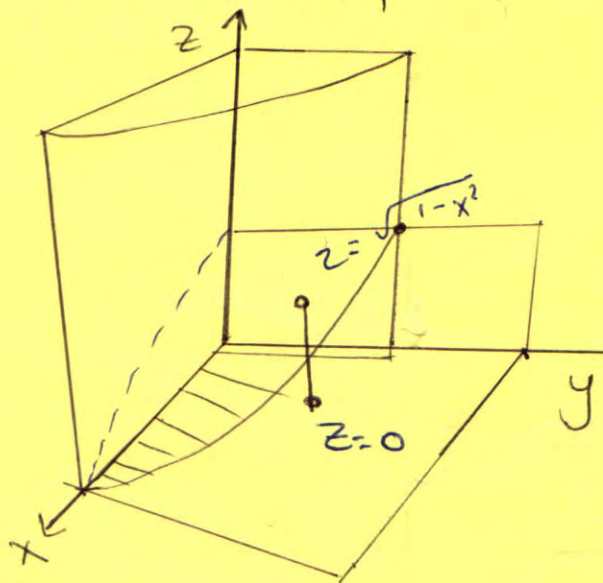H.W

① Find the volume of the region Common to the interior of cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$

② Evaluate $\int_0^1 \int_0^1 \int_{x^2}^1 12x z e^{zy^2} \, dy \, dx \, dz$

③ Find the average value of $(x, y, z) = xyz$ over the cube bounded by the coordinate planes and the planes $x = 2$, $y = 2$ and $z = 2$ in the first octant

④ The region of integration of the integral

$\int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} dz \, dy \, dx$ show in figure
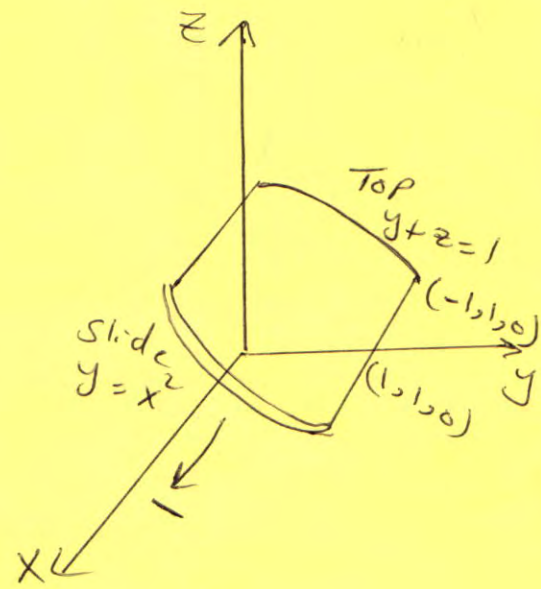
rewrite the integral as an equivalent interated
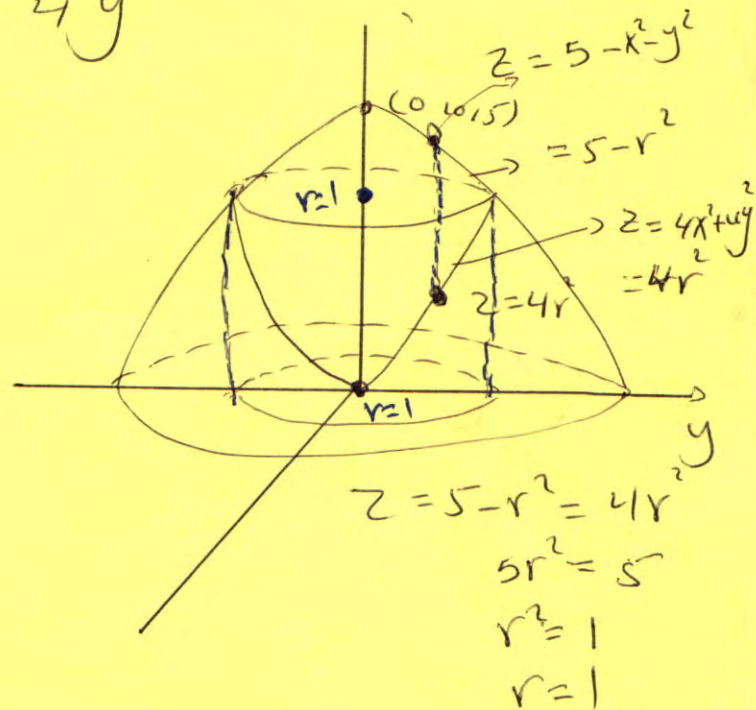
in the order (a) $dy\,dz\,dx$

(b) $dy\,dx\,dz$

(c) $dx\,dy\,dz$

(D) $dx\,dz\,dy$.

(e) $dz\,dx\,dy$

TOP
$y + z = 1$
$(-1, 1, 0)$
Slide
$y = x^2$
$(1, 1, 0)$

(5) Find the volume of the region bounded above by the Paraboloid $z = 5 - x^2 - y^2$ and below by the Paraboloid $z = 4x^2 + 4y^2$

$z = 5 - x^2 - y^2$
$(0, 0, 5)$
$= 5 - r^2$
$r = 1$
$z = 4x^2 + 4y^2$
$= 4r^2$
$z = 4r^2$
$r = 1$

$z = 5 - r^2 = 4r^2$
$5r^2 = 5$
$r^2 = 1$
$r = 1$

H.W   Double integrals
_____

① Evaluate the following integrals

$$\int_0^1 \int_x^1 \frac{1}{y} \sin\frac{x}{y} \cos\frac{x}{y} \, dy \, dx$$

② Find $\displaystyle\int_{-\infty}^{\infty} \int_0^{\infty} \frac{dy \, dx}{(4+3x^2+3y^2)^6}$

③ Evaluate $\displaystyle\int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} (x^2+y^2)^{3/2} \, dy \, dx$

④ Evaluate $\displaystyle\int_0^1 \int_{\sqrt{y}}^1 e^{5x^3} \, dx \, dy$

⑤ Evaluate $\displaystyle\int_0^1 \int_{x^2+1}^2 \frac{xy\,e^y}{y-1} \, dy \, dx$

⑥ Evaluate $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 \cos(4x^3+5) \, dx \, dy$

⑦ Evaluate $\displaystyle\int_0^{\pi} \int_0^{\sqrt{\pi-y^2}} \frac{x^2 y}{\sqrt{x^2+y^2}} \, dx \, dy$

⑧ Evaluate $\displaystyle\int_{y=0}^4 \int_{\sqrt{y}}^2 e^{x^3} \, dx \, dy$

⑨ Find volume in the first octant bounded by $z = 9-x^2$ from above and by $z=0$ from the

below and latterall by $y^2 = 3x$

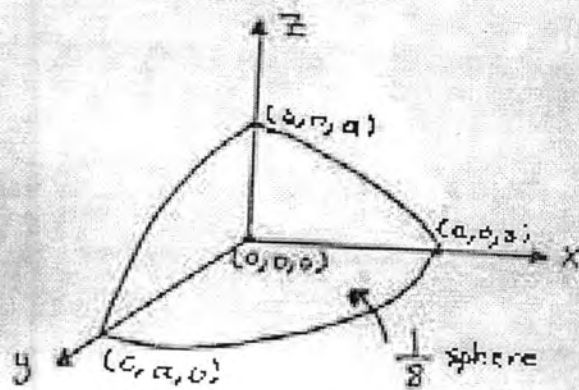below and latterall by $y^2 = 3x$

Double Integrals :

The equation of surface is $f(x,y,z)=0$ (or $z=f(x,y)$ which may be 1st order or 2nd order.
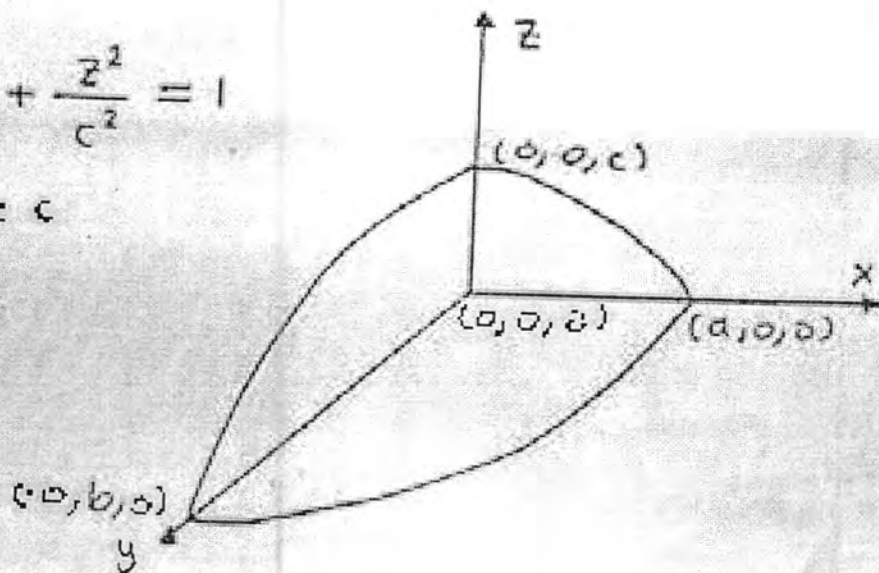
The Equations of Some Geometric Figures

1. Sphere

$$x^2 + y^2 + z^2 = a^2$$

$\frac{1}{8}$ Sphere

2. Ellipisoid

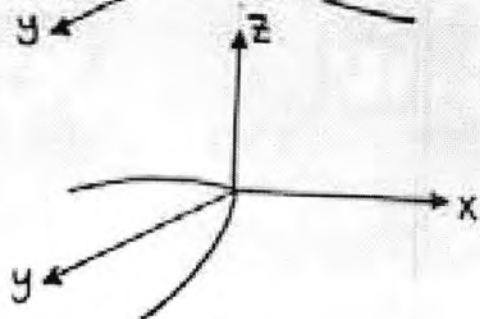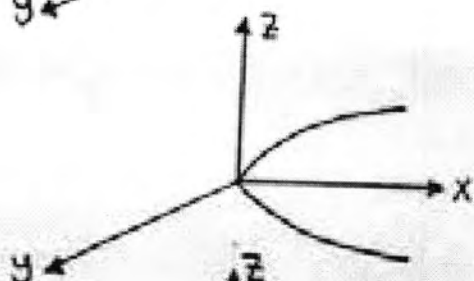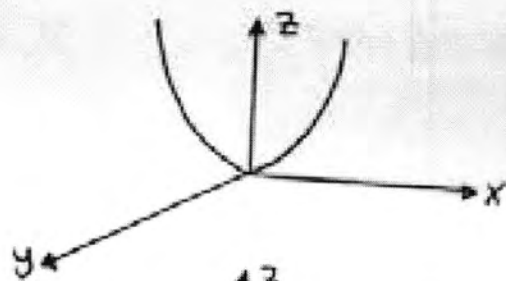$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$a \neq b \neq c$$

3. Paraboloid

$$z = x^2 + y^2 \quad ; \quad z \geqslant 0$$

$$x = z^2 + y^2 \quad ; \quad x \geqslant 0$$
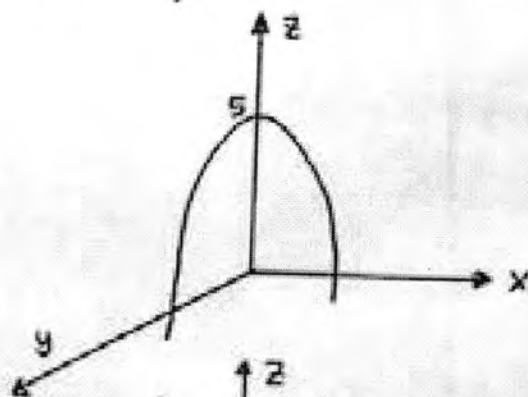
$$y = z^2 + x^2 \quad ; \quad y \geqslant 0$$
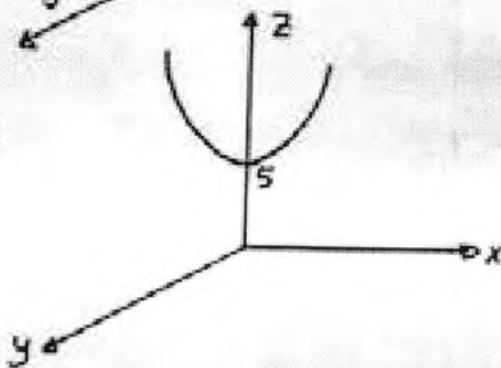
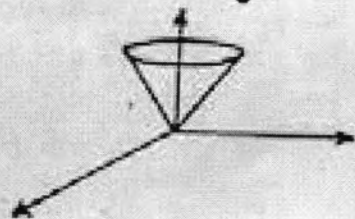ex. sketch $z = 5 - x^2 - y^2$

$$x^2 + y^2 = 5 - z$$

$$5 - z \geqslant 0$$

$$z \leqslant 5$$

ex. sketch $z = 5 + x^2 + y^2$

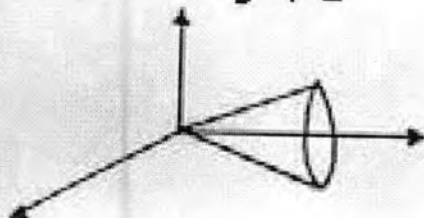$$z - 5 \geqslant 0 \quad \therefore \quad z \geqslant 5$$

4. Cone

$$z^2 = x^2 + y^2 \qquad x^2 = y^2 + z^2 \qquad y^2 = x^2 + z^2$$