Chapter Five: Controller Tuning

Chapter Eighteen in the textbook

The selection of a controller type (P, PI, and PID) and its parameters (intimately related to the model of the process to be controlled. The adjustment of the controller parameters to achieve satisfactory control is called tuning. The selection of the controller parameters is essentially an optimization problem in which the designer of the control system attempts to satisfy some criterion of optimality, the result of which is often referred to as "good" control. The process of tuning can vary from atrial-and-error attempt to find suitable control parameters for good control to an elaborate optimization calculation based on a model of the process and a specific criterion of optimal control. In many applications, there is no model of the process, and the criterion for good control is only vaguely defined. A typical criterion for good control is that the response of the system to a step change in set point or load has minimum overshoot and a one-quarter decay ratio. Other criteria may include minimum rise time and minimum settling time.

Controller Tuning

- Tuning is the adjustment of the controller parameters to obtain a specified closed-loop response. Controller tuning is the process of setting controller gains to achieve desired performance.
- After a control system is installed, the controller settings must usually be adjusted until the control system performance is considered to be satisfactory.
- The tuning goal is to determine the gains that optimize system response. A high gain increases responses but moves the system closer to instability.
- A low gain improves the stability but the system becomes sluggish.

Design and Tuning of a Control System

1. What should type of feedback controller be used to control a given process?

Some engineers select PI and others select PID. In all cases, the selection of K_c and τ_I has an important effect on the response of the controlled process.

2. How do we select the best values of the adjustable parameters of the feedback controller?

This is known as the controller tuning problem. We need to have a quantitative measure to compare the alternatives and then select the best type of controller and its parameters.

3. What performance criteria should be used for selecting and tuning the controller?

A variety of performance criteria should be used to keep the max derivation (Error) as small as possible by achieving a short settling time, minimizing the integral of errors until the process has settled to its set point, and keeping a low decay ratio.

Simple Performance Criteria

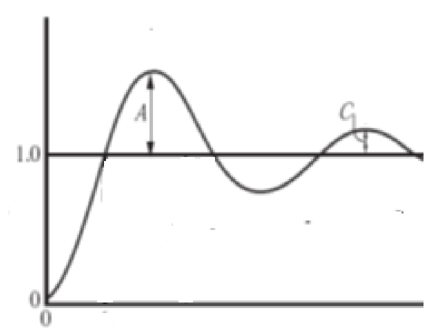
1. **Steady-state performance criteria**

The principal steady-state performance criteria usually are zero. So, the P control cannot achieve steady-state error while a PI can.

2. **Dynamic response performance criteria**

a. The criteria that use only a few points of the response are simpler but approximate.
b. The criteria that use the entire closed–loop response from t = 0 to very large are more precise but more cumbersome to use.
c. For the best criteria, the decay ratio, and the optimum value of DR is.

$$\frac{C}{A} = \frac{1}{4}$$



It is the reasonable tradeoff between a fast rise time and a reasonable settling time.

This criterion is usually known as the one-quarter decay ratio.

Example 1: Select the gain of a proportional controller (Kc) using the one-quarter decay ratio criterion (DR=0.25). The process is described by:

$$G_P(s) = \frac{10}{(s+2)(2s+1)}, \qquad \text{Assume } G_m(s) = G_f(s) = 1$$

Answer:

$$Y(s) = \frac{G_P G_v G_c}{1 + G_P G_v G_c G_m}$$

$$Gc = Kc$$

$$Y(s) = \frac{10Kc/[(s+2)(2s+1)}{1 + 10Kc/[(s+2)(2s+1)} Y_{sp}(s)$$

$$Y(s) = \frac{10Kc}{(s+2)(2s+1) + 10Kc} Y_{sp}(s)$$

$$Y(s) = \frac{10Kc}{2s^2 + 5s + 1 + 10Kc} Y_{sp}(s)$$

$$Y(s) = \frac{\frac{10Kc}{1+10Kc}}{\frac{2s^2}{1+10Kc} + \frac{5s}{1+10Kc} + 1} Y_{sp}(s)$$

$$\tau^2 = \frac{2}{1+10Kc} \quad \Rightarrow \quad \tau = \sqrt{\frac{2}{1+10Kc}}$$

$$2\,\xi \sqrt{\frac{2}{1+10Kc}} = \frac{5}{1+10Kc}$$

$$\xi = \frac{5}{2\sqrt{2}\sqrt{1+10Kc}} \rightarrow \xi = \frac{10K_c}{1+10K_c}$$

$$DR = 0.25 = \exp\left(\frac{-2\pi\psi}{\sqrt{1-\psi^2}}\right) \rightarrow \xi = 0.2154$$

$$\sqrt{1+10Kc} = 8.3848$$

Kc =6.93

Example 2: Find the gain of a proportional controller that produces a closed-loop response for a second-order system with decay ratio equal to 1/4. The process is described by:

$$G_P(s) = \frac{1}{s^2 + 3s + 1} \quad , \quad G_m(s) = G_f(s) = 1$$

Answer:

$$Y(s) = \frac{G_P G_v G_c}{1 + G_P G_v G_c G_m}$$

$$Y(s) = \frac{K_c \frac{1}{s^2 + 3s + 1}}{1 + K_c \frac{1}{s^2 + 3s + 1}}$$

$$Y(s) = \frac{\frac{K_c}{s^2 + 3s + 1}}{\frac{s^2 + 3s + 1 + K_c}{s^2 + 3s + 1}}$$

$$Y(s) = \frac{K_c}{s^2 + 3s + 1 + K_c}$$

$$Y(s) = \frac{\frac{K_c}{1 + K_c}}{\frac{s^2}{1 + K_c} + \frac{3}{1 + K_c} s + 1}$$

$$\tau^2 = \frac{1}{1 + K_c} \rightarrow \tau = \frac{1}{\sqrt{1 + K_c}}$$

$$2\xi\tau = \frac{3}{1 + K_c}$$

$$\xi = \frac{3\sqrt{1 + K_c}}{2(1 + K_c)} = \frac{3}{2\sqrt{1 + K_c}}$$

For DR=1/4, the $\xi = 0.2154$

$$0.2154 = \frac{3}{2\sqrt{1 + K_c}}$$

$$\sqrt{1 + K_c} = 6.9621 \rightarrow K_c = 47.4718$$

Controller Tuning

Process Reaction Curve (PRC) Method

It is an empirical rule which has been proven in practice and is known as the Cohen and Coon method. It applies only to open-loop processes that are inherently stable and suggests a method to first model the process in the open loop and then pick the appropriate control parameters. Noted that most responses have a "sigmoidal" response to a step change. The procedure of this method is to open the control system by disconnection the controller from the final control element, and introduce a step change of the magnitude M in the variable X which actuates the final control element to find the Transfer function between Y and X as follows:

$$G_{PRC} = \frac{Y(s)}{X(s)} = G_v(s)\, G_p(s) H(s)$$

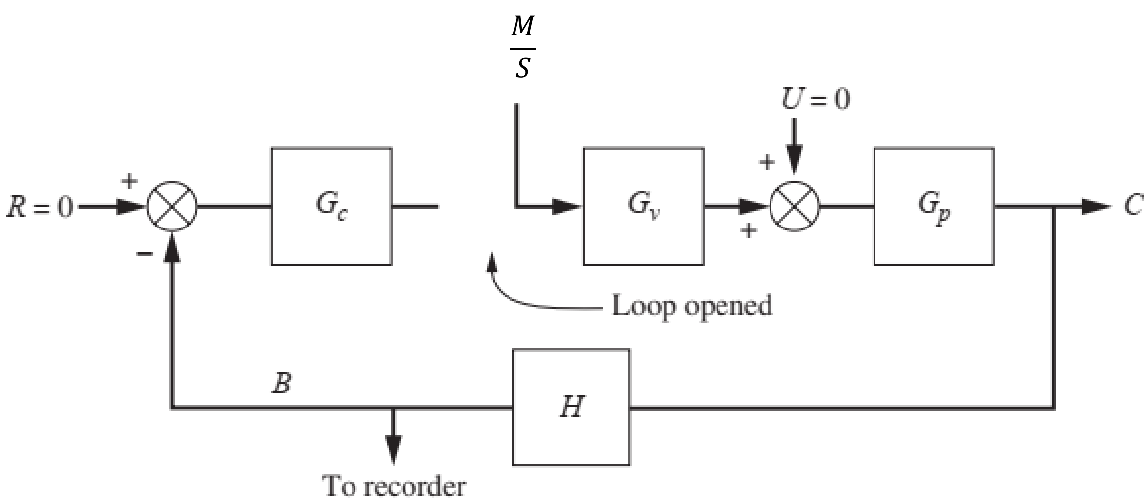


Figure 1: Block diagram of a control loop for measurement of a process reaction curve.

Cohen and coon suggested estimating the response of the above equation as a first order with time delay.

1. After the process reaches a steady state at the normal level of operation, switch the controller to manual. In a modern controller, the controller output will remain at the same value after switching as it had before switching. (This is called "bumpless" transfer.)

2. With the controller in the manual, introduce a small step change in the controller output that goes to the valve and records the transient, which is the process reaction curve (Figure 2).
3. Draw a straight line tangent to the curve at the point of inflection, as shown in Figure 2. The intersection of the tangent line with the time axis is the apparent transport lag t_d; the apparent first-order time constant τ is obtained from

$$\tau = \frac{B}{S} = \frac{\text{Output (at steady state)}}{\text{Slope}}$$

where B is the ultimate value and S is the slope of the tangent line. The steady-state gain that relates B to A in Figure 2 is given by

$$K_P = \frac{B}{A} = \frac{\text{Output (at steady state)}}{\text{Intput (at steady state)}}$$

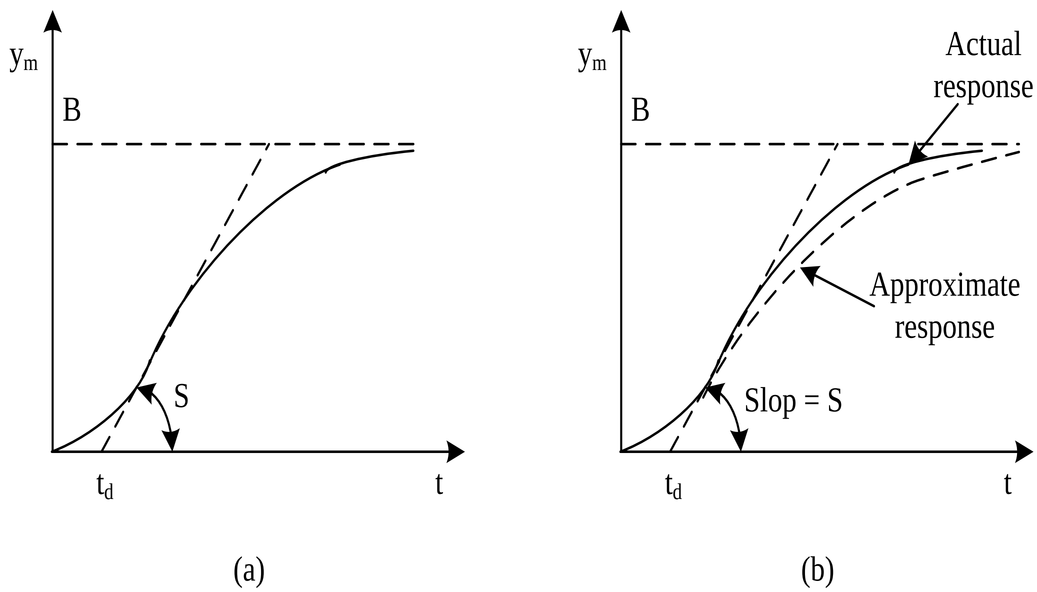


Figure 2: Typical process reaction curve showing graphical construction to determine first-order with transport lag model.

4. Using the values of K_P, T, and t_d from step 3, the controller settings are found from the relations given in equations below. All the controller settings are a function of the

dimensionless group t_d/τ, the ratio of the apparent transport lag to the apparent time constant. Also, K_c is inversely proportional to K_P.

According to Cohen and Coon, most processes will have a response to this change that may be approximated as a first-order system with dead time. This model is First order pulse dead-time (FOPDT):

$$G_{PRC} = \frac{Y(s)}{X(s)} \approx \frac{Ke^{-t_d s}}{\tau s + 1}$$

Cohen and coon used the approximated model of Equation 2 and estimated $K_P, \tau,$ and t_d. So, the derived expression of the best controller setting using the load change and variation criteria, such as one-quarter decay ratio, minimum offset, and minimum square error (ISE).

Cohen-Coon controller settings are as follows:

For Proportional controller:

$$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(1 + \frac{t_d}{3\tau}\right)$$

For Proportional-Integral controller:

$$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(0.9 + \frac{t_d}{12\tau}\right)$$

$$\tau_I = t_d\left(\frac{30 + 3t_d/\tau}{9 + 20t_d/\tau}\right)$$

For Proportional-Integral-Derivative controller:

$$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(\frac{4}{3} + \frac{t_d}{4\tau}\right)$$

$$\tau_I = t_d\left(\frac{32 + 6t_d/\tau}{13 + 8t_d/\tau}\right)$$

$$\tau_D = t_d\left(\frac{4}{11 + 2t_d/\tau}\right)$$

Advantages of the C-C or process reaction curve method:

- Only a single experimental test is required.
- It does not require trial and error.
- The controller settings are easily calculated.

Disadvantages of the C-C or process reaction curve method:

- The experimental test is performed under open-loop conditions. Thus if a significant load change occurs during the test, no corrective action is taken and the test results may be significantly distorted.
- It may be difficult to determine the slope at the inflection point accurately, especially if the measurement is noisy and a small recorder is used.
- The method tends to be sensitive to controller calibration errors. By contrast, the Z-N method is less sensitive to controller errors in Kp since the controller gain is adjusted during the experimental test.
- This method is not recommended for processes that have oscillatory open-loop responses.

Example 3: Consider the system with the following Transfer functions:

$G_p = \frac{1}{(5s+1)(2s+1)}$ $G_m = \frac{1}{10s+1}$ $G_f = 1$

Determine the optimum control elements using the PRC method.

$$G_{PRC} = G_f G_p G_m = \frac{1}{(5s+1)(2s+1)} \times \frac{1}{(10s+1)} \times 1 \Rightarrow \frac{1}{(5s+1)(2s+1)(10s+1)}$$

Solution:

Take Laplace inverse to the equation above and then draw Y(t) against t using partial fraction of the above equations.

For unit step change in X(s)

$$X(s) = \frac{1}{s}$$

$$Y(s) = \frac{1}{s}\frac{1}{(2s+1)(5s+1)}\frac{1}{(10s+1)} \qquad (3)$$

Solving equation (3) by partial fraction

$$\frac{1}{s(2s+1)(5s+1)(10s+1)} = \frac{\alpha_o}{s} + \frac{\alpha_1}{2s+1} + \frac{\alpha_2}{5s+1} + \frac{\alpha_3}{10s+1}$$

$$1 = \alpha_o(2s+1)(5s+1)(10s+1) + \alpha_1 s(5s+1)(10s+1) + \alpha_2 s(2s+1)(10s+1) + \alpha_3 s(2s+1)(5s+1)$$

For s=0 then $\alpha_o = 1$

For s=-1/2 then $1 = \alpha_1(-\frac{1}{2})(-5/2+1)(-10/2+1)$

$$1 = \alpha_1(-\frac{1}{2})(-\frac{3}{2})(-4)$$

$$\alpha_1 = -\frac{1}{3}$$

For s=-1/5 then $1 = \alpha_2 s(2s+1)(10s+1)$

$$1 = \alpha_2(-\frac{1}{5})(-\frac{2}{5}+1)(-\frac{10}{5}+1)$$

$$1 = \alpha_2(-\frac{1}{5})(\frac{3}{5})(-1)$$

$$\alpha_2 = \frac{25}{3}$$

For s=-1/10 then $1 = \alpha_3 s(2s+1)(5s+1)$

$$1 = \alpha_3(-\frac{1}{10})(-\frac{2}{10}+1)(-\frac{5}{10}+1)$$

$$1 = \alpha_3(-\frac{1}{10})(\frac{8}{10})(\frac{5}{10})$$

$$\alpha_3 = -25$$

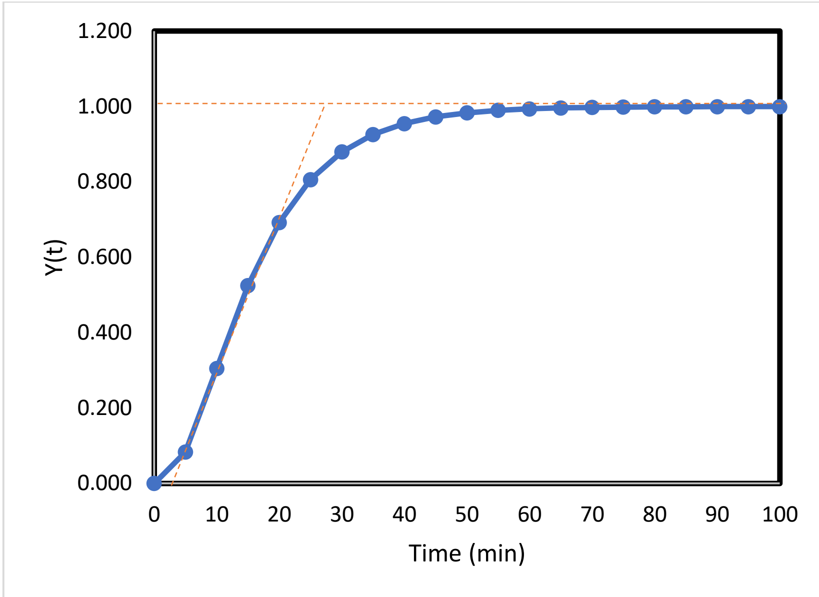
$$Y(s) = \frac{1}{s} - \frac{\frac{1}{3}}{2s+1} + \frac{\frac{25}{3}}{5s+1} - \frac{25}{10s+1}$$

$$Y(s) = \frac{1}{s} - \frac{\frac{1}{6}}{s+\frac{1}{2}} + \frac{\frac{5}{3}}{s+\frac{1}{5}} - \frac{2.5}{s+\frac{1}{10}}$$

$$Y(t) = 1 - \frac{1}{6}e^{-\frac{t}{2}} + \frac{5}{3}e^{-\frac{t}{5}} - 2.5e^{-\frac{t}{10}}$$

Then draw Y(t) against t by using partial fraction of the above eqn.

Time (min)	Y(t)
0	0.000
5	0.083
10	0.305
15	0.525
20	0.692
25	0.806
30	0.880
35	0.926
40	0.955
45	0.972
50	0.983
55	0.990
60	0.994
65	0.996
70	0.998
75	0.999
80	0.999
85	0.999
90	1.000
95	1.000
100	1.000



Then $G_{PRC} = \frac{Ke^{-t_d s}}{\tau s+1}$ From the figure above,

$K = \text{gain} = \frac{B}{A} = \frac{1}{1} = 1$

S = slope at the inflection point = 0.05

B = Ultimate response = 1.0

τ = effective time constant = $\frac{B}{S} = \frac{1}{0.05} = 20$

t_d = delay time = 2.5

$$\therefore G_{PRC} = \frac{1.0e^{-2.5s}}{20s+1}$$

Using Cohen and Coon settings

For Proportional controller:

$$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(1 + \frac{t_d}{3\tau}\right) = \frac{1}{1} \times \frac{20}{2.5}\left(1 + \frac{2.5}{3 \times 20}\right) = 8.33$$

For PI controller:

$$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(0.9 + \frac{t_d}{12\tau}\right) = \frac{1}{1} \times \frac{20}{2.5}\left(0.9 + \frac{2.5}{12\times 20}\right) = 7.28$$

$$\tau_I = t_d\left(\frac{30+3t_d/\tau}{9+20t_d/\tau}\right) = 2.5\left(\frac{30+3\times 2.5/20}{9+20\times 2.5/20}\right) = 6.6$$

For PID controller:

$$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(\frac{4}{3} + \frac{t_d}{4\tau}\right) = 10.9$$

$$\tau_I = t_d\left(\frac{32 + 6t_d/\tau}{13 + 8t_d/\tau}\right) = 5.85$$

$$\tau_D = t_d\left(\frac{4}{11 + 2t_d/\tau}\right) = 0.89$$

Example 4: Table 1 shows the experimental process reaction curve of the open loop system with a PI controller. Using the data in given in the Table 1:

Table 1: Input and output variables with respect to time

Time (min)	-10	-5	0	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90
Manipulated input	10	10	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
Measurement of output	0.650	0.650	0.650	0.651	0.652	0.668	0.735	0.817	0.881	0.979	1.075	1.151	1.213	1.239	1.262	1.311	1.329	1.338	1.350	1.350	1.350

Table 2: Cohen and Coon Controller Settings.

	P	PI	PID
K_c	$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(1 + \frac{t_d}{3\tau}\right)$	$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(0.9 + \frac{t_d}{12\tau}\right)$	$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(\frac{4}{3} + \frac{t_d}{4\tau}\right)$
τ_I	-	$\tau_I = t_d\left(\frac{30 + 3t_d/\tau}{9 + 20t_d/\tau}\right)$	$\tau_I = t_d\left(\frac{32 + 6t_d/\tau}{13 + 8t_d/\tau}\right)$
τ_D	-	-	$\tau_D = t_d\left(\frac{4}{11 + 2t_d/\tau}\right)$

Solution:

$A = 15 - 10 = 5$

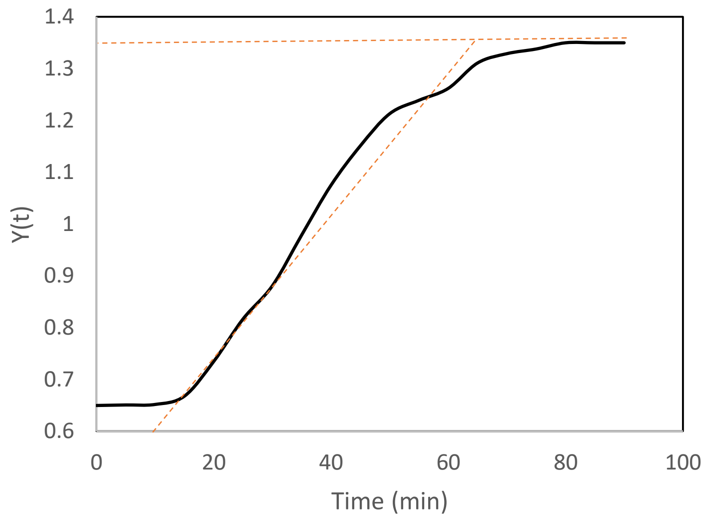
$B = 1.35 - 0.65 = 0.7$

$K = \frac{B}{A} = \frac{0.7}{5} = 0.14$

$S = \frac{1.35 - 0.65}{60 - 14} = 0.0152$

$\tau = \frac{B}{S} = \frac{0.7}{0.0152} = 46 \text{ min}$

$t_d = 14 \text{ min}$



$$G_{PRC} = \frac{0.14e^{-14s}}{46\,s + 1}$$

$$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(0.9 + \frac{t_d}{12\tau}\right) = \frac{1}{0.14} \times \frac{46}{14}\left(0.9 + \frac{14}{12 \times 46}\right) = 21.71$$

$$\tau_I = t_d\left(\frac{30 + 3t_d/\tau}{9 + 20t_d/\tau}\right) = 14\left(\frac{30 + 3 \times 14/46}{9 + 20 \times 14/46}\right) = 28.68 \text{ min}$$

Example 5: Consider the feedback control system for the stirred-tank blending process shown in Fig. 1 and the following step test. The controller was placed in manual, and then its output was suddenly changed from 30% to 43%. The resulting process reaction curve is shown in Figure (2). Thus, after the step change occurred at t = 0, the measured exit composition changed from 35% to 55% (expressed as a percentage of the measurement span), which is equivalent to the mole fraction changing from 0.10 to 0.30. Select the PID controller settings using the Cohen-Coon technique.

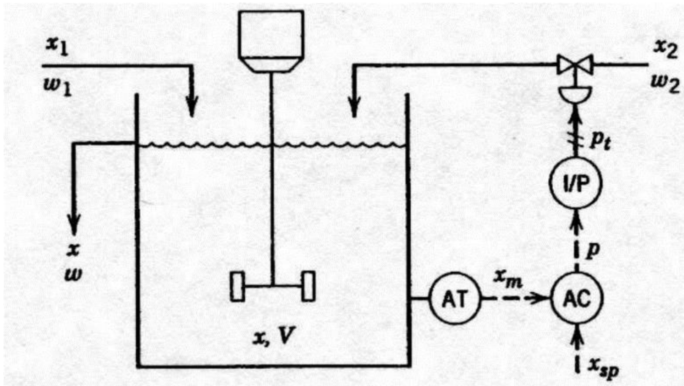


Figure 1: Composition control system for a stirred-tank blending process.

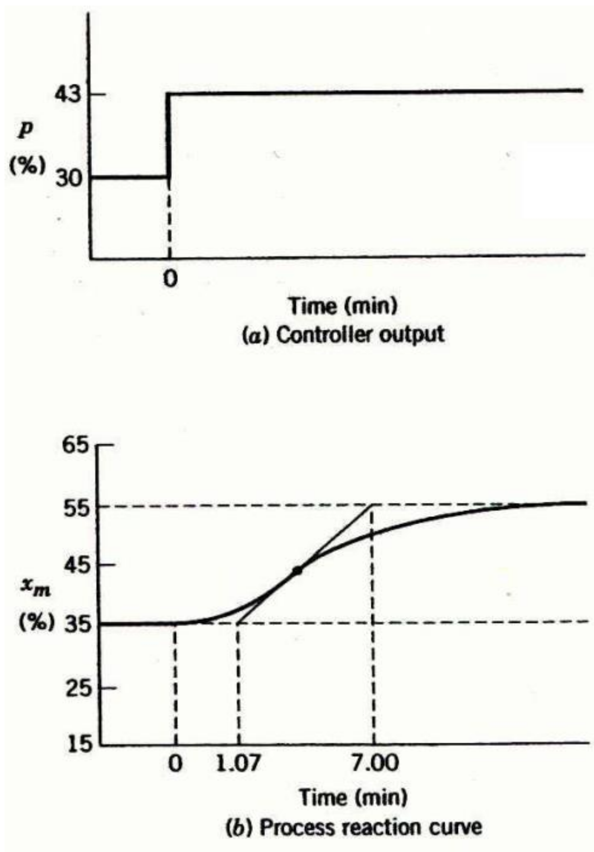


Figure 2: Process reaction curve.

Cohen and Coon Controller Settings

	P	PI	PID
K_c	$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(1+\frac{t_d}{3\tau}\right)$	$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(0.9+\frac{t_d}{12\tau}\right)$	$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(\frac{4}{3}+\frac{t_d}{4\tau}\right)$
τ_I	-	$\tau_I = t_d\left(\frac{30+3t_d/\tau}{9+20t_d/\tau}\right)$	$\tau_I = t_d\left(\frac{32+6t_d/\tau}{13+8t_d/\tau}\right)$
τ_D	-	-	$\tau_D = t_d\left(\frac{4}{11+2t_d/\tau}\right)$

- t_d = time delay

Solution:

A first-order-plus-time-delay model can be developed from the process reaction curve in Figure (2) using the graphical method. The tangent line through the inflection point intersects the horizontal lines for the initial and final composition values at 1.07 min and 7.00 min, respectively.

$$S = \frac{\Delta Y}{\Delta X} = \frac{55 - 35}{7 - 1.07} = 3.3726\ \%/\text{min}$$

$$K = \frac{\text{Output}}{\text{Input}} = \frac{B}{A} = \frac{55 - 35}{43 - 30} = \frac{20}{13} = 1.5384\ \%/\%$$

$$\tau_d = 1.07\ \text{min}$$

$$\tau = \frac{B}{S} = \frac{20}{3.3726} = 5.93\ \text{min}$$

The resulting empirical process model can be expressed as:

$$\frac{X(s)}{P(s)} = G_{prc} = \frac{1.538e^{-1.07s}}{5.93s + 1}$$

$$K_c = \frac{1}{K}\frac{\tau}{t_d}\left(\frac{4}{3} + \frac{t_d}{4\tau}\right) = \frac{1}{1.538} \times \frac{5.93}{1.07}\left(\frac{4}{3} + \frac{1.07}{4 \times 5.93}\right) = 4.967$$

$$\tau_I = t_d\left(\frac{32 + 6t_d/\tau}{13 + 8t_d/\tau}\right) = 1.07\left(\frac{32 + 6 \times 1.07/5.93}{13 + 8 \times 1.07/5.93}\right) = 2.54\ \text{min}$$

$$\tau_D = t_d\left(\frac{4}{11 + 2t_d/\tau}\right) = 1.07\left(\frac{4}{11 + 2 \times 1.07/5.93}\right) = 0.3767\ \text{min}$$

Ziegler-Nichols (Z-N) Method

These rules were first proposed by Ziegler and Nichols (1942), who were engineers for a major control hardware company in the United States (Taylor Instrument Co.). Based on their experience with the transients from many types of processes, they developed a closed-loop tuning method still used today in one form or another. The method is described as a closed-loop method because the controller remains in the loop as an active controller in automatic mode. This closed-loop method will be contrasted with an open-loop tuning method to be discussed later. We have already discussed the ZieglerNichols rules in Chapter 16 as a natural consequence of our study of frequency response. Ziegler and Nichols did not suggest that the ultimate gain Kcu and ultimate period Pu be computed from frequency response calculations based on the model of the process. They intended that Kcu and Pu be obtained from a closed-loop test of the actual process. When the rules were first proposed, frequency response methods and process models were not generally available to the control engineers. The rules are presented below and are in the form that one would use for actual application to a real process.

1. After the process reaches a steady state at the normal level of operation, remove the integral and derivative modes of the controller, leaving only proportional control. On some PID controllers, this requires that the integral time tI be set to its maximum value and the derivative time t_D to its minimum value. On computer-based controllers, the integral and derivative modes can be removed completely from the controller.
2. Select a value of proportional gain Kc, disturb the system, and observe the transient response. If the response decays, select a higher value of Kc and again observe the response of the system. Continue increasing the gain in small steps until the response first exhibits a sustained oscillation. The value of gain and the period of oscillation that correspond to the sustained oscillation are the ultimate gain Kcu and the ultimate period Pu.

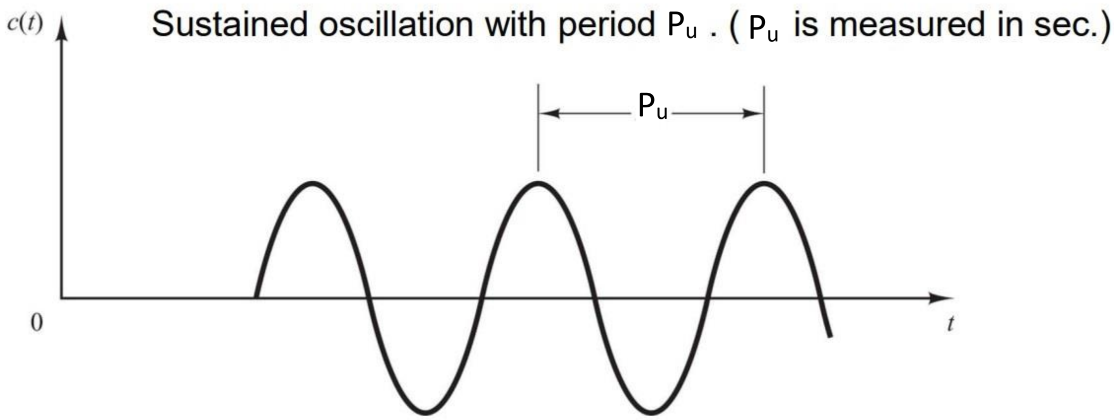
Some very important precautions to take in applying this step of the tuning method are given in the next section.

3. From the values of Kcu and Pu found in step 2, use the Ziegler-Nichols rules given in Table 2 to determine controller settings (K_c, t_I, and t_D). (This table is the same as Table 16.1.)

Although variations in the tuning rules given in Table 2 are used by the industry, the same approach of using Kcu and Pu to obtain controller parameters is used. The Ziegler-Nichols rules generally provide conservative (and safe) controller settings. The Z-N settings should be considered as only approximate settings for satisfactory control. Fine-tuning of the controller settings is usually required to get an improved control response. Ziegler-Nicholas (Z-N) method is a popular method for tuning P, PI, and PID controllers. The Z-N controller tuning method is developed by J.G. Zielger and N.B. Nichols, and is pseudo-standard in the control field. The ZN setting are benchmarks against which the performances of other controller settings are compared in many studies. This method starts by zeroing the integral and differential gains and then raising the proportional gain until the system is unstable. The value of Kp at the point of instability is called K max the frequency of oscillation is f0. The method of Ziegler and Nichols known as the continuous cycling method. In this method, integration and derivative terms of the controller are disabled and the proportional gain is increased until a continuous oscillation occurs at ultimate gain K_{cu} for the closed loop system. Considering K_{cu} and its related oscillating ultimate period, P_u , the controller parameters can be calculated from the equation bellow.

$$K_{cu} \Rightarrow \text{from the procedure of the continuous cycling method}$$

$$P_u = \frac{2\pi}{w} \qquad \text{when } (w) = \text{frequency}$$



For Proportional controller:

$$K_c = \frac{K_{cu}}{2}$$

For Proportional-Integral controller:

$$K_c = \frac{K_{cu}}{2.2}$$

$$\tau_I = \frac{P_u}{1.2}$$

For Proportional-Integral-Derivative controller:

$$K_c = \frac{K_{cu}}{1.7}$$

$$\tau_I = \frac{P_u}{2}$$

$$\tau_D = \frac{P_u}{8}$$

Also, Ziegler-Nichols used another method bode diagram of two graphs: one is a plot of the logarithm of the magnitude of sinusoidal transfer function; the other is a plot of phase angle; both are plotted against the frequency on a logarithm scale as shown in Figure (2). Gain margin (GM) and crossover frequency (ω) can be found from two plots therefore, the ultimate gain and period of oscillation are calculated from following:

$K_u = 20\log(GM)$

$P_u = \frac{2\pi}{\omega}$

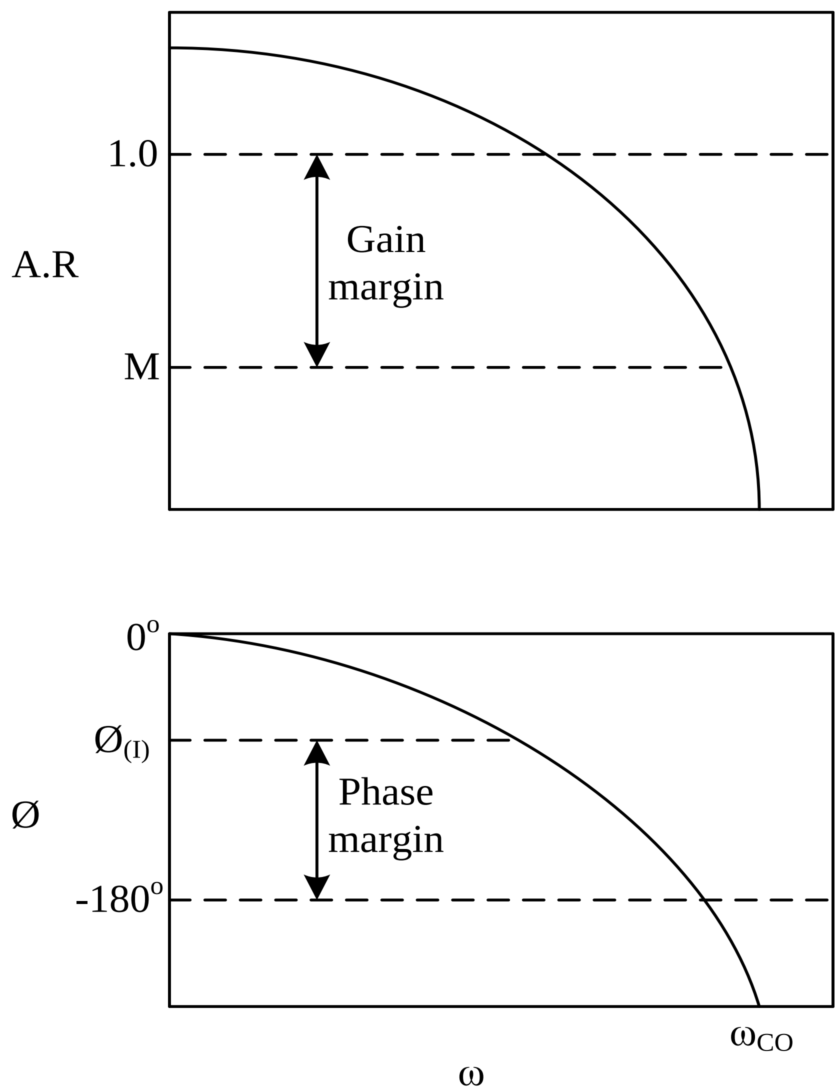


Figure 2: Definition of gain and phase margins.

Example 6: Determine the Ziegler – Nichols tuning parameters for a PID controller, with the given transfer functions. Assume that the time constants have units of minutes.

$G_v = \frac{5}{2S+1}$ $G_m = \frac{0.4}{5S+1}$ $G_p = \frac{2}{S+1}$

Solution:

Continuous cycling method

$1 + G_{loop} = 0$

$1 + G_v G_m G_p G_c = 0$

Put $G_c = K_{cu}$

$$1 + \left(\frac{5}{2S+1} \times \frac{0.4}{5S+1} \times \frac{2}{S+1} \times K_{cu}\right) = 0$$

$$1 + \left(\frac{4\,K_{cu}}{(2S+1)(5S+1)(S+1)}\right) = 0 \rightarrow \frac{(2S+1)(5S+1)(S+1)}{(2S+1)(5S+1)(S+1)} + \frac{4\,K_{cu}}{(2S+1)(5S+1)(S+1)} = 0$$

$10\,S^3 + 17\,S^2 + 8\,S + 1 + 4\,K_{cu} = 0$

Put S=iw, and i= -1

$10\,(iw)^3 + 17\,(iw)^2 + 8\,(iw) + 1 + 4\,K_{cu} = 0$

$-10\,iw^3 - 17\,w^2 + 8\,iw + 1 + 4\,K_{cu} = 0$

Real: $-17\,w^2 + 1 + 4\,K_{cu} = 0$

Imaginary: $-10\,iw^3 + 8\,iw = 0$

$10\,iw^3 = 8\,iw$

$w^2 = 0.8 \rightarrow w = 0.894$

$-17 \times 0.8 + 1 + 4\,K_{cu} = 0$

$K_{cu} = 3.15$

$$P_u = \frac{2\pi}{w} = \frac{2\pi}{0.894} = 7.03 \text{ min}$$

For K_c, τ_I, and τ_D

$$K_c = \frac{K_{cu}}{1.7} = \frac{3.15}{1.7} = 1.85$$

$$\tau_I = \frac{P_u}{2} = \frac{7.03}{2} = 3.52 \text{ min}$$

$$\tau_D = \frac{P_u}{8} = \frac{7.03}{8} = 0.879 \text{ min}$$

Tuning Relations Based on Integral Error Criteria

- Controller tuning relations have been developed that optimize the closed-loop response for a simple process model and a specified disturbance or set-point change.
- The optimum settings minimize an integral error criterion.
- Three popular integral error criteria are:

  The main three methods of the integral error performance criteria used in terms of:

✓ **Integral of the absolute value of the error (IAE)**

$$\mathrm{IAE} = \int_0^{\infty} |\mathrm{e(t)}| \, \mathrm{dt}$$

where the error signal e(t) is the difference between the set point and the measurement.

✓ **Integrated Square Error (ISE)**

This error uses the square of the error, thereby penalizing large errors more than small errors. This gives more conservative response (faster return to set point).

$$ISE = \int_0^{\infty} e^2 \, dt$$

✓ **Integrated Time-Weighted Absolute Error (ITAE)**

This criterion is based on the integral of the absolute value of the error multiplied by time. It results in errors existing over time being penalized even though may be small, which results in a more heavily damped response.

$$ITAE = \int_0^{\infty} t \, |e| \, dt$$

If the performance indices increases, control system can perform poorly and even become unstable. So it needs to tune the controller parameters to achieve good control performance with the proper choice of tuning constants. Also, they derived expression for the best controller setting using load change and variation criteria, such as **One quarter decay Ratio, Minimum offset** and **Minimum square error (ISE)**