A pressure gauge is fixed at the base of the tank to be used to determine the level of styrene.

a. Determine the gage pressure when the tank is full of styrene.
b. If the tank is to be used for storage of liquid hexane (sp. gr. 0.659), will the same pressure gage calibration be adequate ? What is the risk in using the same calibration to determine the level of hexane in the tank.
c. What will be the new pressure with hexane to indicate that the tank is full.

Solution

a. The liquid in full tank will exert a gage pressure at the bottom equal to 15.0 m of styrene. The tank has to operate with atmospheric pressure on it and in it, or it will break on expansion at high pressure or collapse at lower pressure.

$$p = h\,\rho\,g$$

$$= 15.0\ \text{m}\ \frac{0.909\ \text{g styrene/cm}^3}{1.0\ \text{g H}_2\text{O/cm}^3}\bigg|\ \frac{1.0\ \text{g H}_2\text{O/cm}^3}{}\ \bigg|\ \frac{10^3\ \text{kg/m}^3}{1\ \text{g/cm}^3}\bigg|\ \frac{9.80\ \text{m/s}^2}{}\ \bigg|\ \frac{1\ \text{Pa}}{1\ (\text{kg})(\text{m})^{-1}(\text{s})^{-2}}$$

$$= 134 \times 10^3\ \text{Pa} = \mathbf{134\ kPa\ gage}$$

b. Hexane is a liquid of specific gravity lower than that of styrene; therefore a tank full of hexane would exert a proportionally lower pressure. If the same calibration is used the tank may overflow while the pressure gage was indicating only a partially full tank.

c. New $p = h\,\rho\,g$

$$= 15.0\ \text{m}\ \begin{array}{c|c|c|c|c} 0.659\ \text{g hexane/cm}^3 & 1.0\ \text{g H}_2\text{O/cm}^3 & 10^3\ \text{kg/m}^3 & 9.8\ \text{m/s}^2 & 1\ \text{Pa} \\ \hline 1.0.0\ \text{g H}_2\text{O/cm}^3 & & & & 1(\text{kg})(\text{m})^{-1}(\text{s})^{-2} \end{array}$$

$$= \quad 96900\ \text{Pa} = \mathbf{96.9\ kPa}$$

Problem 2

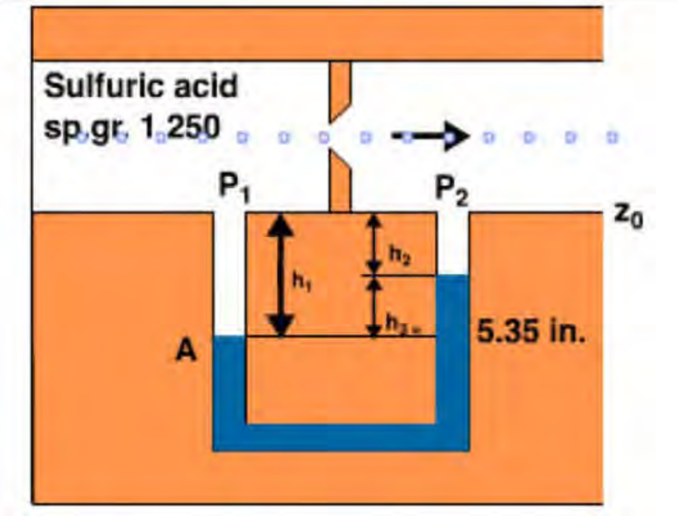
A U-tube manometer is used to determine the pressure drop across an orifice meter. The liquid flowing in the pipe line is a sulfuric acid solution having a specific gravity (60°/60°) of 1.250. The manometer liquid is mercury, with a specific gravity (60°/60°) of 13.56. The manometer reading is 5.35 inches, and all parts of the system are at a temperature of 60°F. What is the pressure drop across the orifice meter in psi.

Solution

First we calculate density of acid and mercury.

$$\rho_{acid} \quad = \quad \begin{array}{c|c|c} 1.250 & 62.4\ \text{lb/ft}^3 & 1\ \text{ft}^3 \\ \hline & & 1.728 \times 10^3\ \text{in}^3 \end{array} \quad = \quad 0.0451\ \text{lb/in}^3$$

$$\rho_{Hg} \quad = \quad \begin{array}{c|c|c} 13.56 & 62.4\ \text{lb/ft}^3 & 1\ \text{ft}^3 \\ \hline & & 1.728 \times 10^3\ \text{in}^3 \end{array} \quad = \quad 0.490\ \text{lb/in}^3$$


Sulfuric acid
sp.gr. 1.250
P1
P2
z0
h1
h2
h3 =
5.35 in.
A

left column *right column*

$$\text{At } z_o \quad p_1 + \rho_a h_1 g = p_2 + \rho_a h_2 g + \rho_{Hg} h_3 g$$

$$p_1 - p_2 + \rho_a (h_1 - h_2) g = \rho_{Hg} h_3 g$$

$$p_1 - p_2 + \rho_a h_3 g = \rho_{Hg} h_3 g$$

$$p_1 - p_2 = (\rho_{Hg} - \rho_a) h_3 g$$

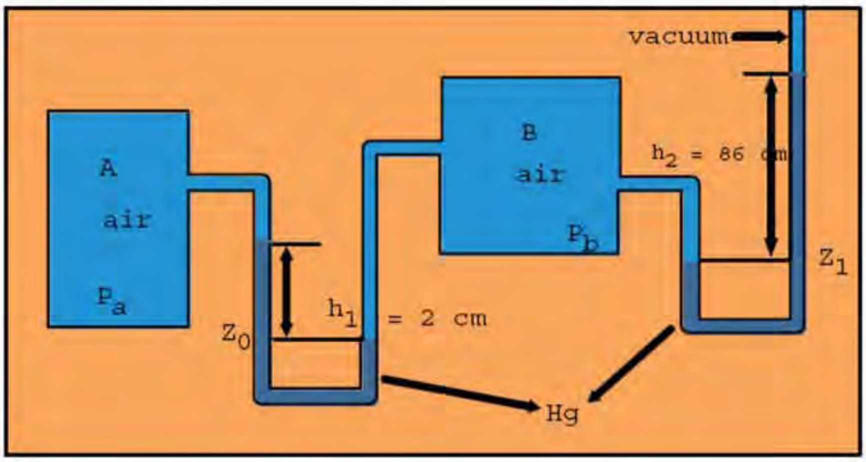
$$p_1 - p_2 = \frac{(0.490 - 0.0451)\ \text{lb}_f}{\text{in}^2} \left| \frac{(5.35)\ \text{in}}{} \right| \frac{32.2\ \text{ft/s}^2}{32.174\ (\text{ft})(\text{lb}_m)/(\text{s}^2)(\text{lb}_f)} = \mathbf{2.38\ lb_f/in^2\ (psi)}$$

Problem 3

The pressure difference between two air tanks A and B is measured by a U - tube manometer, with mercury as the manometer liquid. The barometric pressure is 700 mm Hg.

a. What is the absolute pressure in the tank A ?
b. What is the gauge pressure in the tank A ?

Solution



a. At Z_0 $\quad p_a + h_1 \rho_{Hg} g = p_b$ (neglecting the effect of air in the U - tube) (1)

at Z_1 $\quad p_b = h_2 \rho_{Hg} g$ (2)

Eliminate p_b from the equations

$$p_a + h_1 \rho_{Hg} g = h_2 \rho_{Hg} g$$

$$p_a = (h_2 - h_1) \rho_{Hg} g$$

$= 840$ mm Hg absolute

The pressure measured by this manometer system is the absolute pressure because the reference (pressure above the mercury) in the vertical tube is a vacuum.

b. $p_a = 840 - 700 = 140$ mm Hg

Chapter 6

Introduction to Material Balances

6.1 The Concept of a Material Balance

A **material balance** is nothing more than the application of the law of the **conservation of mass**:

"Matter is neither created nor destroyed"

6.2 Open and Closed Systems

a. System

By **system** we mean any arbitrary portion of or a whole **process** that you want to consider for analysis. You can define a **system** such as a **reactor**, a **section of a pipe**. Or, you can define the **limits** of the **system** by drawing the **system boundary**, namely a line that encloses the portion of the process that you want to analyze.

b. Closed System

Figure 6.1 shows a two-dimensional view of a three-dimensional vessel holding **1000 kg of H_2O**. **Note that material neither enters nor leaves the vessel**, that is, **no material crosses the system boundary**. Changes can take place **inside the system**, but for a **closed system**, **no mass exchange occurs with the surroundings**.

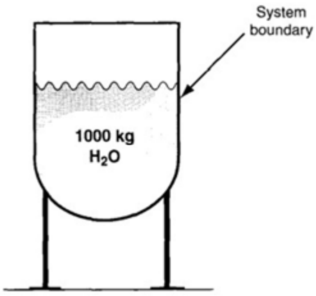


Figure 6.1 A closed system.

c. Open System

Figure 6.2 is an example of an **open system** (also called a **flow system**) because material crosses the system boundary.

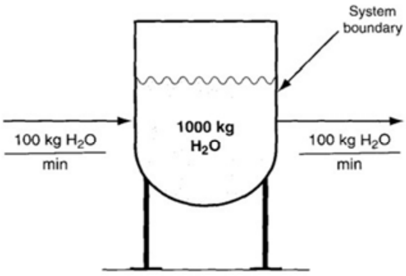


Figure 6.2 An open steady–state system.

6.3 Steady-State and Unsteady-State Systems

a. Steady–State System

Because the rate of addition of water is equal to the rate of removal, the amount of water in the vessel shown in **Figure 6.2** remains constant at its original value (**1000 kg**). We call such a **process** or **system** a **steady–state process** or a **steady–state system** because

1. The **conditions** inside the process (specifically the amount of water in the vessel in Figure 6.2) **remain unchanged with time**, and
2. The **conditions** of the flowing streams **remain constant with time**.

✶ Thus, in a **steady-state process**, by definition all of the conditions in the process **(e.g., temperature, pressure, mass of material, flow rate, etc.)** remain constant with time. A **continuous process** is one in which material enters and/or leaves the system without interruption.

b. Unsteady–State System

Because the amount of water in the system **changes with time (Figure 6.3)**, the **process** and **system** are deemed to be an **unsteady–state** (**transient**) **process or system**.

✶ For an **unsteady-state process**, not all of the **conditions** in the **process (e.g., temperature, pressure, mass of material, etc.)** remain constant with time, and/or the **flows** in and out of the **system** can **vary with time**.

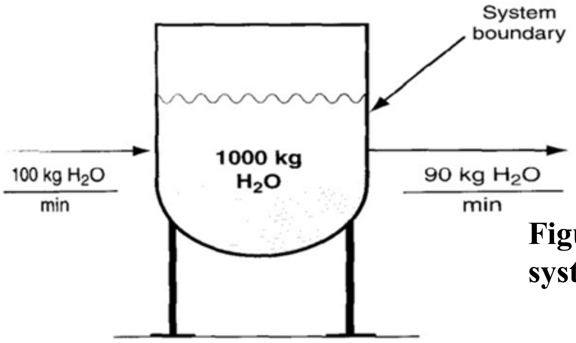


Figure 6.3 Initial conditions for an open unsteady–state system with accumulation.

- Figure 6.4 shows the system after 50 minutes of accumulation (Fifty minutes of accumulation at 10 kg/min amounts to 500 kg of total accumulation).

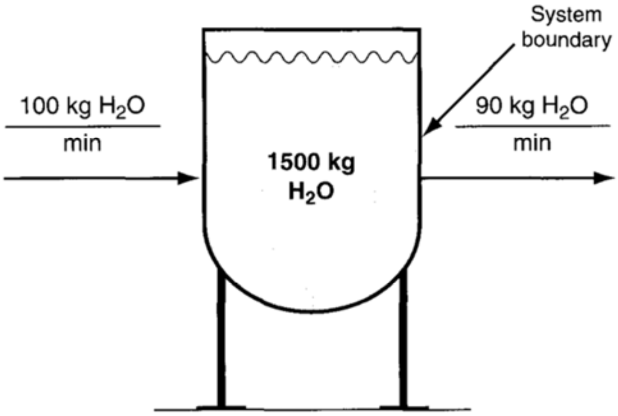


Figure 6.4 The condition of the open unsteady–state system with accumulation after 50 minutes.

- ✶ **Figures 6.5** and **6.6** demonstrate **negative accumulation**.

  Note that the amount of water in the system decreases with time at the rate of **10 kg/min**. Figure 6.6 shows the system after **50 minutes** of operation.

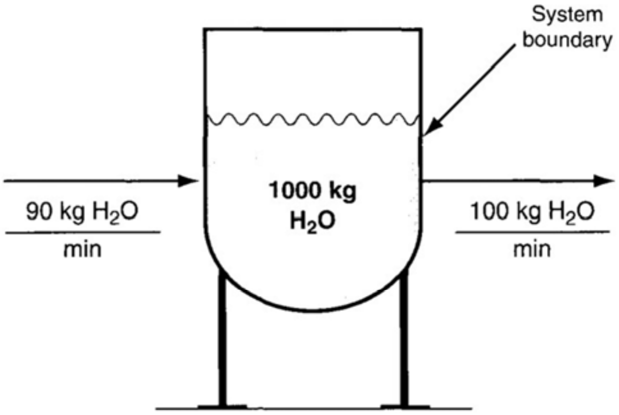


Figure 6.5 Initial conditions for an unsteady–state process with negative accumulation.

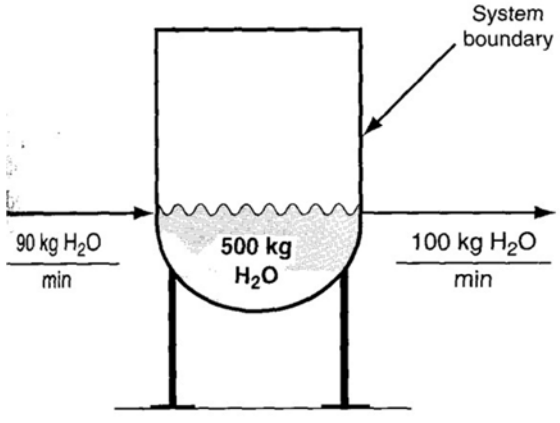


Figure 6.6 Condition of the open unsteady–state system with negative accumulation after 50 minutes.

- **The material balance for a single component process is**

$$\begin{Bmatrix}\text{Accumulation of material}\\ \text{within the system}\end{Bmatrix} = \begin{Bmatrix}\text{Total flow into}\\ \text{the system}\end{Bmatrix} - \begin{Bmatrix}\text{Total flow out}\\ \text{of the system}\end{Bmatrix} \quad \ldots 6.1$$

Equation 6.1 can apply to **moles** or any **quantity** that is **conserved**. As an example, look at **Figure 6.7** in which we have converted all of the mass quantities in **Figure 6.2** to their equivalent values in moles.

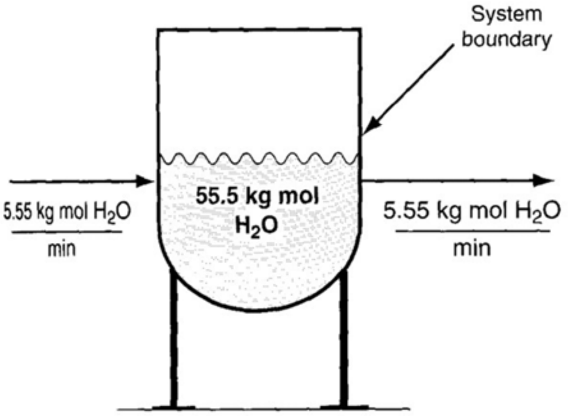


Figure 6.7 The system in Figure 6.2 with the flow rates shown in kg mol.

If the process is in the **steady state**, the **accumulation** term by definition is **zero**, and **Equation 6.1** simplifies to a famous truism

What goes in must come out (In = Out) …6.2

If you are analyzing an unsteady-state process, the accumulation term over a time interval can be calculated as

$$\{\text{Accumulation}\} = \begin{Bmatrix}\text{Final material}\\ \text{in the system}\end{Bmatrix} - \begin{Bmatrix}\text{Initial material}\\ \text{in the system}\end{Bmatrix} \quad (6.3)$$

The **times** you select for the final and initial conditions can be anything, but you usually select an **interval** such as **1 minute** or **1 hour** rather than specific times.

- When you combine **Equations 6.1 and 6.3** you get the **<u>general material balance</u>** for a component in the system in the **<u>absence of reaction</u>**

$$\left\{\begin{matrix}\text{Final material}\\ \text{in the system}\\ \text{at } t_2\end{matrix}\right\}-\left\{\begin{matrix}\text{Initial material}\\ \text{in the system}\\ \text{at } t_1\end{matrix}\right\}=\left\{\begin{matrix}\text{Flow into}\\ \text{the system}\\ \text{from } t_1 \text{ to } t_2\end{matrix}\right\}-\left\{\begin{matrix}\text{Flow out of}\\ \text{the system}\\ \text{from } t_1 \text{ to } t_2\end{matrix}\right\} \quad \ldots 6.4$$

Example 6.1

Will you save money if instead of buying premium 89 octane gasoline at $1.269 per gallon that has the octane you want, you blend sufficient 93 octane supreme gasoline at $l.349 per gallon with 87 octane regular gasoline at $1.149 per gallon?

Solution

Choose a **basis** of **1 gallon of 89 octane gasoline**, the desired product. The system is the gasoline tank.

- For simplicity, assume that **no gasoline exists** in the tank at the start of the blending, and **one gallon exists** in the tank at the end of the blending.
- This arrangement corresponds to an **unsteady-state process**. Clearly it is an **open system**.

The **initial number of gallons** in the system is **zero** and the **final number of gallons** is **one**.

Let **x** = the number of gallons of **87** octane gasoline added, and
y = the number of gallons of **93** octane added to
the blend. Since **x + y = 1** is the total flow into the
tank,

∴ y = 1 – x

According to Equation (6.4) the balance on the octane number is

$$\overset{\textit{Accumulation}}{\left|\frac{89\ \text{octane}}{1\ \text{gal}}\right|\frac{1\ \text{gal}}{}} - 0 = \overset{\textit{Inputs}}{\left|\frac{87\ \text{octane}}{1\ \text{gal}}\right|\frac{x\ \text{gal}}{} + \left|\frac{93\ \text{octane}}{1\ \text{gal}}\right|\frac{(1-x)\ \text{gal}}{}}$$

The solution is x = 2/3 gal and thus y = 1 – x = 1/3 gal.

The cost of the blended gasoline is (2/3) ($l.l49) + (1/3) ($l.349) = $ 1.216 A value less than the cost of the 89 octane gasoline ($l.269).

6.4 Multiple Component Systems

Suppose the input to a vessel contains **more than one component**, such as 100 kg/min of a 50% water and 50% sugar (sucrose, $C_{12}H_{22}O_{11}$, MW = 342.3) mixture (see Figure 6.8). The mass balances with respect to the **sugar and water**, balances that we call **component balances**.

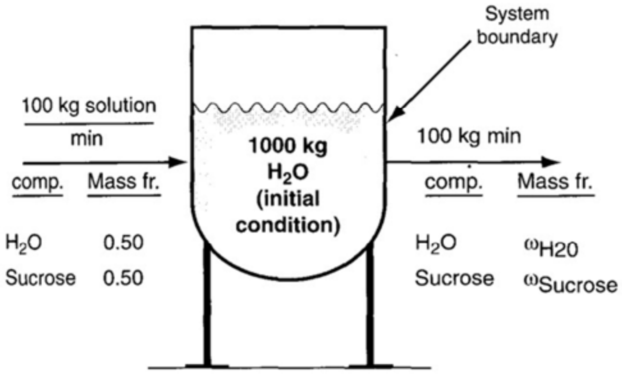


Figure 6.8 An open system involving two components.

For Example, look at the mixer shown in Figure 6.9, an apparatus that mixes two streams to increase the concentration of NaOH in a dilute solution. **The mixer is a steady–state open system**. Initially the mixer is empty, and after 1 hour it is empty again.

Basis = 1 hour for convenience. As an alternate to the **basis** we selected, you could select **F_1 = 9000 kg/hr as the basis, or F_2 = 1000 kg/hr as the basis;** the **numbers** for this example would not change – just the **units** would change. Here are the components and total balances in **kg**:

	Flow in			
Balances	F_1	F_2	Flow out	Accum.
NaOH	450	500	950	= 0
H_2O	8,550	500	9,050	= 0
Total	9,000	1,000	10,000	= 0

We can convert the kg shown in Figure 6.9 to kg moles by dividing each compound by its respective molecular weight (NaOH = 40 and H_2O = 18).

$$\text{NaOH:}\quad \frac{450}{40} = 11.25 \qquad \frac{500}{40} = 12.50 \qquad \frac{950}{40} = 23.75$$

$$H_2O\text{:}\quad \frac{8550}{18} = 475 \qquad \frac{500}{18} = 27.78 \qquad \frac{9050}{18} = 502.78$$

Then the component and total balances in **kg mol** are:

	Flow in			
Balances	F_1	F_2	Flow out	Accum.
NaOH	11.25	12.50	23.75	= 0
H_2O	475	27.78	502.78	= 0
Total	486.25	40.28	536.53	= 0

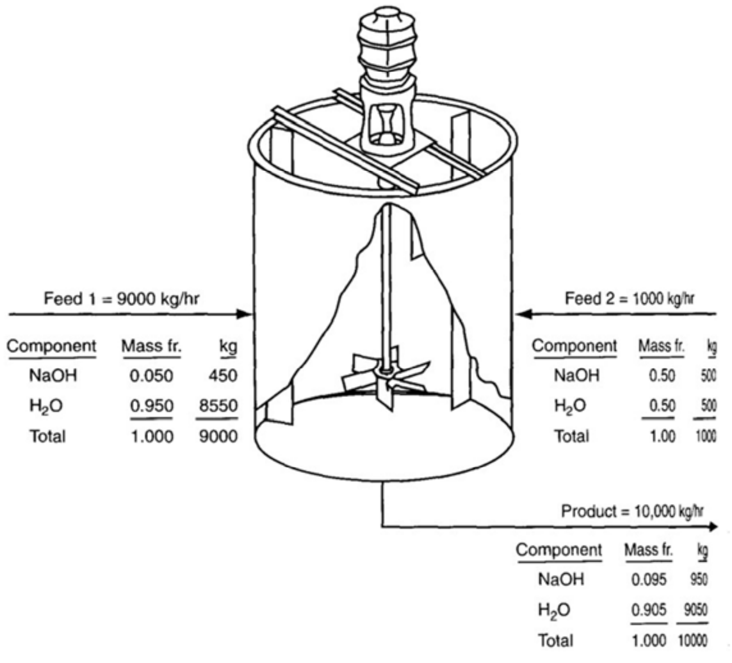


Figure 6.9 Mixing of a dilute stream of NaOH with a concentrated stream of NaOH. Values below the stream arrows are based on 1 hour of operation.

Example 6.2

Centrifuges are used to separate particles in the range of 0.1 to 100 μm in diameter from a liquid using centrifugal force. Yeast cells are recovered from a broth (a liquid mixture containing cells) using a tubular centrifuge (a cylindrical system rotating about a cylindrical axis). Determine the amount of the cell-free discharge per hour if 1000 L/hr is fed to the centrifuge, the feed contains 500 mg cells/L, and the product stream contains 50 wt.% cells. Assume that the feed has a density of 1 g/cm^3.

Solution

This problem involves a **steady state**, **open (flow)** system **without reaction**.

Basis = 1 hour

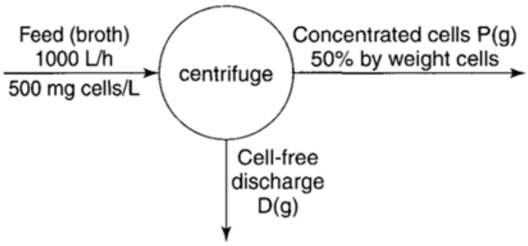


Figure E6.2

M.B. on cells

In (mass) = Out (mass)

$$\frac{1000\text{ L feed}}{} \left| \frac{500\text{ mg cells}}{1\text{ L feed}} \right| \frac{1\text{ g}}{1000\text{ mg}} = \frac{0.5\text{ g cells}}{1\text{ g }P} \left| \frac{P\text{ g}}{} \right.$$

P = 1000 g

M.B. on fluid

In (mass) = Out (mass)

$$\frac{1000\text{ L}}{} \left| \frac{1000\text{ cm}^3}{1\text{ L}} \right| \frac{1\text{ g fluid}}{1\text{ cm}^3} = \frac{1000\text{ g }P}{} \left| \frac{0.50\text{ g fluid}}{1\text{ g }P} \right. + D\text{ g fluid}$$

$D = (10^6 - 500)$ g

6.5 Accounting for Chemical Reactions in Material Balances

Chemical reaction in a system requires the augmentation of **Equation 6.4** to take into account the **effects of the reaction**. To illustrate this point, look at **Figure 6.10**, which shows a steady–state system in which **HCl** reacts with **NaOH** by the following reaction:

$$\mathbf{NaOH + HCl \longrightarrow NaCl + H_2O}$$

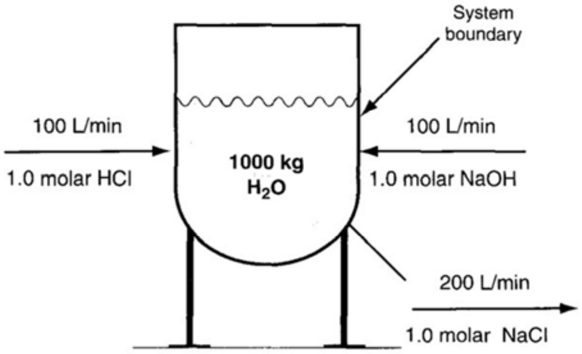


Figure 6.10 Reactor for neutralizing HCl with NaOH.

Equation 6.4 must be augmented to include terms for the **generation** and **consumption** of components by the **chemical reaction** in the system as follows

$$\left\{\begin{matrix}\text{Accumulation}\\\text{within the}\\\text{system}\end{matrix}\right\}=\left\{\begin{matrix}\text{Input}\\\text{through}\\\text{the system}\\\text{boundaries}\end{matrix}\right\}-\left\{\begin{matrix}\text{Output}\\\text{through}\\\text{the system}\\\text{boundaries}\end{matrix}\right\}+\left\{\begin{matrix}\text{Generation}\\\text{within the}\\\text{system}\end{matrix}\right\}-\left\{\begin{matrix}\text{Consumption}\\\text{within the}\\\text{system}\end{matrix}\right\} \quad ...6.5$$

6.6 Material Balances for Batch and Semi-Batch Processes

- A **batch process** is used to process a **fixed amount** of material each time it is operated. **Initially**, the material to be processed is charged into the system. After processing of the material is complete, the products are **removed**.
- **Batch processes** are used **industrially** for specialty processing applications (e.g., producing **pharmaceutical** products), which typically operate at relatively **low production rates**.
- Look at Figure 6.11a that illustrates what occurs at the start of a batch process, and after thorough mixing, the final solution remains in the system (Figure 6.11b).

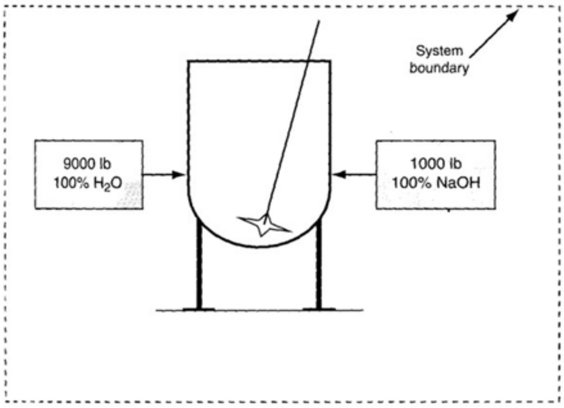


Figure 6.11a The initial state of a batch mixing process.

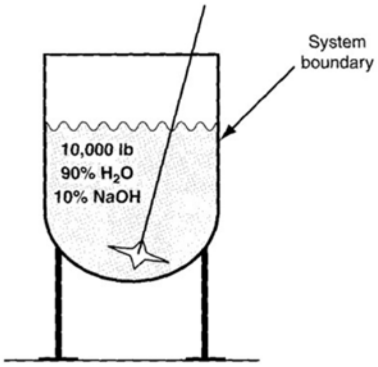


Figure 6.11b The final state of a batch mixing process.

- We can summarize the **hypothetical operation** of the **batch** as a flow system (open system) as follows (**Figure 6.12**):

Final conditions: All values = 0 **Flows out:**

$NaOH = 1,000$ lb

$H_2O = 9,000$ lb

Total = 10,000 lb

Initial conditions: All value = 0 **Flows in:**

$NaOH = 1,000$ lb

$H_2O = 9,000$ lb

Total 10,000 lb

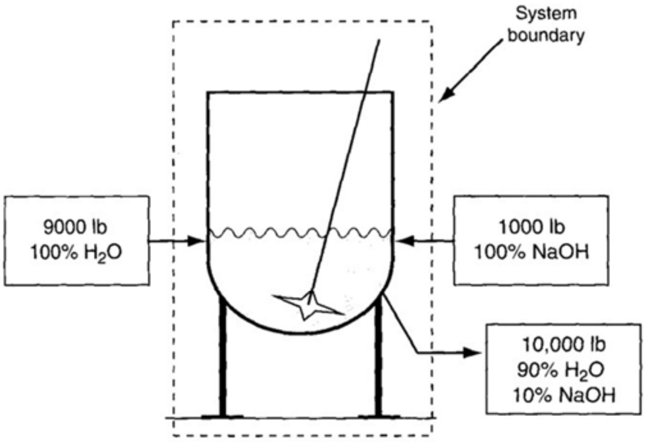


Figure 6.12 The batch process in Figure 6.11 represented as an open system.

☒ In a **semi-batch process** material **enters** the process during its operation, but does **not leave**. Instead, mass is allowed to accumulate in the process vessel. Product is **withdrawn** only after the process is over.

☒ **A Figure 6.13 illustrates** a **semi-batch mixing process**. Initially the vessel is empty (**Figure 6.13a**). **Figure 6.13b** shows the **semi-batch system** after **1 hour** of operation. **Semi-batch processes** are **open** and **unsteady – state**.

☒ Only flows **enter** the systems, and **none leave**, hence the system is an **unsteady state** – one that you can treat as having **continuous flows**, as follows:

Final conditions: **Flows out:** All values = 0

$NaOH = 1,000$ lb

H_2O = 9,000 lb
Total = 10,000 lb

Flows in:

NaOH = 1,000 lb
H_2O = 9,000 lb
Total = 10,000 lb

Initial conditions: All values = 0

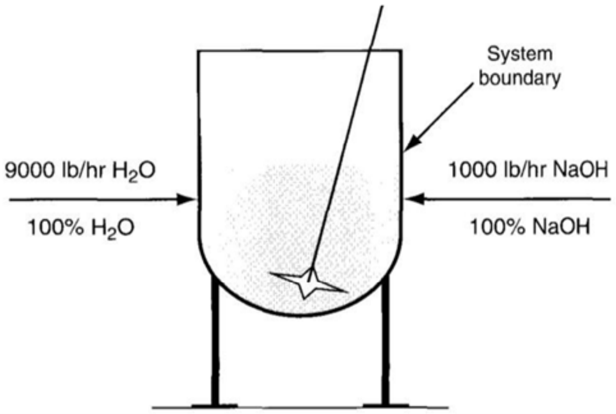


Figure 6.13a Initial condition for the semi-batch mixing process. Vessel is empty.

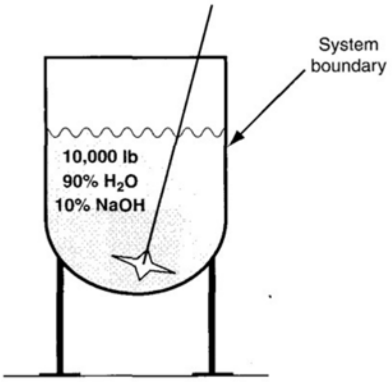


Figure 6.13b Condition of a semi-batch mixing process after 1 hour of operation.

Example 6.3

A measurement for water flushing of a steel tank originally containing motor oil showed that 0.15 percent by weight of the original contents remained on the interior tank surface. What is the fractional loss of oil before flushing with water, and the pounds of discharge of motor oil into the environment during of a 10,000 gal tank truck that carried motor oil? (The density of motor oil is about 0.80 g/cm^3).

Solution

Basis: 10,000 gal motor oil at an assumed 77°F

The initial mass of the motor oil in the tank was

(10000 gal)(3.785 lit/1 gal)(1000 cm^3/1 lit)(0.8 g/cm^3)(1 lb/454 g) = 66700 lb

The mass fractional loss is **0.0015**. The oil material balance is

Initial		unloaded		residual discharged on cleaning
66,700	=	66,700 (0.9985)	+	66,700 (0.0015)

Thus, the discharge on flushing is **66,700 (0.00 15) = 100 lb**.

Questions

1. Is it true that if no material crosses the boundary of a system, the system is a closed system?

2. Is mass conserved within an open process?
3. Can an accumulation be negative? What does a negative accumulation mean?
4. Under what circumstances can the accumulation term in the material balance be zero for a process?
5. Distinguish between a steady-state and an unsteady-state process.
6. What is a transient process? Is it different than an unsteady-state process?

7. Does Equation 6.4 apply to a system involving more than one component?
8. When a chemical plant or refinery uses various feeds and produces various products, does Equation 6.4 apply to each component in the plant?
9. What terms of the general material balance, Equation (6.5), can be deleted if
    a. The process is known to be a steady-state process.
    b. The process is carried out inside a closed vessel.
    c. The process does not involve a chemical reaction.
10. What is the difference between a batch process and a closed process?
11. What is the difference between a semi-batch process and a closed process?
12. What is the difference between a semi-batch process and an open process?

Answers:

1. Yes
2. Not necessarily – accumulation can occur
3. Yes; depletion
4. No reaction (a) closed system, or (b) flow of a component in and out are equal.
5. In an unsteady-state system, the state of the system changes with time, whereas with a steady-state system, it does not.
6. A transient process is an unsteady-state process.
7. Yes
8. Yes
9. (a) Accumulation; (b) flow in and out; (c) generation and consumption
10. None
11. A flow in occurs
12. None, except in a flow process, usually flows occur both in and out

Problems

1. Here is a report from a catalytic polymerization unit:

Charge:	**Pounds per hour**
Propanes and butanes	15,500
Production:	

Propane and lighter	5,680
Butane	2,080
Polymer	missing

What is the production in pounds per hour of the polymer?

2. A plant discharges 4,000 gal/min of treated wastewater that contains 0.25 mg/L of PCB, (polychloronated biphenyls) into a river that contains no measurable PCBs upstream of the discharge. If the river flow rate is 1,500 cubic feet per second, after the discharged water has thoroughly mixed with the river water, what is the concentration of PCBs in the river in mg/L?

Answers:

1. 7740 lb/hr
2. $1.49 * 10^{-3}$ mg/L

Supplementary Problems (Chapter Six):

Problem 1

b. The input is 1.5 kg in one hour.

c. The output is 1.2 kg in one hour.

d. Assume the process is unsteady state. Then the accumulation in the soil is 0.3 kg in one hour.

e. Assume unsteady state. If not, the accumulation would be zero and perhaps some leak from the closed system occurred (as would likely occur in the field).

Problem 2

Problem 6.2

If 1 L of

thy alcohol is mixed with 1 L of water, how many kilograms of solution result?

How many liter

n?

Solution

The den

sities of alcohol and water at 20°C are 0.789 and 0.998 g/cm³, respectively.

Basis: 1 L of each compound

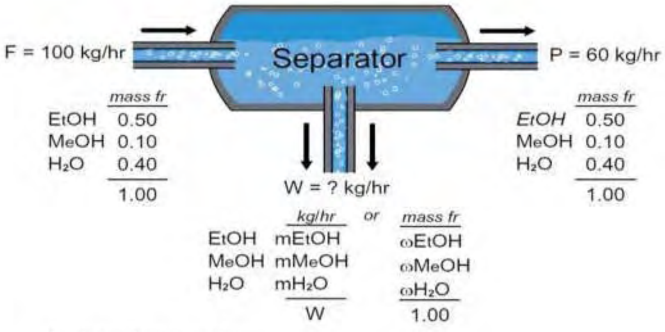
1000 cm³	= 789 g		0.998 g / cm³	1000 cm³	= 998 g		0.789 g / cm³

The total kg are

789 + 998 = 1787 g.

The volumes are not additive. For a 789/1787 = 0.442 mass fraction solution of alcohol in water, the density at 20°C is 0.929 g/cm³.

$$\frac{1787\ \text{g}}{} \Big| \frac{\text{cm}^3}{0.929\ \text{g}} = 1923\ \text{cm}^3$$



An obvious basis is one hour.

Problem 3

The variables whose values are unknown are either (a) m_{EtOH}, m_{MeOH}, and m_{H_2O} plus W, or (b) ω_{EtOH}, ω_{MeOH}, and ω_{H_2O} plus W. Either set of four is acceptable as they are equivalent. We have four unknowns, and need four independent equations.

$$\begin{array}{lllllllllll}
\text{Total:} & F & = & P & + & W & & F & = & P + & W \\
\text{EtOH:} & 0.50F & = & 0.80P & + & m_{EtOH} & & 0.50F & = & 0.80P & + \omega_{EtOH}W \\
\text{MeOH:} & 0.10F & = & 0.15P & + & m_{MeOH} & \text{or} & 0.10F & = & 0.15P & + \omega_{MeOH}W \\
H_2O\text{:} & 0.40F & = & 0.05P & + & m_{H_2O} & & 0.40F & = & 0.05P & + \omega_{H_2O}W
\end{array}$$

In addition you know one more independent equation holds for the components in W

$$m_{EtOH} + m_{MeOH} + m_{H2O} = W \qquad \text{or} \qquad \omega_{EtOH} + \omega_{MeOH} + \omega_{H2O} = 1$$

The solution of the equations is (using the total and first two component balances)

	m_i *(kg/hr)*	ω_i *(mass fr)*
EtOH	2	0.050
MeOH	1	0.025
H_2O	37	0.925
	40	1.00

As a check, we will use the third component balance, the one for H_2O, a redundant equation

$$\begin{array}{rcl@{\qquad}c@{\qquad}rcl}
0.40(100) & \overset{?}{=} & 0.05(60) + 37 & \text{or} & 0.40(100) & = & 0.05(60) + 0.925(40) \\
40 & = & 3 + 37 & & 40 & = & 3 + 37
\end{array}$$

7.1 Problem Solving

An orderly method of analyzing problems and presenting their solutions represents training in logical thinking that is of considerably greater value than mere knowledge of how to solve a particular type of problem.

7.2 The Strategy for Solving Problems

1. Read and understand the problem statement.
2. Draw a sketch of the process and specify the system boundary.
3. Place labels for unknown variables and values for known variables on the sketch.
4. Obtain any missing needed data.
5. Choose a basis.
6. Determine the number of unknowns.
7. Determine the number of independent equations, and carry out a degree of freedom analysis.
8. Write down the equations to be solved.
9. Solve the equations and calculate the quantities asked for.
10. Check your answer.

Example 7.1

A thickener in a waste disposal unit of a plant removes water from wet sewage sludge as shown in Figure E7.l. How many kilograms of water leave the thickener per 100 kg of wet sludge that enter the thickener? The process is in the steady state.

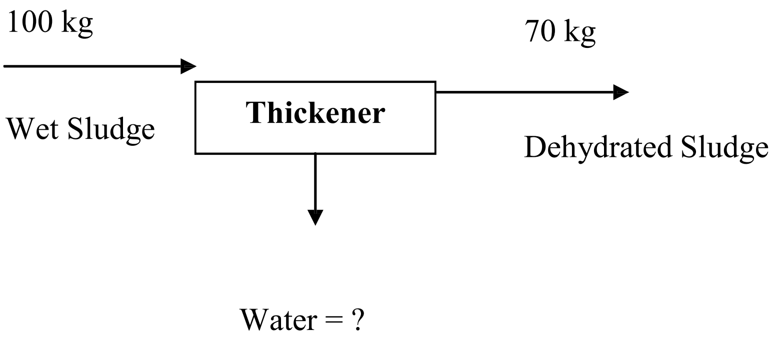


Figure E7.1

Solution

Basis: 100 kg wet sludge

The system is the thickener (an open system). No accumulation, generation, or consumption occurs. The total mass balance is

$$\underline{\text{In}} = \underline{\text{Out}}$$

$$100 \text{ kg} = 70 \text{ kg} + \text{kg of water}$$

Consequently, the water amounts to 30 kg.

Example 7.2

A continuous mixer mixes NaOH with H_2O to produce an aqueous solution of NaOH. Determine the composition and flow rate of the product if the flow rate of NaOH is 1000 kg/hr, and the ratio of the flow rate of the H_2O to the product solution is 0.9. For this process,

1. Sketch of the process is required.
2. Place the known information on the diagram of the process.
3. What basis would you choose for the problem?
4. How many unknowns exist?
5. Determine the number of independent equations.
6. Write the equations to be solved.
7. Solve the equations.
8. Check your answer.

Solution

1. The process is an open one, and we assume it to be steady state.

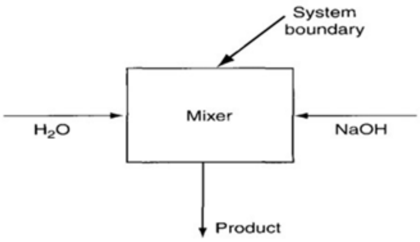
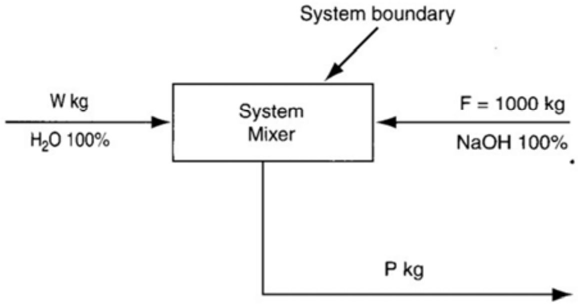


Figure E7.2

2. Because no contrary information is provided about the composition of the H_2O and NaOH streams, we will assume that they are 100% H_2O and NaOH, respectively.



Component	kg	ω (add if useful)
NaOH	P_{NaOH}	$\omega^P_{NaOH} = \frac{P_{NaOH}}{P}$
H_2O	P_{H_2O}	$\omega^P_{H_2O} = \frac{P_{H_2O}}{P}$
Total	P	1.00

3. Basis (1000 kg or 1 hour or 1000 kg/hr) (all are equivalent)

4. We do not know the values of four variables: W, P, P_{NaOH} and P_{H2O}.

5. You can write three material balances:

- one for the NaOH
- one for the H_2O
- one total balance (the sum of the two component balances)

Only two are independent.

Note: You can write as many independent material balances as there are species involved in the system.

6. Material balance: in = out or in – out = 0

$$\begin{array}{llcll} \text{NaOH balance:} & 1000 = P_{NaOH} & \text{or} & 1000 - P_{NaOH} = 0 & (1) \\ H_2O \text{ balance:} & W = P_{H_2O} & \text{or} & W - P_{H_2Q} = 0 & (2) \\ \text{Given ratio:} & W = 0.9P & \text{or} & W - 0.9P = 0 & (3) \\ \text{Sum of components in } P: & P_{NaOH} + P_{H_2O} = P & \text{or} & P_{NaOH} + P_{H_2O} - P = 0 & (4) \end{array}$$

Could you substitute the total mass balance 1000 + W = P for one of the two component mass balances? Of course In fact, you could calculate P by solving just two equations:

$$\begin{array}{ll} \text{Total balance:} & 1000 + W = P \\ \text{Given ratio:} & W = 0.9P \end{array}$$

7. Solve equations:

W = 0.9 P substitute in total balance 1000 + 0.9 P = P

∴ P = 10000 kg & W = 0.9 * 10000 = 9000 kg (The basis is still 1 hr (F_{NaOH} = 1000 kg))

From these two values you can calculate the amount of H_2O and NaOH in the product

$$\text{From the } \begin{cases} \text{NaOH balance:} \\ \\ H_2O \text{ balance:} \end{cases} \text{ you get } \begin{cases} P_{NaOH} = 1000\text{ kg} \\ \\ P_{H_2O} = 9000\text{ kg} \end{cases}$$

Then

$$\omega^P_{NaOH} = \frac{1000 \text{ kg NaOH}}{10{,}000 \text{ kg Total}} = 0.1$$

$$\omega^P_{H_2O} = \frac{9{,}000 \text{ kg } H_2O}{10{,}000 \text{ kg Total}} = 0.9$$

Note

$$\boxed{\omega^P_{NaOH} + \omega^P_{H_2O} = 1}$$

8. The total balance would have been a redundant balance, and could be used to check the answers

$$P_{NaOH} + P_{H2O} = P$$

$$1,000 + 9,000 = 10,000$$

<u>Note:</u> After solving a problem, use a <u>redundant equation</u> to check your values.

Degree of Freedom Analysis

The phrase degrees of freedom have evolved from the design of plants in which fewer independent equations than unknowns exist. The difference is called the degrees of freedom available to the designer to specify flow rates, equipment sizes, and so on. You calculate the number of degrees of freedom (N_D) as follows:

Degrees of freedom = number of unknowns — number of independent equations

$$N_D = N_U - N_E$$

- ✶ When you calculate the number of degrees of freedom you ascertain the solve ability of a problem. **Three** outcomes exist:

Case	**N_D**	**Possibility of Solution**
$N_U = N_E$	0	Exactly specified (determined); a solution exists
$N_U > N_E$	>0	Under specified (determined); more independent equations required
$N_U < N_E$	<0	Over specified (determined)

For the problem in **Example 7.2**,

$N_U = 4$

$N_E = 4$

So that

$N_D = N_U - N_E = 4 - 4 = 0$

And a **unique** solution exists for the problem.

Example 7.3

A cylinder containing CH_4, C_2H_6, and N_2 has to be prepared containing a CH_4 to C_2H_6 mole ratio of 1.5 to 1. Available to prepare the mixture is (l) a cylinder containing a mixture of 80% N_2 and 20% CH_4, (2) a cylinder containing a mixture of 90% N_2 and 10% C_2H_6, and (3) a cylinder containing pure N_2. What is the number of degrees of freedom, i.e., the number of independent specifications that must be made, so that you can determine the respective contributions from each cylinder to get the desired composition in the cylinder with the three components?

Solution

A sketch of the process greatly helps in the analysis of the degrees of freedom. Look at Figure E7.3.

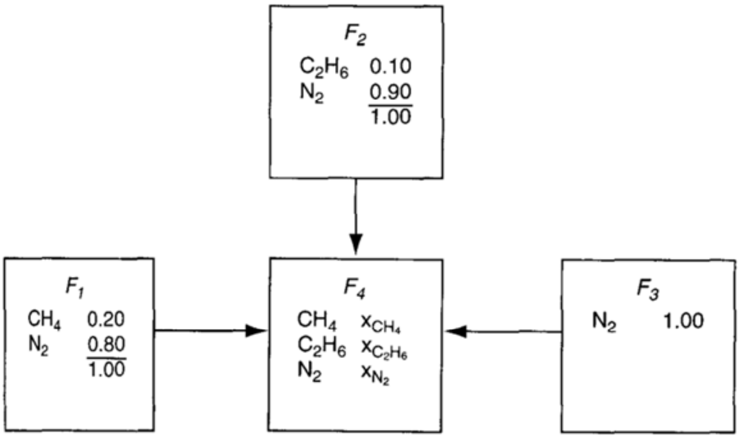


Figure E7.3

Do you count **seven unknowns** — **three** values of $\mathbf{x_i}$ and **four** values of $\mathbf{F_i}$? How many independent equations can be written?

- Three material balances: CH_4, C_2H_6, and N_2
- One specified ratio: moles of CH_4 to C_2H_6 equal 1.5 or $(X_{CH4}/X_{C2H6}) = 1.5$
- One summation of mole fractions: $\sum x_i^{F_4} = 1$

Thus, there are **seven minus five equals two degrees of freedom** ($N_D = N_U - N_E = 7 - 5 = 2$). If you pick a basis, such as $F_4 = 1$, one other value has to be specified to solve the problem to calculate composition of F_4.

Questions

1. What does the concept –solution of a material balance problem‖ mean?
2. (a) How many values of unknown variables can you compute from one independent material balance?

   (b) From three independent material balance equations?