Chapter One: Control System

Chapter Eight in the textbook

Introduction

In previous chapters, the dynamic behavior of several basic systems was examined. With this background, we can extend the discussion to a complete control system and introduce the fundamental concept of feedback. To work with a familiar system, the treatment will be based on a stirred-tank heater.

Figure 1 is a sketch of the apparatus. A liquid stream at a temperature T_i enters an insulated, well-stirred tank at a constant flow rate w (mass/time). It is desired to maintain (or control) the temperature in the tank at T_R by means of the controller. If the measured tank temperature T_m differs from the desired temperature T_R, the controller senses the difference or error $\varepsilon = T_R - T_M$ and changes the heat input in such a way as to reduce the magnitude of ε. If the controller changes the heat input to the tank by an amount that is proportional to ε, we have proportional control.

In Figure 1, it is indicated that the source of heat input q may be electricity or steam. If an electrical source were used, the final control element might be a variable transformer that is used to adjust current to a resistance heating element; if steam were used, the final control element would be a control valve that adjusts the flow of steam. In either case, the output signal from the controller should adjust q in such a way as to maintain control of the temperature in the tank.

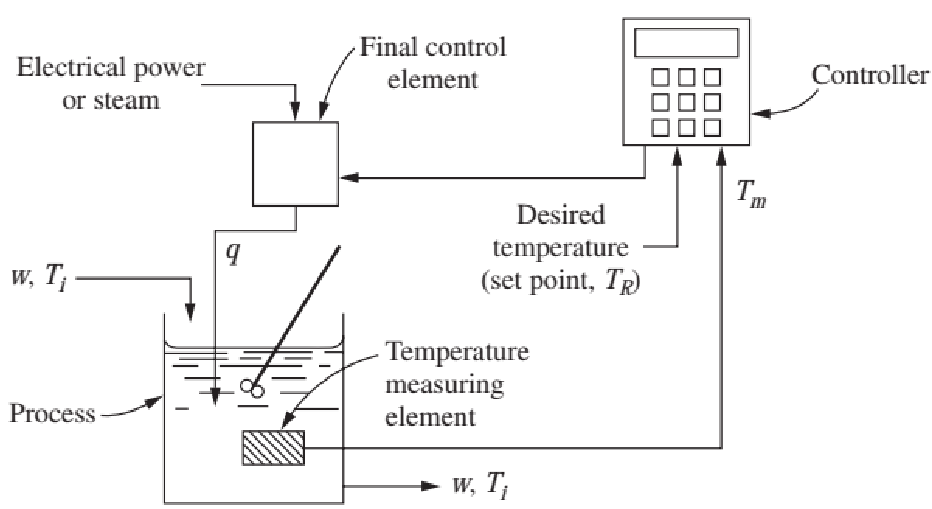


Figure 1: Control system of a stirred tank heater.

Components of a Control System

The system shown in Figure 1 may be divided into the following components:

1. Process (stirred-tank heater).
2. Measuring element (thermometer).
3. Controller.
4. Final control element (variable transformer or control valve).

Each of these components can be readily identified as a separate physical item in the process and will constitute most of the control systems.

Block Diagram

For computational purposes, it is convenient to represent the control system of Figure 1 by means of the block diagram as shown in Figure 2. Such a diagram makes it much easier to visualize the relationships among the various signals. New terms, which appear in Figure 2, are set point and load. The set point is a synonym for the desired value of the controlled variable while the load refers to a change in any variable that may cause the controlled variable of the process to change. In this example, the inlet temperature T_i is a load variable. Other possible loads for this system are changes in flow rate and heat loss from the tank. (These loads are not shown on the diagram.)

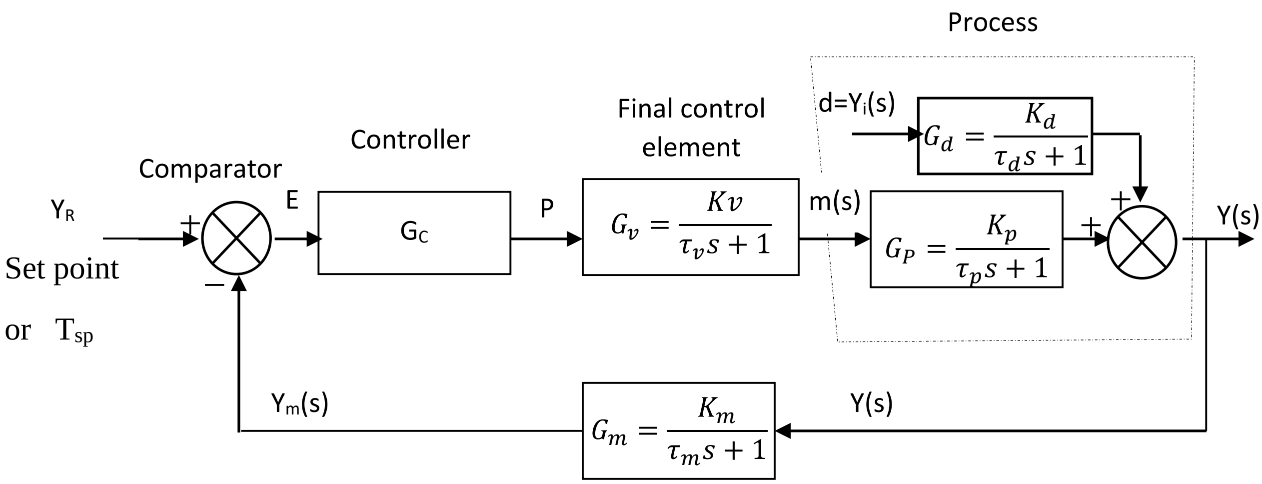


Figure 2: Closed loop block diagram of first order system.

$$Y(s) = G_d\, d(s) + G_p\, m(s)$$

For a stirred tank heater,

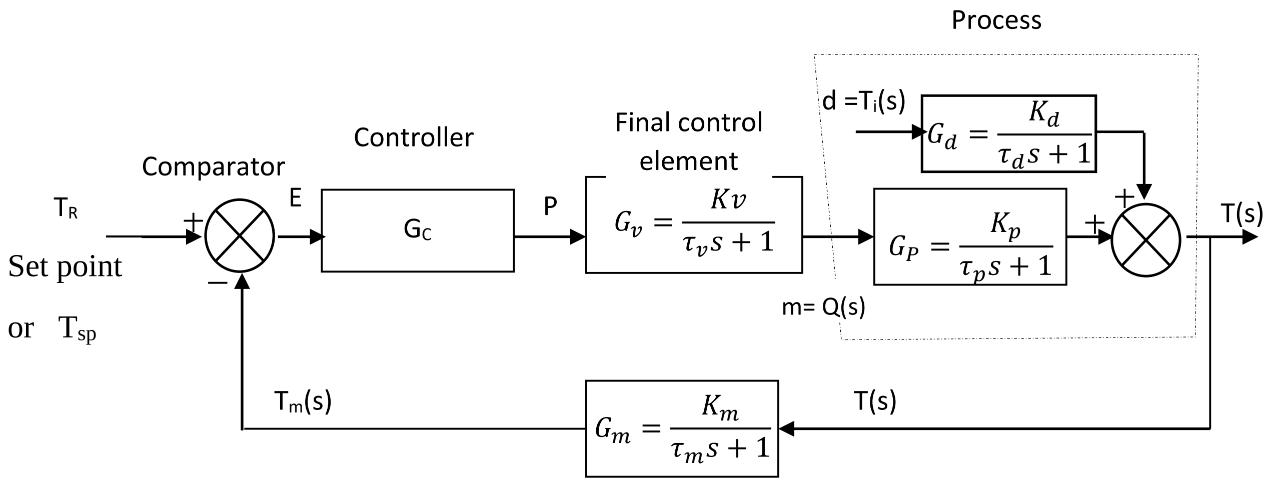


Figure 2: Closed loop block diagram of first order system for a stirred tank heater.

$$T(s) = G_d\, T_i(s) + G_p\, Q(s)$$

$$\text{Controlled variable} = G_d \times \text{Disturbance variable} + G_p \times \text{Manipulated variable}$$

The control system shown in Figure 2 is called a closed-loop system or a feedback system because the measured value of the controlled variable is returned or “fed back” to a device called the comparator. In the comparator, the controlled variable is compared with the desired value or set point. If there is any difference between the measured variable and the set point, an error is generated. This error enters a controller, which in turn adjusts the final control element to return the controlled variable to the set point.

Negative Feedback Versus Positive Feedback

Several terms have been used that may need further clarification. The feedback principle, which is illustrated in Figure 2, involves the use of the controlled variable *T* to maintain itself at a desired value T_R. The arrangement of the apparatus of Figure 2 is often described as negative feedback to contrast with another arrangement called positive feedback. Negative feedback ensures that the difference between T_R and T_m is used to adjust the control element so that the tendency is to reduce the error. For example, assume that the system is at a steady state and that $T = T_m = T_R$. If the load T_i should increase, T and T_m would start to increase, which would cause the error to become negative. With proportional control, the decrease in error would cause the controller and final control element to decrease the flow of heat to the system, with the result that the flow of heat would eventually be reduced to a value such that T approaches T_R. Actually, all the components operate simultaneously, and the only adequate description of what is occurring is a set of simultaneous differential equations.

If the signal to the comparator were obtained by adding T_R and T_m, we would have a positive feedback system, which is inherently unstable. To see that this is true, again assume that the system is at a steady state and that $T = T_m = T_R$. If T_i were to increase, T and T_m would increase, which would cause the signal from the comparator (ε in Figure 2) to increase, with the result that the heat to the system would increase. However, this action, which is just the opposite of that needed, would cause T to increase further. It should be clear that this situation would cause T to "runaway" and control would not be achieved. For this reason, positive feedback would never be used intentionally in the system of Figure 2.

Servo Problem Versus Regulator Problem

The control system of Figure 2 can be considered from the point of view of its ability to handle either of two types of situations. In the first situation, which is called the servomechanism-type (or servo) problem, we assume that there is no change in the load T_i and that we are interested in changing the bath temperature according to some prescribed function of time. For this problem, the set point T_R would be changed in accordance with the desired variation in bath temperature. If the variation is sufficiently slow, the bath temperature may be expected to follow the variation in T_R very closely. There are occasions when a control system in the chemical industry will be operated in this manner. For example, one may be interested in varying the temperature of a reactor according to a prescribed time-temperature pattern. However, the majority of problems that may be described as the servo type come from fields other than the chemical industry. The servo problem can be viewed as trying to follow a moving target (i.e., the changing set point). The other situation will be referred to as the regulator problem. In this case, the desired value T_R is to remain fixed, and the purpose of the control system is to maintain the controlled variable at T_R in spite of changes in load T_i. This problem is very common in the chemical industry, and a complicated industrial process will often have many self-contained control systems, each of which maintains a particular process variable at a desired value. These control systems are of the regulator type.

We will frequently discuss the response of a linear control system to a change in set point (servo problem) separately from the response to a change in load (regulator problem). However, it should be realized that this is done only for convenience. The basic approach to obtaining the response of either type is essentially the same, and the two responses may be superimposed to obtain the response to any linear combination of set point and load changes.

Development of Block diagram

Each block in Figure 2 represents the functional relationship existing between the input and output of a particular component. Finally, the blocks are combined to give the overall block diagram. This is the procedure to be followed in developing Figure 2.

Process

the procedure for developing the transfer function remains the same. An unsteady-state energy balance around the tank gives

$$q + wC(T_i - T_o) - wC(T - T_o) = \rho CV \frac{dT}{dt}$$

At steady state, dT/dt is zero.

$$q_s + wC(T_{i_s} - T_o) - wC(T_s - T_o) = 0$$

$$q - q_s + wC\left[(T_i - T_{i_s}) - (T - T_s)\right] = \rho CV \frac{d(T - T_s)}{dt}$$

$$T_i' = T_i - T_{i_s}$$

$$Q = q - q_s$$

$$T' = T - T_s$$

$$Q + wC(T_i' - T') = \rho CV \frac{dT'}{dt}$$

Taking the Laplace transform

$$Q(s) + wC[T_i'(s) - T'(s)] = \rho CVsT'(s)$$

or

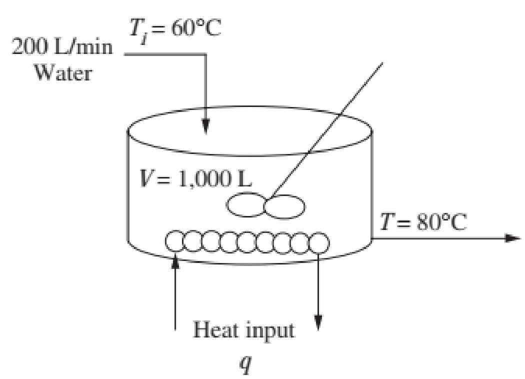
$$T'(s)\left(\frac{\rho V}{w}s + 1\right) = \frac{Q(s)}{wC} + T_i'(s)$$

This last expression can be written for the stirred heater transfer function:

$$T'(s) = \frac{1/wC}{\tau s + 1}Q(s) + \frac{1}{\tau s + 1}T_i'(s)$$

Example 1: Stirred-tank heater model.

a) Determine the response of the outlet temperature of the tank to a step change in the inlet temperature from 60° to 70°C.
b) Determine the response of the outlet temperature of the tank to a step increase in the heat input of 42 kW.
c) Determine the response of the outlet temperature of the tank to a simultaneous step change in the inlet temperature from 60° to 70°C and a step increase in the heat input of 42 kW.



Solution:

The energy balance for the stirred-tank heater is:

$$T'(s) = \frac{1/wC}{\tau s + 1}Q(s) + \frac{1}{\tau s + 1}T_i'(s)$$

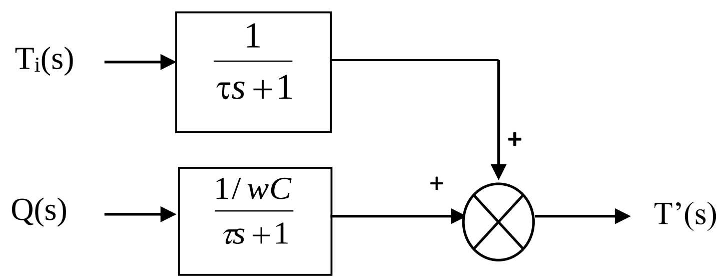
Substituting numerical values for the variables, we obtain the actual transfer unction for this stirred-tank heater.

$$\tau = \frac{\rho V}{w} = \frac{V}{w/\rho} = \frac{V}{v} = \frac{\text{tank volume}}{\text{volumetric flow rate}} = \frac{1{,}000 \text{ L}}{200 \text{ L/min}} = 5 \text{ min}$$

$$\frac{1}{wC} = \frac{1}{\left(\frac{200 \text{ L}}{\text{min}}\right)\left(\frac{1 \text{ kg}}{\text{L}}\right)\left(\frac{4.184 \text{ kJ}}{\text{kg}\cdot°\text{C}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = \frac{1}{14} \frac{°\text{C}}{\text{kJ/s}} = \frac{°\text{C}}{14 \text{ kW}}$$

$$T'(s) = \frac{1}{5s+1} T_i'(s) + \frac{1}{5s+1}\left(\frac{°\text{C}}{14 \text{ kW}}\right) Q(s)$$

The block diagram for the tank is shown in the figure below:



Remember that $T_i'(s)$, T'(s), and Q(s) are deviation variables:

$$T' = T - 80$$
$$T_i' = T_i - 60$$
$$Q = q - q_s$$

The steady-state heat input qs may be found from the steady-state energy balance:

$$q_s + wC(T_{is} - T_o) - wC(T_s - T_o) = 0$$

$$q_s = wC(T_s - T_{is}) = \left(200 \frac{\text{kg}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(4.184 \frac{\text{kJ}}{\text{kg}\cdot°\text{C}}\right)(80°\text{C} - 60°\text{C}) = 280 \text{ kW}$$

So,

$$Q = q - q_s = q - 280 \text{ kW}$$

(a) If the inlet temperature is stepped from 60° to 70°C, then T'(t) = 70 - 60 = 10 °C and T'(s) = $\frac{10}{s}$. Note that Q = 0.

So,

$$T'(s) = \frac{10}{s} \cdot \frac{1}{5s + 1}$$

Taking Laplace inverse,

$$T'(t) = 10(1 - e^{-t/5})$$

and finally, we obtain the expression for T(t), the actual tank outlet temperature.

$$T(t) = T_s + T'(t) = 80 + 10(1 - e^{-t/5})$$

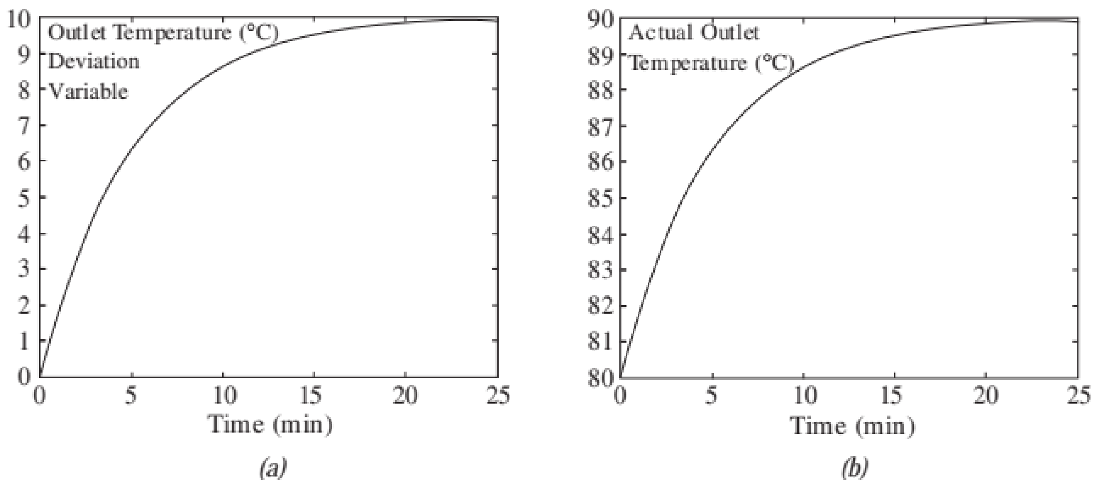


Figure: Outlet temperature from the stirred-tank heater. (a) Deviation variable; (b) actual temperature.

(b) For a step increase in the heat input of 42 kW, Q (s) = $\frac{42}{s}$. Note that T' = 0 for this case.

The expression for T' is given by

$$T'(s) = \frac{42}{s} \cdot \frac{1}{14} \frac{1}{5s + 1}$$

Taking Laplace inverse,

$$T'(t) = 3(1 - e^{-t/5})$$

and finally, we obtain the expression for T(t), the actual tank outlet temperature.

$$T(t) = T_s + T'(t) = 80 + 3(1 - e^{-t/5})$$

(c) If both changes occur simultaneously, the expression for T'(t) is given by:

$$T'(s) = \underbrace{\frac{42}{s} \cdot \frac{1}{14} \frac{1}{5s+1}}_{\Delta T' \text{ due to change in heat input}} + \underbrace{\frac{10}{s} \cdot \frac{1}{5s+1}}_{\Delta T' \text{ due to change in inlet temperature}}$$

Taking Laplace inverse,

$$T'(t) = 13(1 - e^{-t/5})$$

$$T(t) = T_s + T'(t) = 80 + 13(1 - e^{-t/5})$$

Measuring Element

The temperature measuring element, which senses the bath temperature T and transmits a signal Tm to the controller, may exhibit some dynamic to be first-order. In this example, we will assume that the temperature measuring element is a first-order system, for which the transfer function is

$$G_m = \frac{T'_m(s)}{T'(s)} = \frac{1}{\tau_m s + 1}$$

where the input-output variables T' and T_m' are deviation variables, defined as

$$T' = T - T_s$$

$$T'_m = T_m - T_{m_s}$$

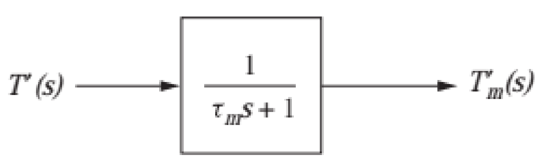


Figure: Block diagram of measuring element.

Example 2:

The temperature sensing element for the stirred-tank heater in Example 1 is a thermocouple. The manufacturer's specifications state that the thermocouple has a response time of 45 s (with the response time defined by the manufacturer as the time required for the thermocouple's reading to be 90 percent complete after a step change). Assuming that the thermocouple behaves as a first-order system, determine the transfer function for the temperature measuring element.

Solution:

The model for the sensor is first-order. If $T'(s) = 1/s$, a unit-step change, then the sensor response is

$$T'_m(s) = \frac{1}{s}\left(\frac{1}{\tau_m s + 1}\right)$$

Taking Laplace inverse,

$$T'_m(t) = (1 - e^{t/\tau_m})$$

Since the ultimate value of T_m' is 1, we know from the manufacturer's specifications that we can expect the response to be 90 percent complete at $t = 45$ s, which enables us to determine τ_m.

$$0.9 = (1 - e^{45\text{ s}/\tau_m})$$

$$\frac{45\text{ s}}{\tau_m} = 2.303$$

$$\tau_m = 19.5\text{ s} = 0.33\text{ min}$$

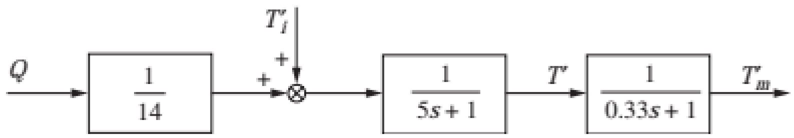
So, Therefore, the transfer function relating the actual temperature in the tank T' to the measured or indicated temperature T' is

$$\frac{T'_m(s)}{T'(s)} = \frac{1}{0.33s + 1}$$

Example 3:

For a step change in the inlet temperature to the stirred tank of 10 ^oC (no change in heat input Q (s)=0), plot the actual tank temperature and the temperature indicated by the thermocouple in Example 2 as a function of time.

Solution:



Block diagram for stirred-tank heater and measuring element.

From Example 2,

$$T'(t) = 10(1 - e^{-t/5})$$

$$T'_m(s) = \frac{10}{s(0.33s+1)(5s+1)} = \frac{0.71}{s+3.03} - \frac{10.71}{s+0.2} + \frac{10}{s}$$

Taking Laplace inverse,

$$T'_m(t) = 10 + 0.71e^{-3.03t} - 10.71e^{-0.2t}$$

Plotting T' and T_m', we obtain the figure below. From the graphs, it is clear that T_m', lags behind T'.

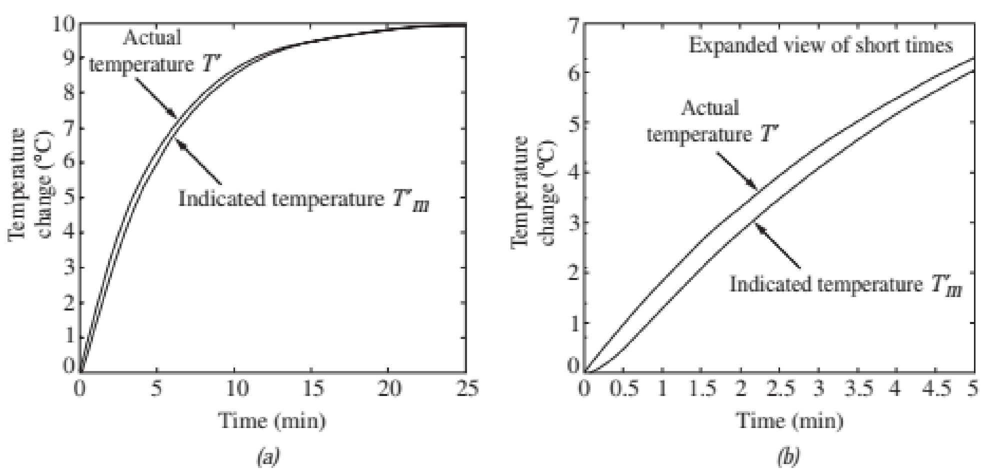


Figure: Comparison of temperature response for measured and actual temperatures in the stirred- tank heater. (*a*) Complete response; (*b*) expanded time scale for short times.

Problems

8.1. The two-tank heating process shown in Fig. P8–1 consists of two identical, well-stirred tanks in series. A flow of heat can enter tank 2. At time $t = 0$, the flow rate of heat to tank 2 suddenly increases according to a step function to 1,000 Btu/min, and the temperature of the inlet water T_i drops from 60 to 52°F according to a step function. These changes in heat flow and inlet water temperature occur simultaneously.

(*a*) Develop a block diagram that relates the outlet temperature of tank 2 to the inlet temperature to tank 1 and the flow of heat to tank 2.

(*b*) Obtain an expression for $T_2'(s)$ where T_2' is the deviation in the temperature of tank 2. This expression should contain numerical values of the parameters.

(*c*) Determine $T_2(2)$ and $T_2(\infty)$.

(*d*) Sketch the response $T_2'(t)$ versus t.

Initially, $T_i = T_1 = T_2 = 60$°F and $q = 0$. The following data apply:
$w = 250$ lb/min
Holdup volume of each tank = 5 ft^3
Density of fluid = 50 lb/ft^3
Heat capacity of fluid = 1 Btu/(lb · °F)

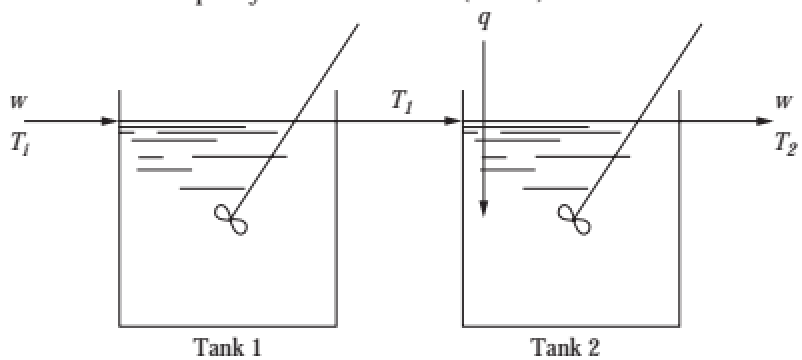


FIGURE P8–1

8.2. The two-tank heating process shown in Fig. P8–2 consists of two identical, well-stirred tanks in series. At steady state, $T_a = T_b = 60°F$. At time $t = 0$, the temperature of each stream entering the tanks changes according to a step function, that is, $T_a' = 10u(t)$ and $T_b' = 20u(t)$ where T_a' and T_b' are deviation variables.

(*a*) Develop the block diagram that relates T_2', the deviation in temperature in tank 2, to T_a' and T_b'.

(*b*) Obtain an expression for $T_2'(s)$.

(c) Determine $T_2(2)$.

The following data apply:

$w_1 = w_2 = 250$ lb/min

Holdup volume of each tank = 10 ft^3

Density of fluid = 50 lb/ft^3

Heat capacity of fluid = 1 Btu/(lb · °F)

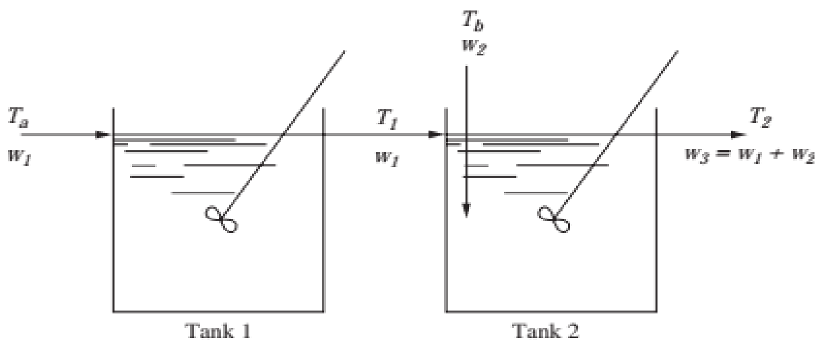


FIGURE P8–2

8.3. The heat transfer equipment shown in Fig. P8–3 consists of two tanks, one nested inside the other. Heat is transferred by convection through the wall of the inner tank. The contents of each tank are well mixed. The following data and information apply:

1. The holdup volume of the inner tank is 1 ft^3. The holdup of the outer tank is 1 ft^3.
2. The cross-sectional area for heat transfer between the tanks is 1 ft^2.
3. The overall heat-transfer coefficient for the flow of heat between the tanks is 10 Btu/(h · ft^2 · °F).
4. The heat capacity of fluid in each tank is 1 Btu/(lb · °F). The density of each fluid is 50 lb/ft^3.

Initially the temperatures of the feed stream to the outer tank and the contents of the outer tank are equal to 100°F. The contents of the inner tank are initially at 100°F. At time zero, the flow of heat to the inner tank Q is changed according to a step change from 0 to 500 Btu/h.

(*a*) Obtain an expression for the Laplace transform of the temperature of the inner tank $T(s)$.

(*b*) Invert $T(s)$ and obtain T for time = 0, 5 h, 10 h, and ∞.

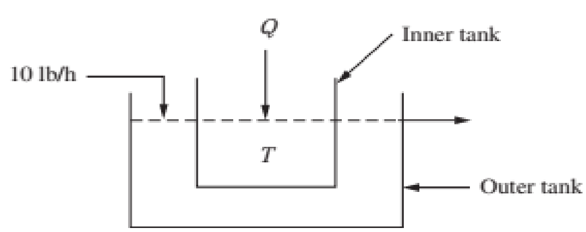


FIGURE P8–3