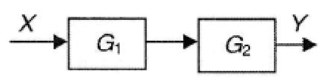
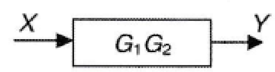
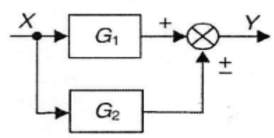
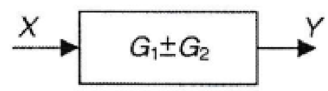
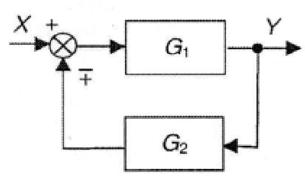
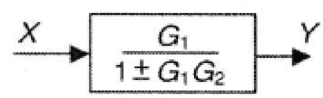
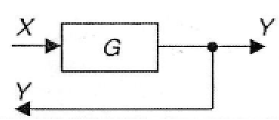
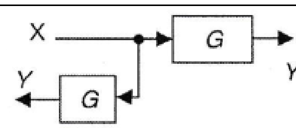
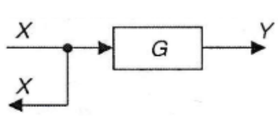
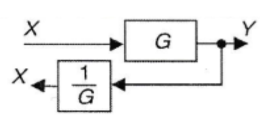
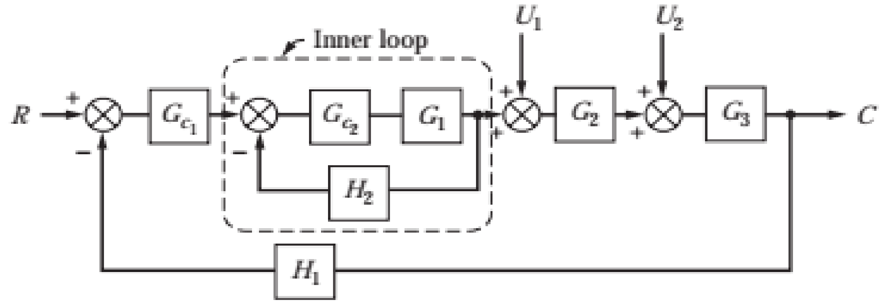
Chapter six: Block Diagram Reduction of a Control System

Chapter Eleven in the textbook

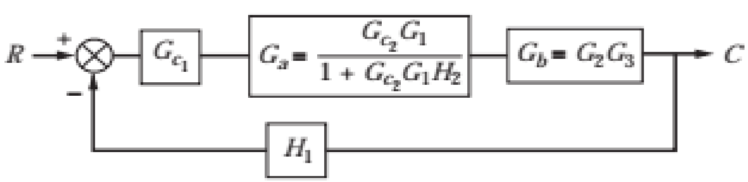
This chapter will discuss reducing the block diagram of a control system to a single block that relates one input to one output variable. The procedure consists of writing, directly from the block diagram, a sufficient number of linear algebraic equations and solving them simultaneously for the transfer function of the desired pair of variables.

Transformation	Equation	Block Diagram	Equivalent Block Diagram
Combining blocks in series	$Y=(G_1G_2)\,X$		
Combining blocks in parallel	$Y=G_1X \pm G_2X$		
Eliminating a feedback loop	$Y=G_1(X \pm G_2Y)$		
Moving a point ahead a block	$Y=GX$		
Moving a point beyond a block	$Y=GX$		

Example 1: Determine the transfer function C/R for the system shown in the following figure. This block diagram represents a cascade control system, which will be discussed later.



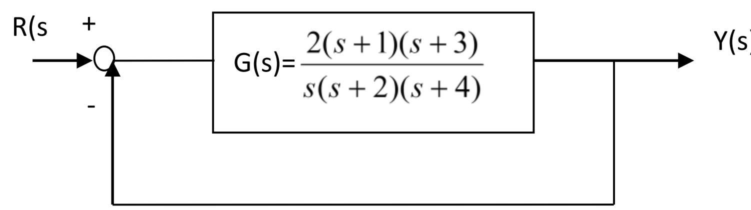
(a)



(b)

(c)

Example 2: A single-loop control system is shown in the figure below. Determine the closed-loop transfer function $\frac{Y(s)}{R(s)}$.



Solution:

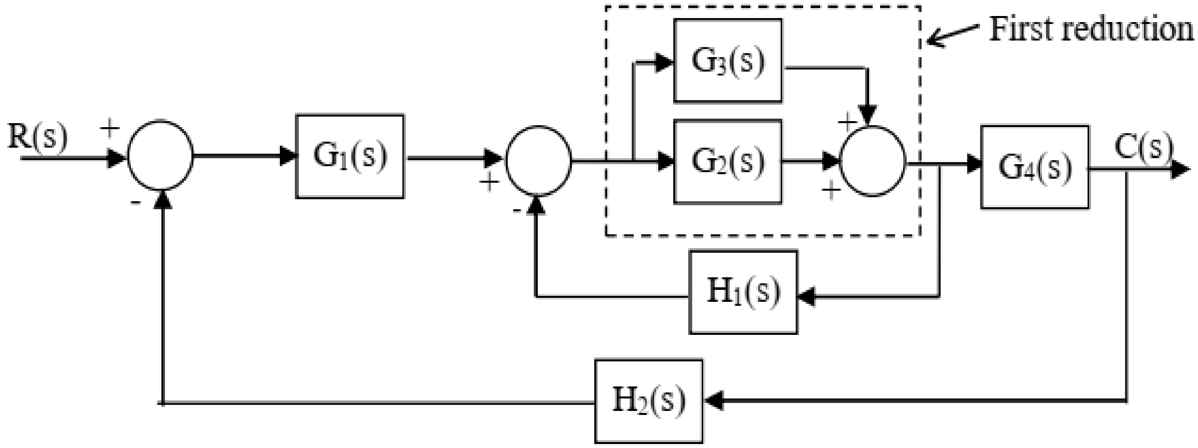
Transfer function $\frac{Y(s)}{R(s)} = \frac{G}{1+GH}$

$$\frac{Y(s)}{R(s)} = \frac{\frac{2(s+1)(s+3)}{s(s+2)(s+4)}}{1+\frac{2(s+1)(s+3)}{s(s+2)(s+4)}*1} = \frac{\frac{2(s+1)(s+3)}{s(s+3)(s+4)}}{\frac{s(s+2)(s+4)+2(s+1)(s+3)}{s(s+2)(s+4)}}$$

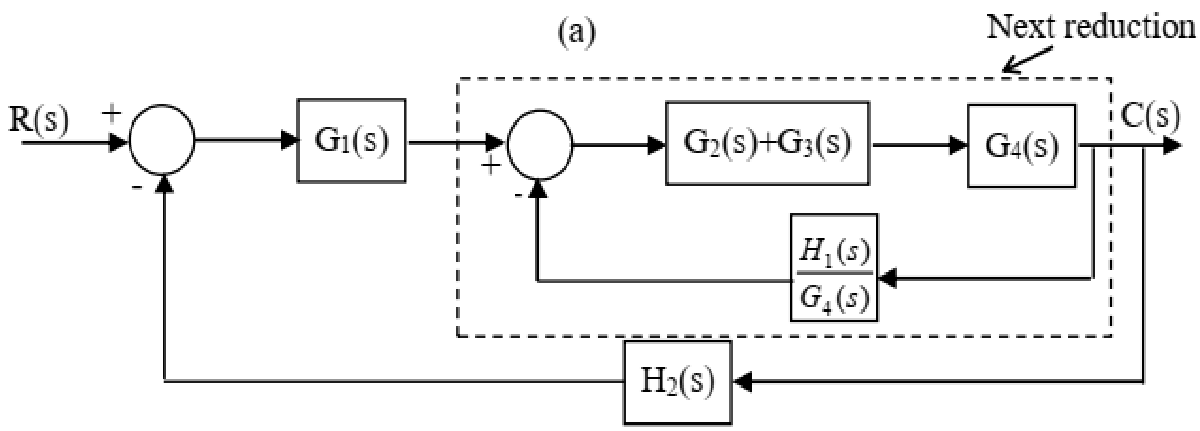
$$\frac{2(s+1)(s+3)}{s(s+2)(s+4)+2(s+1)(s+3)} = \frac{2s^2+6s+2s+6}{s^3+4s^2+2s^2+8s+2s^2+6s+2s+6}$$

$$\therefore \frac{Y(s)}{R(s)} = \frac{2s^2+8s+6}{s^3+8s^2+16s+6} \rightarrow \frac{Y(s)}{R(s)} = \frac{\frac{1}{3}s^2+\frac{4}{3}s+1}{\frac{1}{6}s^3+\frac{4}{3}s^2+\frac{8}{3}s+1}$$

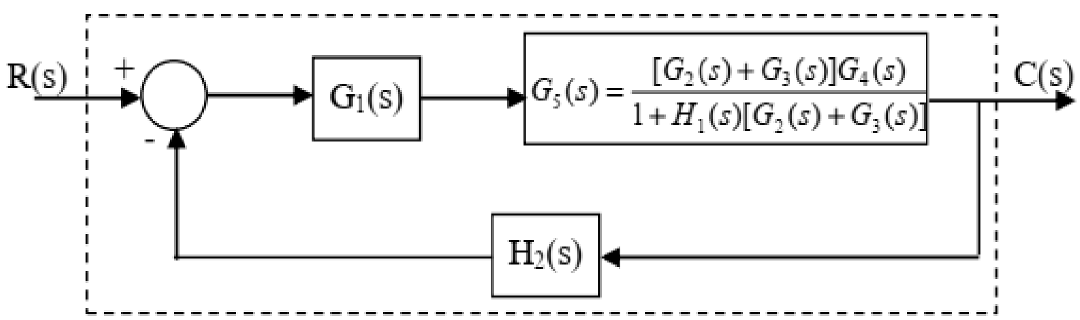
Example 3: Reduce the following block diagram.



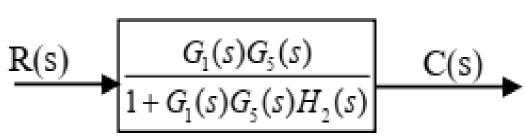
(a)



(b)

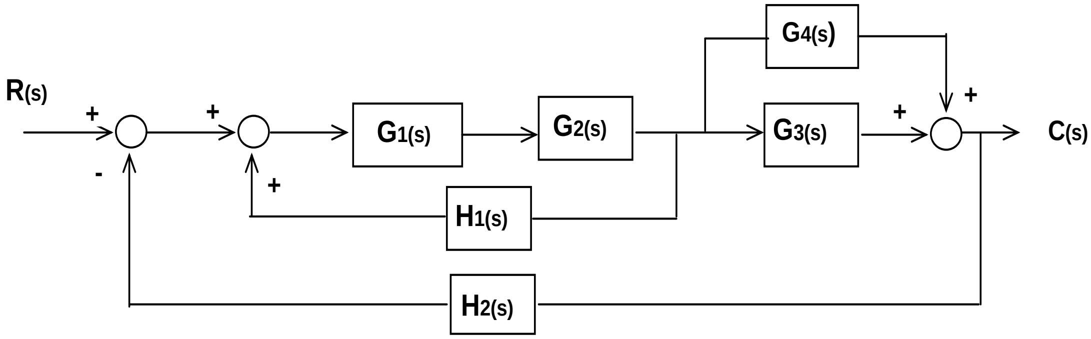


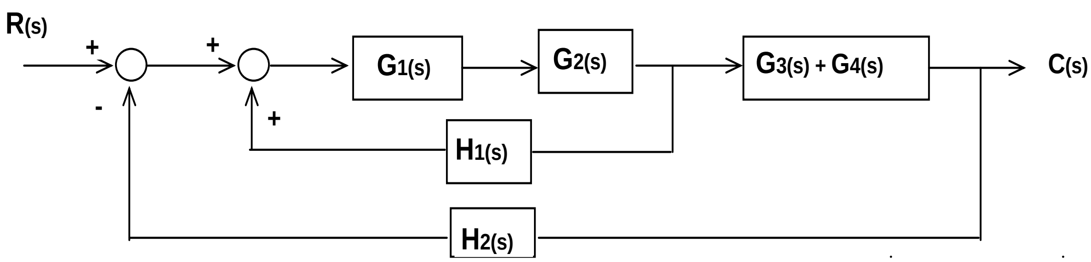
(c)

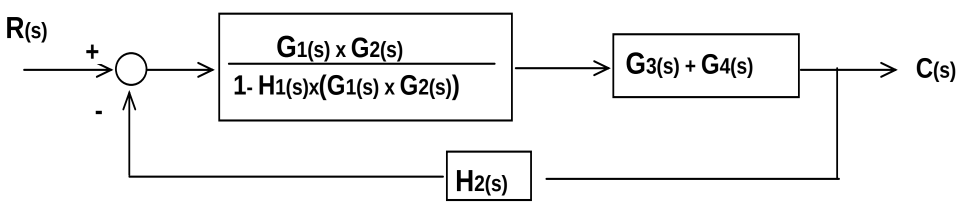


(d)

Example 4: Find the transfer function C/R of the block diagram shown in the figure below.





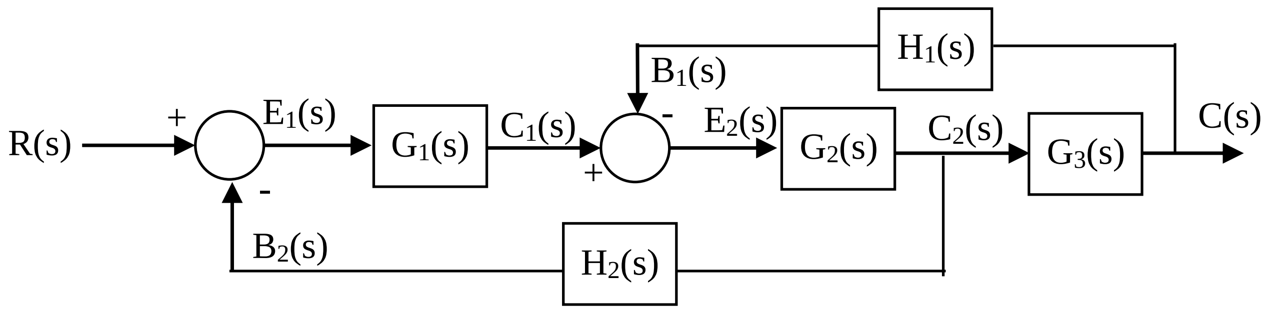


$$\frac{C(s)}{R(s)} = \frac{\dfrac{G_1(s).G_2(s)\left(G_3(s)+G_4(s)\right)}{1-H_1(s).G_1(s).G_2(s)}}{1+H_2(s)\left[\dfrac{G_1(s).G_2(s).\left(G_3(s)+G_4(s)\right)}{1-H_1(s).G_1(s).G_2(s)}\right]}$$

$$= \frac{\dfrac{G_1(s).G_2(s)\left(G_3(s)+G_4(s)\right)}{1-H_1(s).G_1(s).G_2(s)}}{\dfrac{1-H_1(s).G_1(s).G_2(s)}{1-H_1(s)\cdot G_1(s)\cdot G_2(s)}+H(s)\dfrac{G_1(s).G_2(s)\left(G_3(s)+G_4(s)\right)}{1-H_1(s).G_1(s).G_2(s)}}$$

$$\frac{C(s)}{R(s)} = \frac{\left(G_1(s)\cdot G_2(s)\right)\times\left(G_1(s)+G_4(s)\right)}{\left(1-H_1(s)\cdot G_1(s)\cdot G_2(s)\right)+\left(H_2(s)\cdot G_1(s)\cdot G_2(s)\right)\times\left(G_3(s)+G_4(s)\right)}$$

Example 5: Derive the overall transfer function of the control system shown in the following figure:



Solution:

$$E_1(s) = R(s) - B_2(s)$$
$$E_2(s) = C_1(s) - B_1(s)$$
$$C_1(s) = G_1(s)E_1(s)$$
$$C_2(s) = G_2(s)E_2(s)$$
$$C(s) = G_3(s)C_2(s)$$
$$B_1(s) = H_1(s)C(s)$$
$$B_2(s) = H_2(s)C_2(s)$$

Substituting the sub-transfer functions,

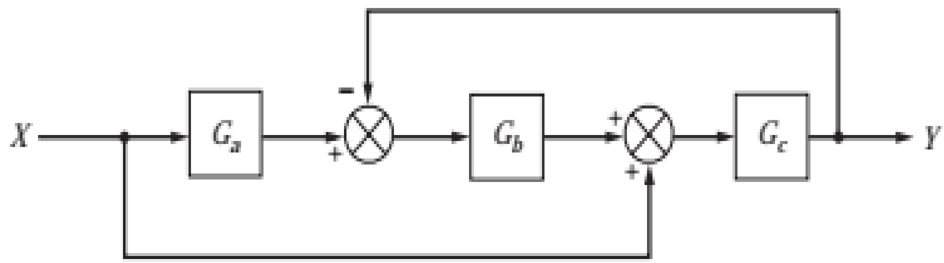
$$C(s) = G_3(s)C_2(s)$$
$$C(s) = G_3(s)G_2(s)E_2(s)$$
$$C(s) = G_3(s)G_2(s)[C_1(s) - B_1(s)]$$
$$C(s) = G_3(s)G_2(s)[G_1(s)E_1(s) - H_1(s)C(s)]$$
$$C(s) = G_3(s)G_2(s)[G_1(s)(R(s) - B_2(s)) - H_1(s)C(s)]$$
$$C(s) = G_3(s)G_2(s)[G_1(s)R(s) - G_1(s)H_2(s)C_2(s) - H_1(s)C(s)]$$
$$C(s) = G_3(s)G_2(s)[G_1(s)R(s) - G_1(s)H_2(s)\frac{C(s)}{G_3(s)} - H_1(s)C(s)]$$
$$C(s) = G_3(s)G_2(s)G_1(s)R(s) - G_3(s)G_2(s)G_1(s)H_2(s)\frac{C(s)}{G_3(s)} - G_3(s)G_2(s)H_1(s)C(s)$$
$$[1 + G_2(s)G_1(s)H_2(s) + G_3(s)G_2(s)H_1(s)]C(s) = G_3(s)G_2(s)G_1(s)R(s)$$

Finally, the overall transfer function

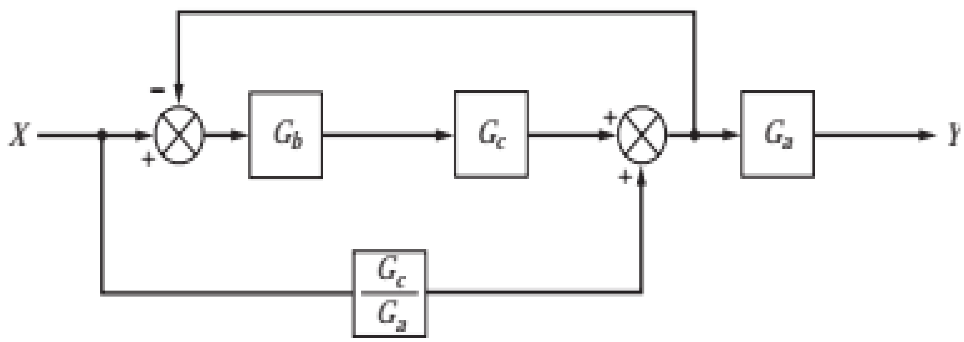
$$\frac{C(s)}{R(s)} = \frac{G_1(s)G_2(s)G_3(s)}{1 + G_1(s)G_2(s)H_2(s) + G_2(s)G_3(s)H_1(s)}$$

Problems

11.1. Determine the transfer function $Y(s)/X(s)$ for the block diagrams shown in Fig. P11–1. Express the results in terms of G_a, G_b, and G_c.



(a)



(b)

FIGURE P11–1

11.2. Find the transfer function $Y(s)/X(s)$ of the system shown in Fig. P11–2.

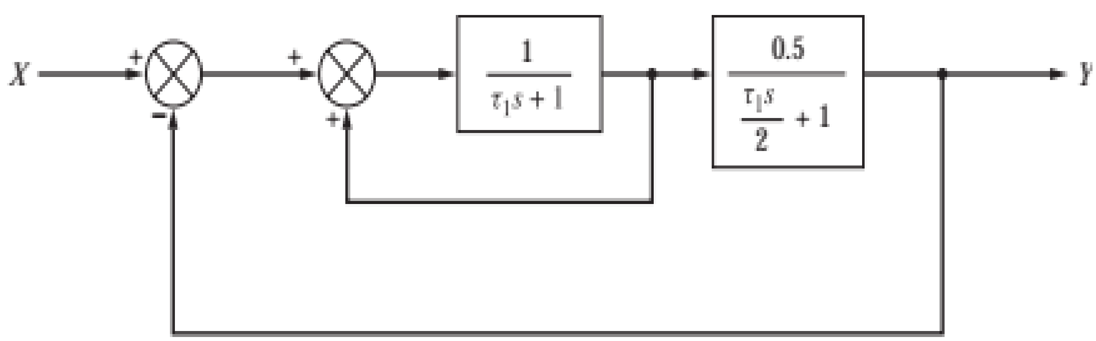


FIGURE P11–2

11.3. For the control system shown in Fig. P11–3 determine the transfer function $C(s)/R(s)$.

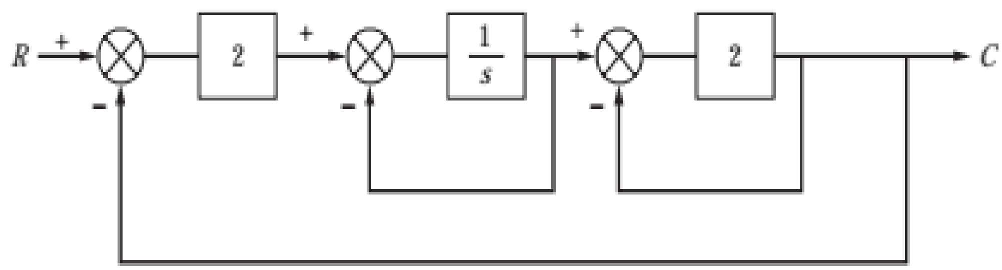


FIGURE P11–3

11.4. Derive the transfer function Y/X for the control system shown in Fig. P11–4.

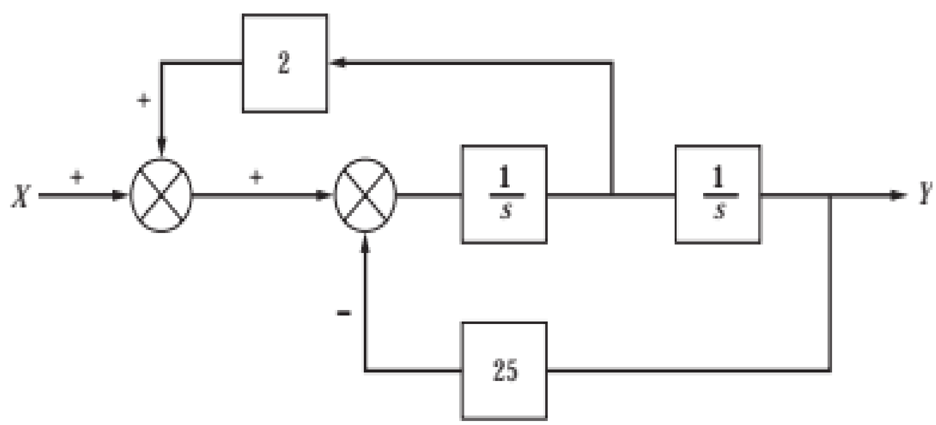


FIGURE P11–4

11.5. Derive the transfer function T/T_R for the temperature control system shown in Fig. 8–16.

11.6. Derive the transfer functions C_2/C_0 and C_2/C_R for the reactor control system shown in Fig. P10–3.