Homework 1

Q1: Find the Laplace transform of:

a) $L\{\cos 2t + e^{7t}\}$

b) $L\{5e^{-t} + \cos 3t\}$.

c) $L\{2\sin 3t - 4e^{-4t}\}$.

d) $L\{9te^{-7t}\}$.

e) $L\{e^{4t}\cos 2t\}$

f) $L\{t\sin 3t\}$

g) $L\{te^{-2t}\sin 3t\}$

Q2: Find the inverse Laplace Transform of each of the following:

a) $\frac{27}{s^2+81}$

b) $\frac{5s\quad 8}{s(s\quad 4)}$

c) $\frac{2s\quad 6}{(s\quad 2)(s\quad 4)}$

d) $\frac{5s+1}{s^2\quad s\quad 12}$

e) $\frac{5s+3}{s^2+2s+5}$

f) $\frac{(s+4)}{(s+1)^2}$

g) $\frac{s}{(s+2)^3}$

h) $\frac{s(s+1)}{(s+2)(s+3)(s+4)}$

i) $\frac{s(s+1)}{(s+2)(s+3)(s^2+4)}$

Q3: Use the Partial fraction expansions to find the Inverse Laplace's Transform of the following:

a) $F(s) = \frac{20}{(s^2+6s+25)(s+1)}$

b) $F(s) = \frac{12(s+1)}{s(s+2)^2(s+3)}$

c) $F(s) = \frac{100}{(s^2+25)(s+2)}$

Homework 2

Q1: By employing convolution theorem, evaluate the following:

(1) $L^{-1}\ \dfrac{1}{(s+1)(s^2+1)}$

(2) $L^{-1}\ \dfrac{s}{(s+1)^2(s^2+1)}$

(3) $L^{-1}\ \dfrac{1}{(s^2+a^2)^2}$

(4) $L^{-1}\ \dfrac{s^2}{(s^2+a^2)(s^2+b^2)},\ a \neq b$

(5) $L^{-1}\ \dfrac{1}{s^2(s+1)^2}$

(6) $L^{-1}\ \dfrac{4s+5}{(s-1)^2(s+2)}$

Q2: Use Laplace transforms to solve the following Differential Equations:

a) $y' + 3y = 13\sin 2t,\ y(0) = 6$

b) $y' + y = e^{-3t}\cos 2t,\ y(0) = 0$

c) $y'' + 9y = \sin 3t,\ y(0) = 1,\ y'(0) = 0$

d) $y'' - 3y' + 2y = t\,e^{t}\ ;\ y(0) = 0,\ y'(0) = 0.$

e) $y'' + 8y' + 25y = 100\ ,\ y(0) = 2,\ y'(0) = 20$

Homework 3

Q1: A thermometer having a time constant of 1 min is initially at 50 deg C. It is immersed in a bath maintained at 100 deg C at t = 0. Determine the temperature reading at 1.2 min.

Q2: In problem Q1, if at t = 1.5 min, the thermometer is removed again from the bath and put in another bath at 75 ^{o}C, determine the maximum temperature indicated by the thermometer. What will be the indicated temperature at t= 20 min?

Q3: A process of unknown transfer function is subjected to a unit impulse input. The output of the process is measured accurately and is found to be represented by the function $Y(t) = te^{-t}$. Determine the unit step response in this process.

Q4: A thermometer having first-order dynamics with a time constant of 1 min is placed in a temperature bath at 100 °F. After the thermometer reaches steady state, it is suddenly placed in a bath at 110 °F at t = 0 and left there for 1 min, after which it is immediately returned to the bath at 100 °F.

(a) Draw a sketch showing the variation of the thermometer reading with time.

(b) Calculate the thermometer reading at t = 0.5 min and at t = 2.0 min.

Q5: Rewrite the sinusoidal response of a first-order system in terms of a cosine wave (Re express the forcing function as a cosine wave, and compute the phase difference between input and output cosine waves).

Homework 4

Q1: A liquid-level system has a cross-sectional area of 3.0 ft^2. The valve characteristics are q=8 h

Where: q = flow rate cfm, h = level above the valve, ft

Calculate the time constant for this system.

Q2: Derive the 1^{st} order transfer function of a liquid-level system shown fig. (1) to determine the response of the outlet flow ($q_o(t)$) to unit pulse change in the inlet flow (q_{in} (t)).

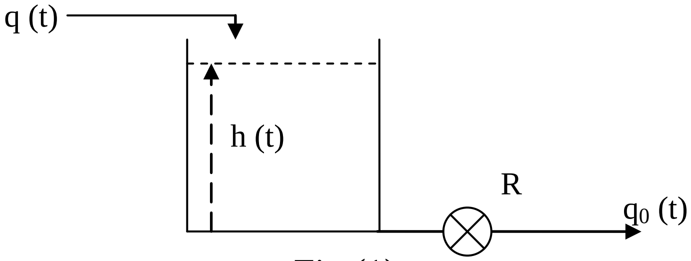

Fig.(1)

Q3: Derive the transfer function *H/Q* for the liquid level system shown in Fig.(2). The resistances are linear. *H* and Q are deviation variables.

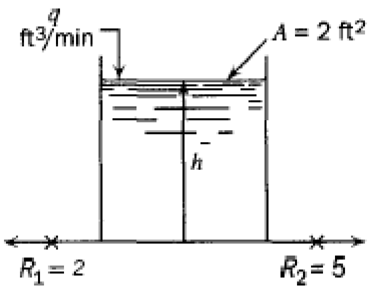

Fig.(2)

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Homework 5

Q1: Find the transfer functions H_2/Q and H_3/Q for the three-tank system shown in Fig. (1), where H_2, H_3 and Q are deviation variables. Tank 1 and Tank 2 are interacting.

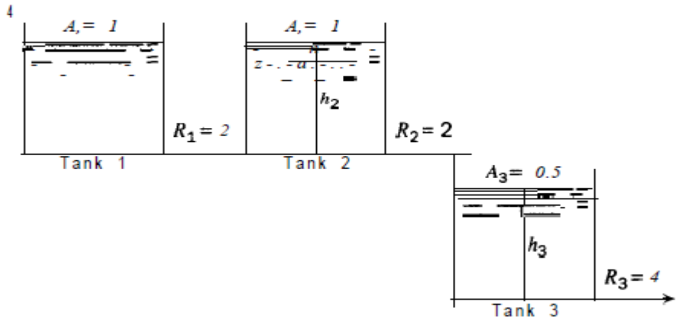


Fig.(1)

Q2: In the two-tank mixing process shown in Fig.(2), x varies from 0 lb salt/ft^3 to 1 lb salt/ft^3 according to a step function. At what time does the salt concentration in tank 2 reach 0.6 lb salt/ft^3? The volume of each tank is 6 ft^3.

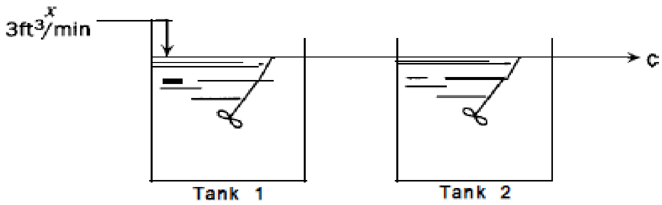


Fig.(2)

Q3: Derive the transfer functions $H_1(s)/Q(s)$ and $H_2(s)/Q(s)$ for the liquid level system shown in Fig.(3). The resistances are linear and $R_1 = R_2 = 1$. Note that two streams are flowing from tank 1, one of which flows into tank 2.

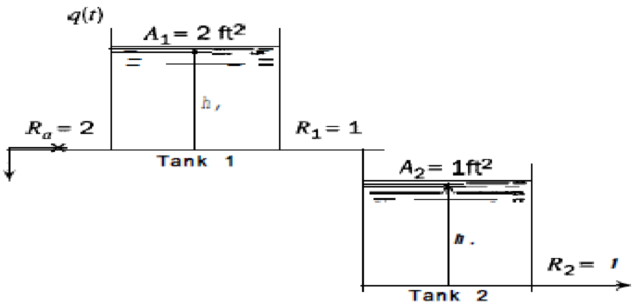


Fig.(3)

Q4: The two-tank mixing process shown in Fig. (4) contains a recirculation loop that transfers solution from tank 2 to tank 1 at a flow rate of αq_o. Develop a transfer function that relates the concentration in tank 2, C_2, to the concentration in the feed, x; i.e. $C_2(s)/X(s)$ where C_2 and X are deviation variables.

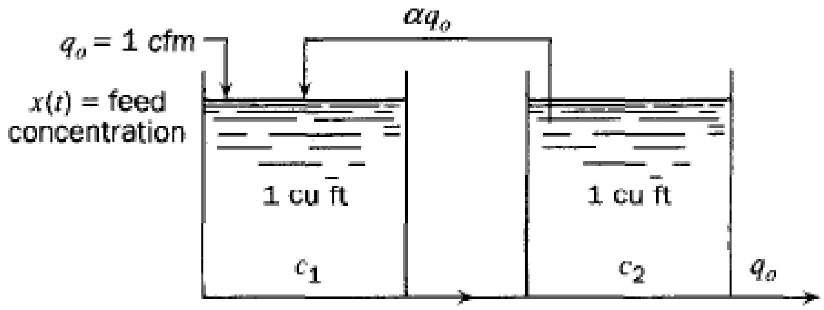


Fig. (4)

Q5: Determine the transfer function H(s)/Q(s) for the liquid-level system shown in Fig. (5) Resistances R_1 and R_2 are linear. The flow rate from tank 3 is maintained constant at b by means of a pump; i.e., the flow rate from tank 3 is independent of head h. The tanks are non-interacting.

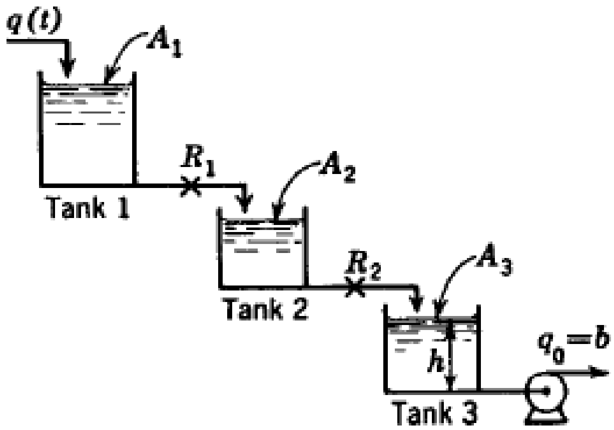


Fig. (5)

Homework 6

Q1: Linearize the function $f(x) = x^3+x^2+8$ at $x = 2$.

Q2: Linearize the following nonlinear equation:

$f(x, y) = x^2 + 8xy + 3y^2$ About the point $P(x_0, y_0) = (3, 10)$.

Q3: Consider the nonlinear differential equation

$$\frac{dy}{dt} = (1+y)y + u + 2$$

a) Derive the linearized differential equation using deviation variables.

b) Derive the transfer function between input u and output y.

Q4: a) Derive the dynamic model (differential equation(s)) for the system in Fig.(1). Linearize the model, if necessary.

b) Determine the transfer function $C_A(s)/T_0(s)$ using the model.

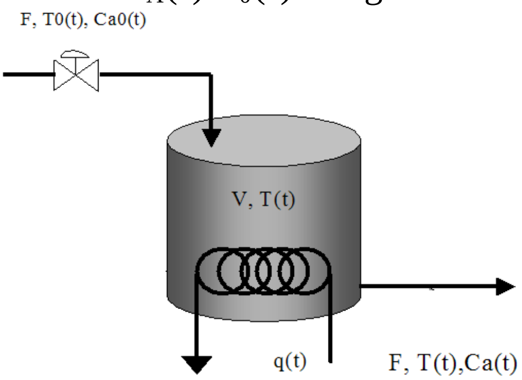


Fig. (1)

Q5: Consider the chemical reactor shows in the fig. (2). The reaction occurring in the reactor is: $A \rightarrow B$, where $ra = -kVC_a^{0.5}$. The following assumptions are adequate to the system: The flow rates and volume are constants and the reactor is isothermal.

a) Formulate the model for the dynamic response of the concentration of A in the reactor Ca(t) and linearize the equation in (a).

b) Solve the linearized equation analytically for a step change (A/s) in the inlet concentration of A, Ca0 and Sketch the dynamic behavior of Ca(t) graphically.

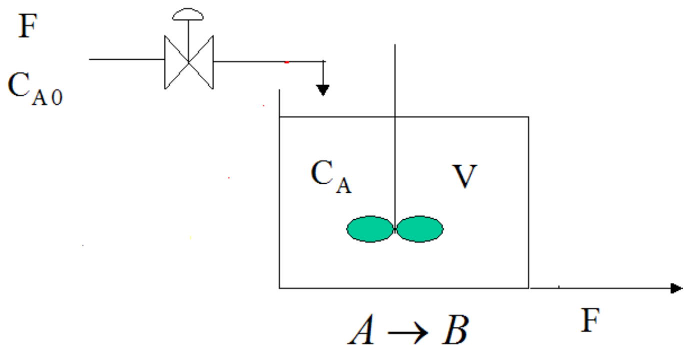


Fig.(2). Isothermal reactor.

Homework 7

Q1: For second-order system with the following transfer function, determine the damping factor. Identify the system as underdamped, critically damped or overdamped.

$$H(s) = \frac{3}{s^2 + 6s + 9}$$

Q2: For each of the second-order systems that follow, find ξ, τ, tp, tr ,%OS and DR.

a) $$Y(s) = \frac{16}{s^2 + 3s + 16}$$

b) $$Y(s) = \frac{0.04}{s^2 + 0.02s + 0.04}$$

Q3: A step change of magnitude 4 is introduced into a system having the transfer function:

$$\frac{Y(s)}{X(s)} = \frac{10}{s^2 + 1.6s + 4}$$

Determine (a) Percent overshoot, (b) Rise time, (c) Maximum value of $Y(t)$, (d) Ultimate value of $Y(t)$, and (e) Period of oscillation.

Q4: The transfer function for a thermometer in a CSTR reactor is given by.

$$\frac{T_{thermometer}(s)}{T_{Reactor}(s)} = \frac{16}{(3s+1)(10s+1)}$$

Estimate the following.

a) find ξ, τ
b) The rise time.
c) The peak time_.
d) The percentage overshoot.

Q5: The Fig.(1) illustrates a step test for a heat exchanger system. Determine the value of ξ and τ from the graph.

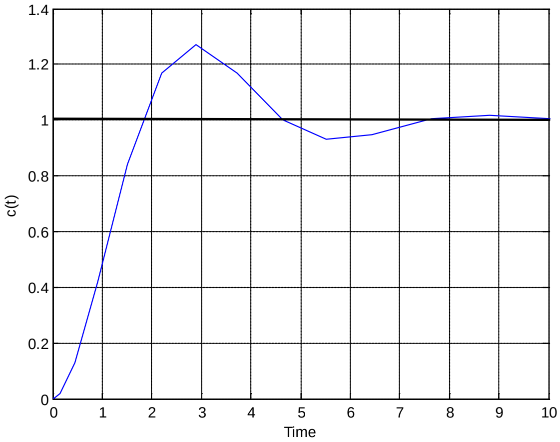


Fig.(1). Heat exchanger step response.

Homework 10

1. (a) For the control system shown in Fig. (1). Obtain the closed-loop transfer function C(s)/U(s).
   (b) Find the value of Kc for which the closed-loop response has a of 2.3.
   (c) Find the offset for a unit-step change in U if Kc = 4.

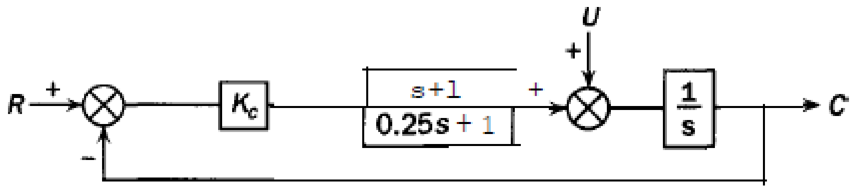


Fig. (1)

2. For the control system shown in Fig. (2), determine:
   a) C(s)/R(s)
   b) C(0.5) and C(∞)
   c) Offset

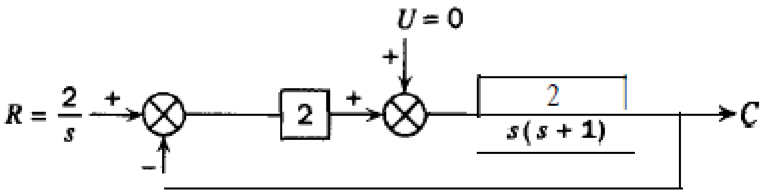


Fig. (2)

3. For the control system shown in Fig. (3), determine an expression for C(t) if a unit-step change occurs in R. Sketch the response C(t) and compute C(2).

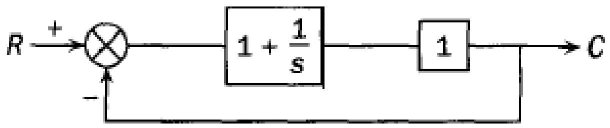


Fig. (3)

4. Compare the responses to a unit-step change in set point for the system shown in Fig. (4) for both negative feedback and positive feedback. Do this for Kc of 0.5 and 1.0. Compare these responses by sketching C(t).

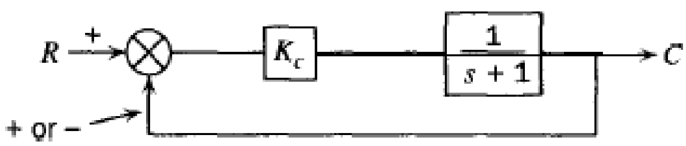


Fig. (4)

Homework 11

Q1: Derive the transfer function Y(s)/X(s) for the control system shown in Fig. (1).

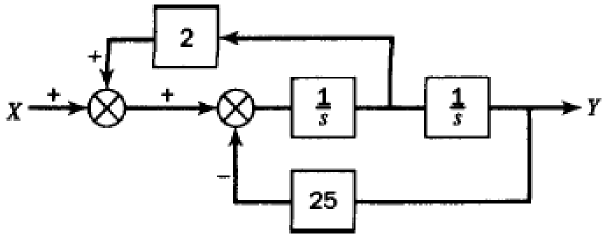


Fig. (1)

Q2: Reduce the block diagram of Fig (2). to single block and find the system transfer function $\frac{Y(s)}{R(s)}$.

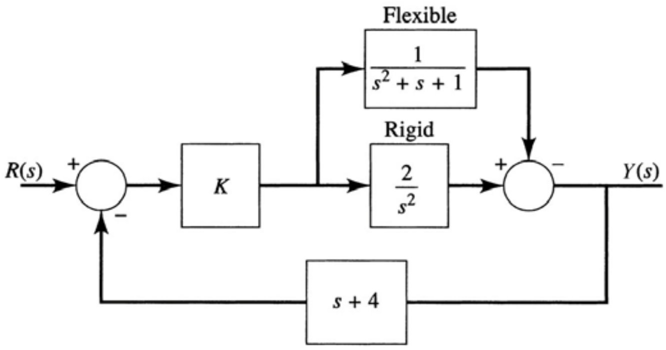


Fig. (2)

Q3: For the control system shown in Figures (3, 4, 5, 6, 7 and 8). Derive the transfer function C(s)/R(s).

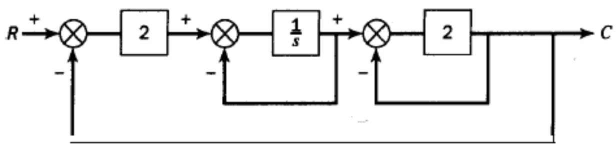


Fig (3)

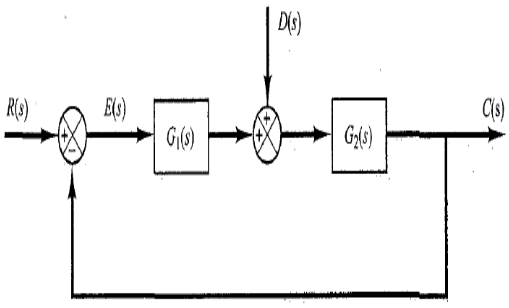


Fig (4)

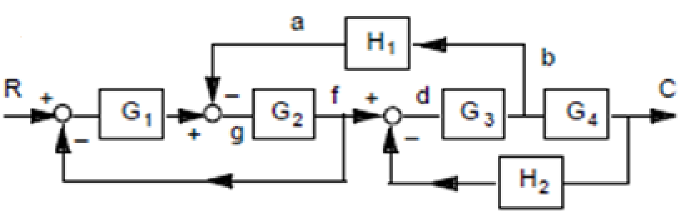


Fig (5)

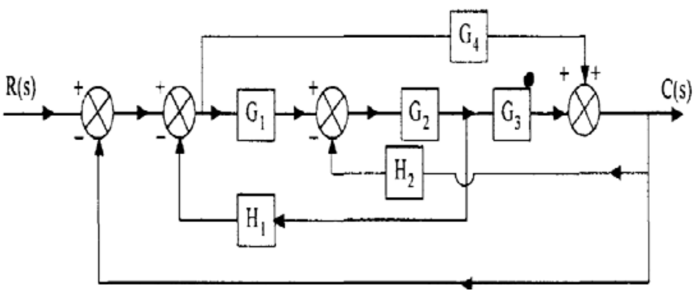


Fig (6)

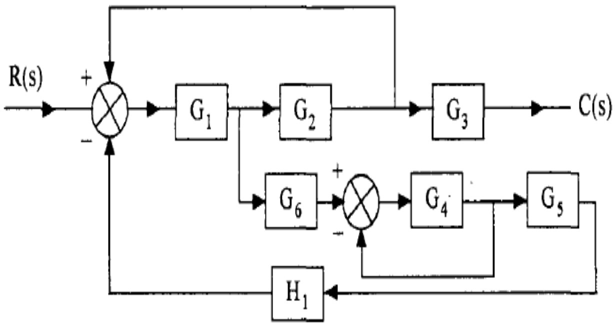


Fig (7)

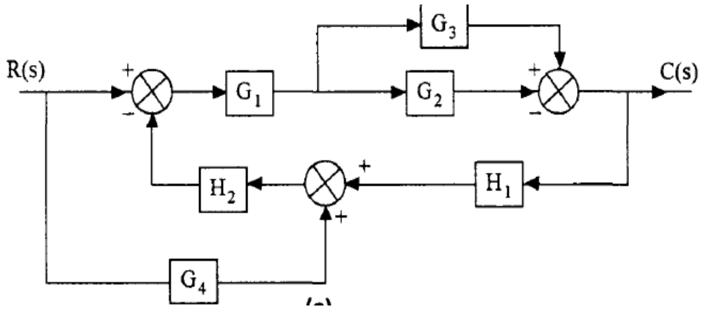


Fig (8)

Homework 12

Q1: Test the stability of control systems represented by the following characteristic equations:

a) $s^4+6s^3+21s^2+36s+20=0$

b) $3s^4+10s^3+5s^2+5s+2=0$

c) $s^5+s^4+2s^3+2s^2+3s+5=0$

d) $s^5+s^4+2s^3+2s^2+3s+5=-0$

e) $s^6+5s^5+4s^4+3s^3+s^2+4s - 4=0$

f) $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$

g) $s^7+9s^2+24s^5+24s^4+24s^2+23s+15=0$

Q2: Find the range of values for K in order to maintain a stable condition in each of the following system:

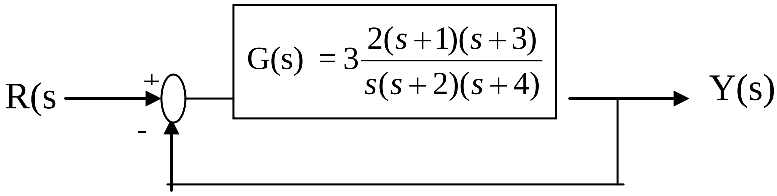
a) $s^3+34.5s^2+7500s+7500K=0$

b) $s^3+3Ks^2+(K+2)s+4=0$

c) $s^4+5s^3+5s^2+4s+K=0$

d) $s^3+3Ks^2+(K+2)s+4=0$

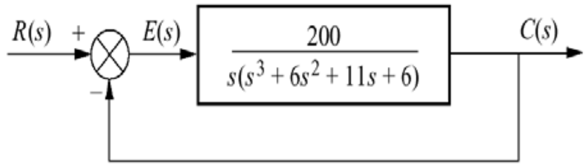
Q3: A single loop control system is shown in figure.

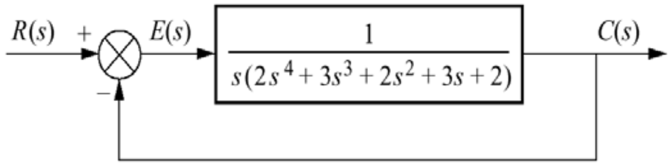


(a)Determine closed-loop transfer function $T(s) = \frac{Y(s)}{R(s)}$

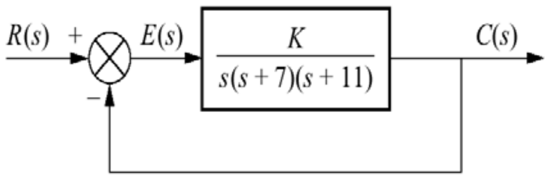
(b)Determine whether or not the systems given below are stable.

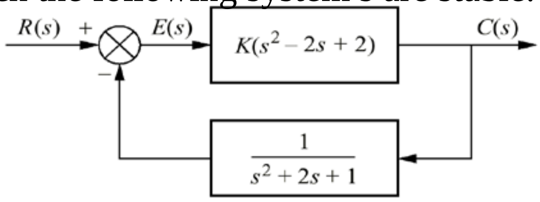
Q4: Determine whether or not the systems given below are stable.





Q5: Determine the range of values of K for which the following system s are stable.





Homework 13

Q1: Sketch Bode plot for the transfer function.

$G(s) = 10 / s(1+0.4s)(1+0.1s)$

Q2: the Bode plots for the following open-loop transfer function.

$G(S) = 0.75(1+0.2S)/ S(1+0.5S)(1+0.1S)$

Q3: Draw the Bode plots for the following open-loop transfer functions:

a) $G(s) = \dfrac{200}{s(1+0.5s)(1+0.1s)}$

b) $G(s) = \dfrac{0.2s^2}{(1+0.4s)(1+0.04s)}$

c) $G(s) = \dfrac{5(0.6s+1)}{s^2(4s+1)}$

d) $G(s) = \dfrac{0.1s(1+0.2s)}{s(s^2+16s+100)}$

e) $G(s) = \dfrac{10(s+3)}{s(2+s)(s^2+s+2)}$