Chapter Three: Physical Examples of First-Ordered Systems

Chapter Five in the textbook

In the first part of this chapter, several physical systems that can be represented by a first-order transfer function will be considered. In the second part, a method for approximating the dynamic response of a nonlinear system by a linear response will be presented. This approximation is called linearization.

Examples of First-Order Systems

Consider the system shown in the following figure, which consists of a tank of uniform cross-sectional area A to which is attached a flow resistance R such as a valve, a pipe, or a weir. Assume that qo, the volumetric flow rate (volume/time) through the resistance, is related to the head h by the linear relationship:

$$q_o = \frac{h}{R} \qquad (1)$$

A resistance that has this linear relationship between flow and head is referred to as a linear resistance.

A time-varying volumetric flow q of liquid of constant density r enters the tank. Determine the transfer function that relates head to flow. We can analyze this system by writing a transient mass balance around the tank:

$$\begin{pmatrix}\text{Rate of}\\ \text{Mass Flow in}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{Mass Flow in}\end{pmatrix} = \begin{pmatrix}\text{Rate of Accumlation}\\ \text{of Mass in Tank}\end{pmatrix}$$

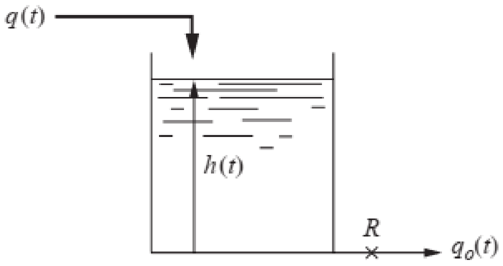
In terms of the variables used in this analysis, the mass balance becomes:

$$\rho q(t) - \rho q_o(t) = \frac{d(\rho A h)}{dt} \qquad (2)$$

$$q(t) - q_o(t) = \frac{d(Ah)}{dt} \qquad (3)$$

$$q(t) - \frac{h}{R} = \frac{Adh}{dt} \qquad (4)$$

At a steady state,



$$q_s - \frac{h_s}{R} = 0$$

$$(q - q_s) - \frac{1}{R}(h - h_s) = A\frac{d(h-h_s)}{dt} \qquad (5)$$

$$(q - q_s) = \frac{1}{R}(h - h_s) + A\frac{d(h-h_s)}{dt}$$

In a deviation form, $Q = q - q_s$ $\quad H = h - h_s$

$Q = \frac{1}{R}H + A\frac{dH}{dt}$ Taking the Laplace transform,

$$Q(s) = \frac{1}{R}H(s) + AsH(s) \rightarrow RQ(s) = H(s) + RAsH(s) \rightarrow RQ(s) = H(s)(1 + RA)$$

$$\frac{H(s)}{Q(s)} = \frac{R}{\tau s+1} \qquad (6)$$

$\tau = AR$ for a level tank.

To find the transfer function relating the inlet flow q(t) to the outlet flow is desired as follows:

$$q_o = \frac{h}{R} \rightarrow \textit{at steady state}\ q_{o,s} = \frac{h_s}{R} \rightarrow \textit{Take Laplace} \rightarrow Q_o(s) = \frac{H(s)}{R} \rightarrow H(s) = RQ_o(s)$$

Sub. in Eq. (6),

$$\frac{RQ_o(s)}{Q(s)} = \frac{R}{\tau s + 1} \rightarrow \frac{Q_o(s)}{Q(s)} = \frac{1}{\tau s + 1}$$

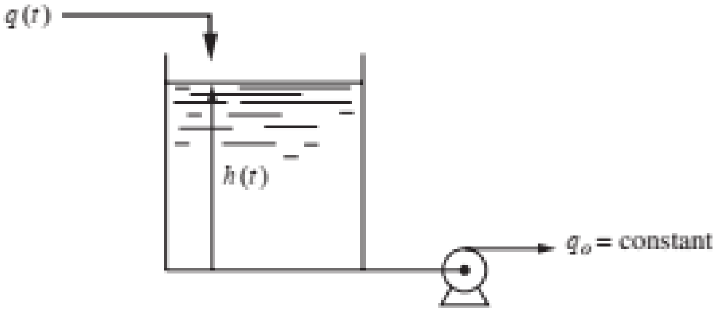
Liquid-Level Process with Constant-Flow Outlet

$$q(t) - q_o(t) = A\frac{dh}{dt}$$

$$q_s - q_o(t) = 0$$

$$Q = q - q_s$$

$$H = h - h_s$$



$$Q = A\frac{dH}{dt} \rightarrow Q(s) = AsH(s) \rightarrow \frac{H(s)}{Q(s)} = \frac{1}{As}$$

Example 5.1. A tank having a time constant of 1 min and a resistance of $\frac{1}{9}$ ft/cfm is operating at steady state with an inlet flow of 10 ft^3/min (or cfm). At time $t = 0$, the flow is suddenly increased to 100 ft^3/min for 0.1 min by adding an additional 9 ft^3 of water to the tank uniformly over a period of 0.1 min. (See Fig. 5–2a for this input disturbance.) Plot the response in tank level and compare with the impulse response.

Before proceeding with the details of the computation, we should observe that as the time interval over which the 9 ft^3 of water is added to the tank is shortened, the input approaches an impulse function having a magnitude of 9.

From the data given in this example, the transfer function of the process is

$$\frac{H(s)}{Q(s)} = \frac{\frac{1}{9}}{s + 1}$$

The input may be expressed as the difference in step functions, as was done in Example 3A.5.

$$Q(t) = 90[u(t) - u(t - 0.1)]$$

The transform of this is

$$Q(s) = \frac{90}{s}\left(1 - e^{-0.1s}\right)$$

Combining this and the transfer function of the process, we obtain

$$H(s) = 10\left[\frac{1}{s(s + 1)} - \frac{e^{-0.1s}}{s(s + 1)}\right] \tag{5.13}$$

The first term in Eq. (5.13) can be inverted as shown in Eq. (4.15) to give $10(1 - e^{-t})$. The second term, which includes $e^{-0.1s}$, must be inverted by use of the theorem on translation of functions given in App. 3A. According to this theorem, the inverse of $e^{-st_0}f(s)$ is $f(t - t_0)u(t - t_0)$ with $u(t - t_0) = 0$ for $t - t_0 < 0$ or $t < t_0$. The inverse of the second term in Eq. (5.13) is thus

$$L^{-1}\left\{\frac{e^{-0.1s}}{s(s + 1)}\right\} = 0 \qquad \text{for } t < 0.1$$

$$= 10\left(1 - e^{-(t-0.1)}\right) \qquad \text{for } t > 0.1$$

or

$$10\left(1 - e^{-(t-0.1)}\right)u(t - 0.1)$$

The complete solution to this problem, which is the inverse of Eq. (5.13), is

$$H(t) = 10\left(1 - e^{-t}\right)u(t) - 10\left(1 - e^{-(t-0.1)}\right)u(t - 0.1) \qquad (5.14)$$

which is equivalent to

$$H(t) = 10\left(1 - e^{-t}\right) \qquad t < 0.1$$

$$H(t) = 10\left[\left(1 - e^{-t}\right) - \left(1 - e^{-(t-0.1)}\right)\right] \qquad t > 0.1$$

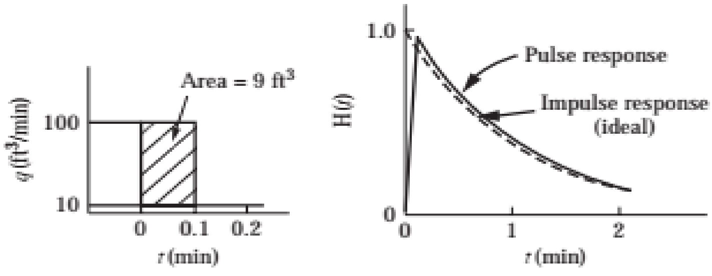
Simplifying this expression for $H(t)$ for $t > 0.1$ gives

$$H(t) = 1.052e^{-t} \qquad t > 0.1$$

From Eq. (4.19), the response of the system to an impulse of magnitude 9 is given by

$$H(t)\big|_{\text{impulse}} = (9)\left(\tfrac{1}{9}e^{-t}\right) = e^{-t}$$

In Fig. 5–2, the pulse response of the liquid-level system and the ideal impulse response are shown for comparison. Notice that the level rises very rapidly during the 0.1 min that additional flow is entering the tank; the level then decays exponentially and follows very closely the ideal impulse response.



(a) (b)

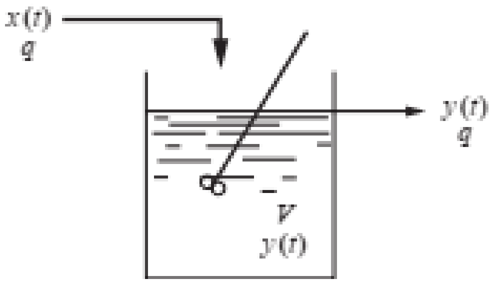
FIGURE 5–2
Approximation of an impulse function in a liquid-level system (Example 5.1). (a) Pulse input; (b) response of tank level.

The responses to step and sinusoidal forcing functions are the same for the liquid-level system as for the mercury thermometer of Chap. 4. Hence, they need not be rederived. This is the advantage of characterizing all first-order systems by the same transfer function.

Mixing Tank

Consider the mixing process shown in the figure in which a stream of solution containing dissolved salt flows at a constant volumetric flow rate q into a tank of the constant holdup volume V. The concentration of the salt in the entering stream x (mass of salt/volume) varies with time. It is desired to determine the transfer function relating the outlet concentration y to the inlet concentration x. If we assume the density of the solution to be constant, the flow rate in must equal the flow rate out, since the holdup volume is fixed. A transient mass balance for the salt is as follows:

$$\begin{pmatrix}\text{Flow Rate of}\\ \text{Salt in}\end{pmatrix} - \begin{pmatrix}\text{Flow Rate of}\\ \text{Salt in}\end{pmatrix} = \begin{pmatrix}\text{Rate of Accumlation}\\ \text{of Salt in Tank}\end{pmatrix}$$



$$qx - qy = \frac{d(Vy)}{dt} = V\frac{dy}{dt}$$

$$qx_s - qy_s = 0$$

$$X = x - x_s \qquad Y = y - y_s$$

$$qX - qY = V\frac{dY}{dt}$$

$$\frac{Y(s)}{X(s)} = \frac{1}{\tau s + 1}$$

$\tau = \frac{V}{q}$ for a mixing tank.

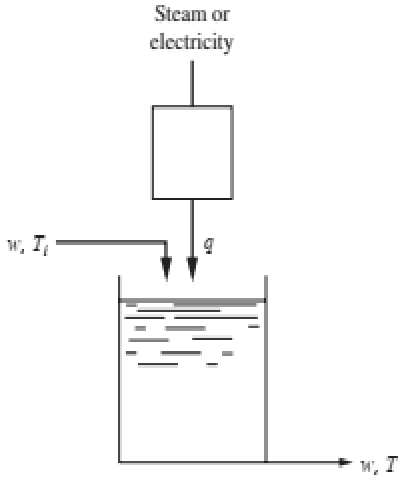
Heating Process

Consider the heating process shown in the following figure. A stream at temperature Ti is fed to the tank. Heat is added to the tank by means of an electric heater. The tank is well mixed, and the temperature of the exiting stream is T. The flow rate to the tank is constant at w lb/h. A transient energy balance on the tank yields

$$\left(\begin{array}{c}\text{Rate of}\\ \text{Energy flow}\\ \text{into Tank}\end{array}\right) - \left(\begin{array}{c}\text{Rate of}\\ \text{Energy flow}\\ \text{out of Tank}\end{array}\right) - \left(\begin{array}{c}\text{Rate of}\\ \text{Energy flow in}\\ \text{from Heater}\end{array}\right) = \left(\begin{array}{c}\text{Rate of Accumlation}\\ \text{of Energy in Tank}\end{array}\right)$$

$$wC(T_i - T_{ref}) - wC(T - T_{ref}) + q = \rho VC\frac{d(T - T_{ref})}{dt} = \rho VC\frac{dT}{dt}$$

$$wC(T_{is} - T_s) + q_s = 0$$



$$wC(T_i - T_{is}) - wC(T - T_s) + q - q_s = \rho VC\frac{d(T - T_s)}{dt}$$

$$T' = T - T_s$$
$$Q = q - q_s$$

$$-wCT' + Q = \rho VC\frac{dT'}{dt}$$

$$-wCT'(s) + Q(s) = \rho VCsT'(s)$$

$$\frac{T'(s)}{Q(s)} = \frac{{}^{1}/_{wc}}{\left({}^{\rho V}/_{w}\right)s + 1} = \frac{K}{\tau s + 1}$$

$\tau = \frac{\rho V}{w}$ for a heating tank.

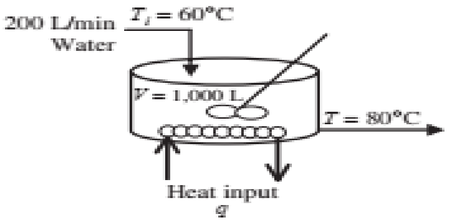
Example 5.2. Consider the mixed tank heater shown in Fig. 5–6. Develop a transfer function relating the tank outlet temperature to changes in the inlet temperature. Determine the response of the outlet temperature of the tank to a step change in the inlet temperature from 60 to 70°C. Before we proceed, intuitively what would we expect to happen? If the inlet temperature rises by 10°C, we expect the outlet temperature to eventually rise by 10°C if nothing else changes. Let's see what modeling the process will tell us.

From Eq. (5.26) we can write the following simplified balance, realizing that $q = q_s$:

$$wC(T_i - T_{is}) - wC(T - T_s) = \rho VC\frac{d(T - T_s)}{dt}$$

In terms of deviation variables, this becomes

$$wCT_i' - wCT' = \rho VC\frac{dT'}{dt}$$



Transforming, we get

$$wCT_i'(s) - wCT'(s) = \rho VCsT'(s)$$

and finally, after rearranging,

$$\frac{T'(s)}{T_i'(s)} = \frac{1}{(\rho V/w)s + 1} = \frac{1}{\tau s + 1}$$

Substituting in numerical values for the variables, we obtain the actual transfer function for this mixed tank heater.

$$\tau = \frac{\rho V}{w} = \frac{V}{w/\rho} = \frac{V}{\upsilon} = \frac{\text{tank volume}}{\text{volumetric flow rate}} = \frac{1{,}000\ \text{L}}{200\ \text{L/min}} = 5\ \text{min}$$

$$\frac{T'(s)}{T_i'(s)} = \frac{1}{5s+1}$$

If the inlet temperature is stepped from 60 to 70°C, $T_i'(t) = 70 - 60 = 10$ and $T_i'(s) = 10/s$. Thus,

$$T'(s) = \frac{10}{s}\left(\frac{1}{5s+1}\right)$$

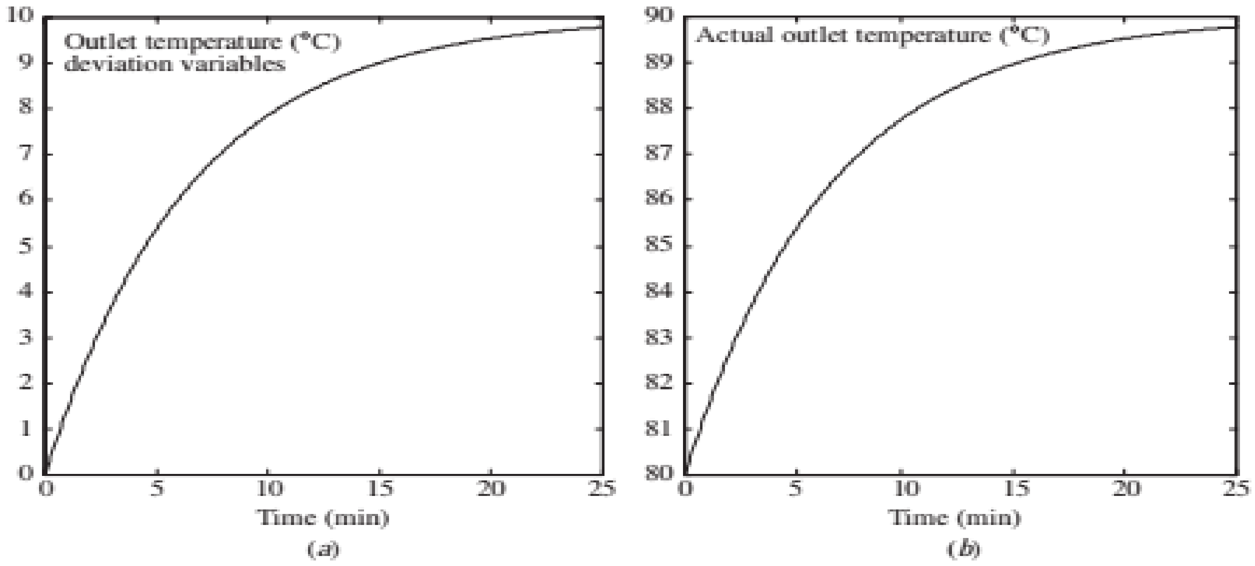
Inverting to the time domain, we obtain the expression for $T'(t)$

$$T'(t) = 10\left(1 - e^{-t/5}\right)$$

and finally, we obtain the expression for $T(t)$, the actual tank outlet temperature.

$$T(t) = T_s + T'(t) = 80 + 10\left(1 - e^{-t/5}\right)$$

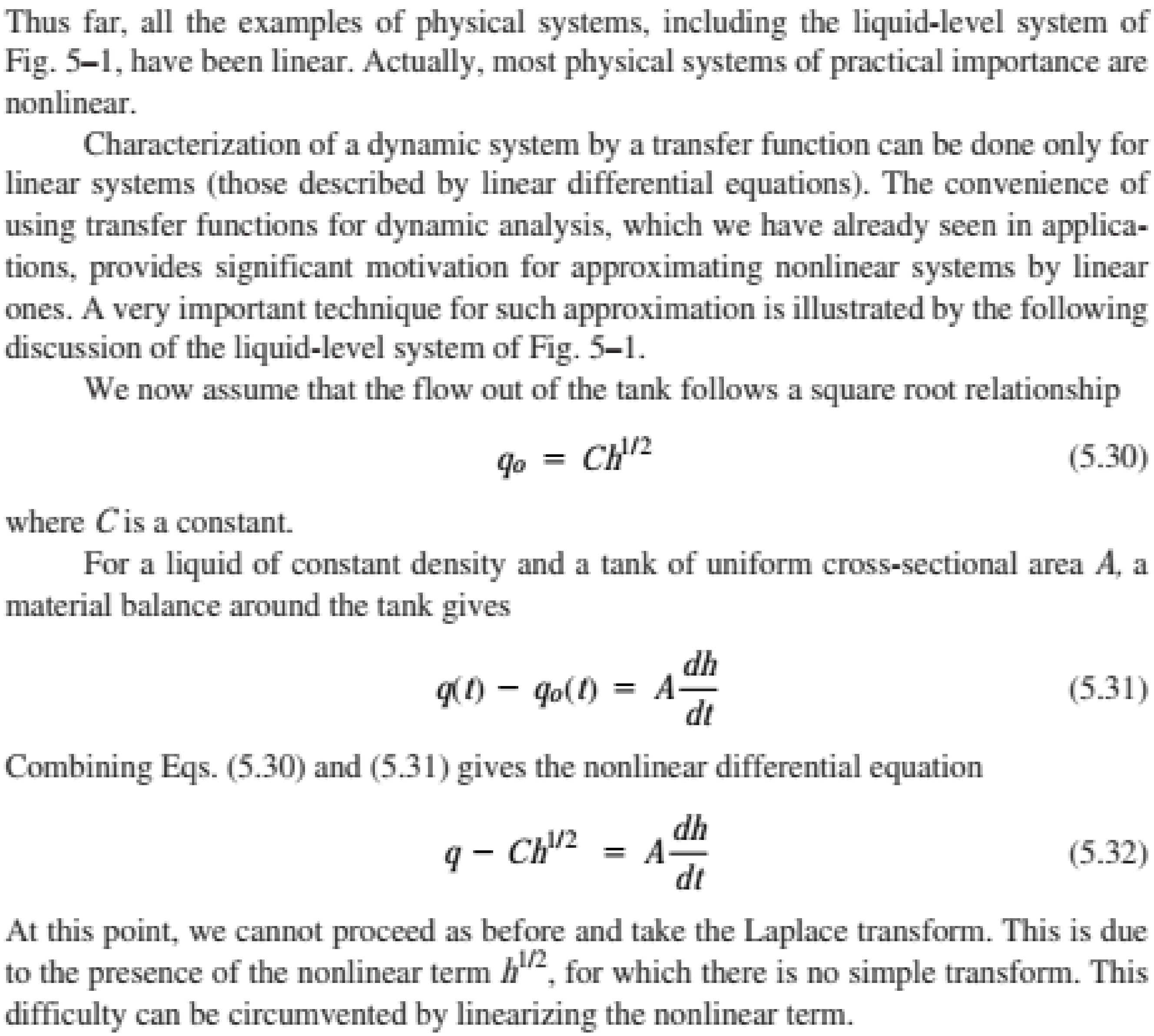
A plot of the outlet temperature (in deviation variables) is shown in the Fig. 5–7*a*. The actual outlet temperature is shown in Fig. 5–7*b*. Note that for the uncontrolled mixing tank, a step change of 10°C in the inlet temperature



(*a*) (*b*)

FIGURE 5–7
(*a*) Tank outlet temperature (deviation variable); (*b*) actual tank outlet temperature.

Linearization



Thus far, all the examples of physical systems, including the liquid-level system of Fig. 5–1, have been linear. Actually, most physical systems of practical importance are nonlinear.

Characterization of a dynamic system by a transfer function can be done only for linear systems (those described by linear differential equations). The convenience of using transfer functions for dynamic analysis, which we have already seen in applications, provides significant motivation for approximating nonlinear systems by linear ones. A very important technique for such approximation is illustrated by the following discussion of the liquid-level system of Fig. 5–1.

We now assume that the flow out of the tank follows a square root relationship

$$q_o = Ch^{1/2} \tag{5.30}$$

where C is a constant.

For a liquid of constant density and a tank of uniform cross-sectional area A, a material balance around the tank gives

$$q(t) - q_o(t) = A\frac{dh}{dt} \tag{5.31}$$

Combining Eqs. (5.30) and (5.31) gives the nonlinear differential equation

$$q - Ch^{1/2} = A\frac{dh}{dt} \tag{5.32}$$

At this point, we cannot proceed as before and take the Laplace transform. This is due to the presence of the nonlinear term $h^{1/2}$, for which there is no simple transform. This difficulty can be circumvented by linearizing the nonlinear term.

By means of a Taylor series expansion, the function $q_o(h)$ may be expanded around the steady-state value h_s; thus

$$q_o = q_o(h_s) + q_o'(h_s)(h - h_s) + \frac{q_o''(h_s)(h - h_s)^2}{2} + \cdots$$

where $q_o'(h_s)$ is the first derivative of q_o evaluated at h_s, $q_o''(h_s)$ is the second derivative, etc. If we keep only the linear term, the result is

$$q_o \cong q_o(h_s) + q_o'(h_s)(h - h_s) \tag{5.33}$$

Taking the derivative of q_o with respect to h in Eq. (5.30) and evaluating the derivative at $h = h_s$ give

$$q_o'(h_s) = \tfrac{1}{2} C h_s^{-1/2}$$

Introducing this into Eq. (5.33) gives

$$q_o = q_{o_s} + \frac{1}{R_1}(h - h_s) \tag{5.34}$$

where $q_{o_s} = q_o(h_s)$ and $1/R_1 = \tfrac{1}{2} C h_s^{-1/2}$.

Substituting Eq. (5.34) into Eq. (5.31) gives

$$q - q_{o_s} - \frac{h - h_s}{R_1} = A\frac{dh}{dt} \tag{5.35}$$

At steady state the flow entering the tank equals the flow leaving the tank; thus

$$q_s = q_{o_s} \tag{5.36}$$

Introducing this last equation into Eq. (5.35) gives

$$A\frac{dh}{dt} + \frac{h - h_s}{R_1} = q - q_s \tag{5.37}$$

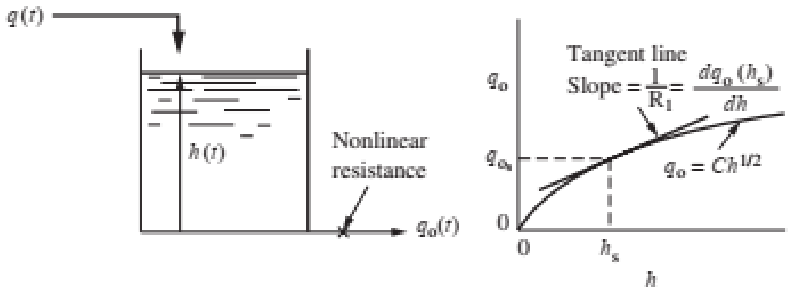
Introducing deviation variables $Q = q - q_s$ and $H = h - h_s$ into Eq. (5.37) and transforming give

$$\frac{H(s)}{Q(s)} = \frac{R_1}{\tau s + 1} \tag{5.38}$$

where

$$R_1 = \frac{2h_s^{1/2}}{C} \qquad \tau = R_1 A$$

We see that a transfer function is obtained that is identical in form with that of the linear system, Eq. (5.8). However, in this case, the resistance R_1 depends on the steady-state conditions around which the process operates. Graphically, the resistance R_1 is the reciprocal of the slope of the tangent line passing through the point (q_{os}, h_s), as shown in



In general, the linearization of a nonlinear function is accomplished using a Taylor series expansion truncated to include only the linear terms. Thus for a single-variable function

$$f(x) = f(x_s) + \left.\frac{df}{dx}\right|_{x_s} (x - x_s) + (\text{higher-order terms}) \tag{5.39}$$

For functions of two variables, we have

$$f(x,y) = f(x_s,y_s) + \left.\frac{\partial f}{\partial x}\right|_{(x_s,y_s)} (x - x_s) + \left.\frac{\partial f}{\partial y}\right|_{(x_s,y_s)} (y - y_s) + (\text{higher-order terms}) \tag{5.40}$$

Consider the differential equation describing the dynamics of a system

$$\frac{dy}{dt} + \underbrace{f(y)}_{\substack{\text{nonlinear}\\ \text{term}}} = x(t) \tag{5.41}$$

Linearizing the nonlinear term gives

$$\frac{dy}{dt} + \underbrace{f(y_s) + \left.\frac{\partial f}{\partial y}\right|_{y_s} (y - y_s)}_{\text{linearized approximation}} = x(t) \tag{5.42}$$

Writing this equation again for the steady-state case gives

$$\frac{dy_s}{dt} + f(y_s) + \left.\frac{\partial f}{\partial y}\right|_{y_s} (y_s - y_s) = x_s \tag{5.43}$$

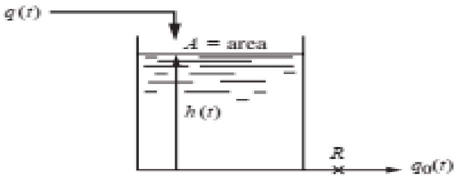
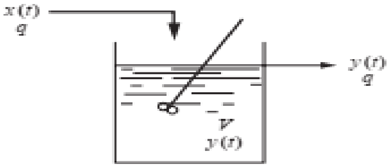
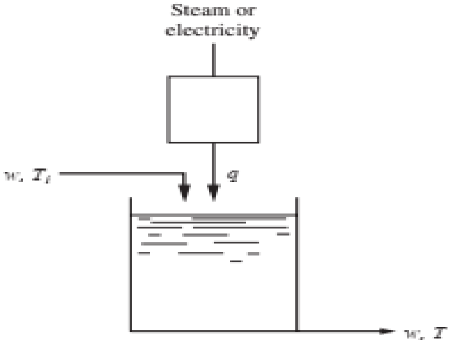
Subtracting the steady-state case in Eq. (5.43) from Eq. (5.42), we can convert the original differential equation to deviation variables:

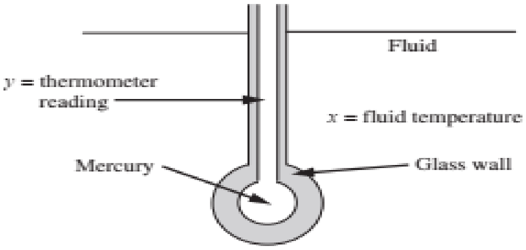
$$\frac{d(y - y_s)}{dt} + \left.\frac{\partial f}{\partial y}\right|_{y_s} (y - y_s) = x - x_s$$

$$\frac{dY}{dt} + \left.\frac{\partial f}{\partial y}\right|_{y_s} Y = X$$

where $X = x - x_s$ and $Y = y - y_s$. Note that the $f(y_s)$ term is eliminated in the process of forming deviation variables, and we are left with only linear terms in the equation which is now amenable to solution using Laplace transforms.

Here are some physical examples of first-order systems:

System		Transfer function
	Figure 5–1 Liquid level system	$\frac{H(s)}{Q(s)} = \frac{R}{ARs + 1}$
	Figure 5–4 Mixing process	$\frac{Y(s)}{X(s)} = \frac{1}{(V/q)s + 1}$
	Figure 5–5 Heating process	$\frac{T'(s)}{Q(s)} = \frac{1/(wC)}{(\rho V/w)s + 1}$

System		Transfer function
	Figure 4–1*a* Thermometer	$\frac{Y(s)}{X(s)} = \frac{1}{(mc/hA)s + 1}$

TAYLOR SERIES EXPANSIONS FOR LINEARIZING NONLINEAR TERMS

Functions of a single variable: $f(x) = f(x_s) + \left.\frac{df}{dx}\right|_{x_s}(x - x_s)$

Functions of two variables:

$$f(x,y) = f(x_s,y_s) + \left.\frac{\partial f}{\partial x}\right|_{(x_s,y_s)}(x - x_s) + \left.\frac{\partial f}{\partial y}\right|_{(x_s,y_s)}(y - y_s)$$

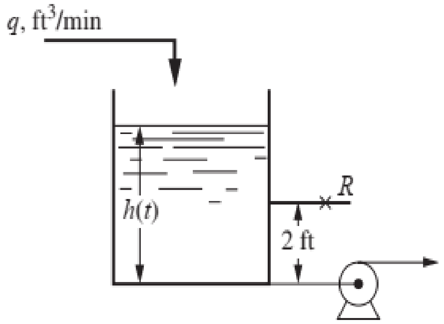
Problems

5.1. Derive the transfer function H(s)/Q(s) for the liquid-level system of the following figure when

(a) The tank level operates about the steady-state value of h_S = 1 ft.

(b) The tank level operates about the steady-state value of h_S = 3 ft.

The pump removes water at a constant rate of 10 cfm (cubic feet per minute); this rate is independent of head. The cross-sectional area of the tank is 1.0 ft^2 and the resistance R is 0.5 ft/cfm.

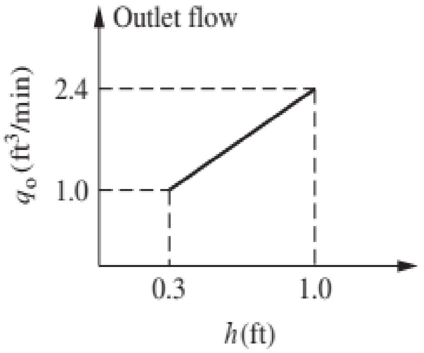


5.2. A liquid-level system, such as the one shown in the figure of the question 5.1, has a cross-sectional area of 3.0 ft^2. The valve characteristics are $q = 8\sqrt{h}$. Where: q = flow rate cfm, h = level above the valve, ft. Calculate the time constant for this system if the average operating level is (a) 3 ft (b) 9 ft

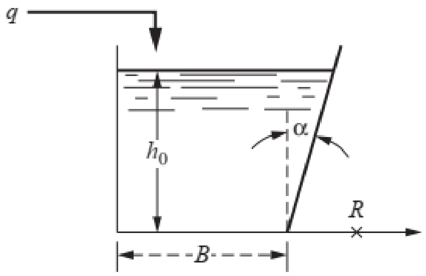
5.3. A tank having a cross-sectional area of 2 ft^2 is operating at steady state with an inlet flow rate of 2.0 cfm. The flow-head characteristics are shown in the following figure.

(a) Find the transfer function H(s)/Q(s).

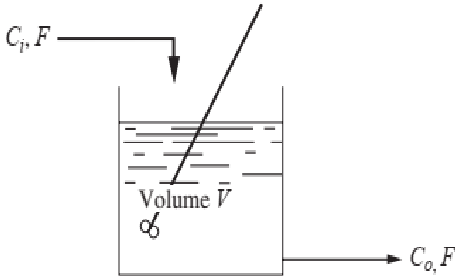
(b) If the flow to the tank increases from 2.0 to 2.2 cfm according to a step change, calculate the level h two minutes after the change occurs.



5.4. Develop a formula for finding the time constant of the liquid-level system shown in the following figure when the average operating level is h. The resistance R is linear. The tank has three vertical walls and one which slopes at an angle (r from the vertical as shown. The distance separating the parallel walls is 1.



5.5. Consider the stirred-tank reactor shown in the following figure. the reaction occurring is $A \rightarrow B$ and it proceeds at a rate $r = kC_o$



Where: r = moles A reacting/ (volume) (time)

k = reaction velocity constant

$C_o(t)$ = concentration of A in reactor, moles/volume

V = volume of mixture in reactor

Further let F = constant feed rate, volume/time

$C_i(t)$ = concentration of A in feed stream

Assuming constant density and constant V, derive the transfer function relating the concentration in the reactor to the feed-stream concentration. Prepare a block diagram for the reactor. Sketch the response of the reactor to a unit-step change in C_i.

5.6. A thermocouple junction of area A, mass m, heat capacity C, and emissivity e is located in a furnace that normally is at Tis, ^{o}C. At these temperatures convective and conductive heat transfer to the junction am negligible compared with radiative heat transfer. Determine the linearized transfer function between the furnace temperature Ti and the junction temperature T_o. For the case m = 0.l g, c = 0.12 cal/(g) (^{o}c), e = 0.7, A = 0.l cm^2, Tis, = 1100 °C. Plot the response of the thermocouple to a 10 °C step change in furnace temperature. Compare this with the true response obtained by integration of the differential equation.

5.7. A liquid-level system has the following properties: Tank dimensions: 10 ft high by 5 ft diameter Steady-state operating characteristics:

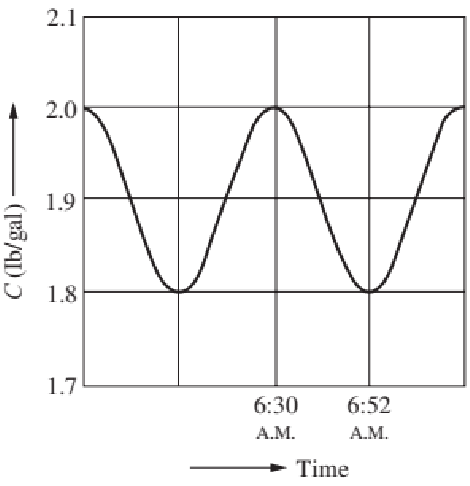
Inflow (gal/hr)	0	5,000	10,000	15,000	20,000	25,000	30,000
Steady state Level (ft)	0	0.7	1.1	2.3	3.9	6.3	8.8

(a) Plot the level response of the tank under the following circumstances: The inlet flow rate is held at 300 gal/min for 1 hr and then suddenly raised to 400 gal/min.

(b) How accurate is the steady-state level calculated from the dynamic response in part (a) when compared with the value given by the table above?

(c) The tank is now connected in series with a second tank that has identical operating characteristics, but which has dimensions 8 ft high by 4 ft diameter. Plot the response of the original tank (which is upstream of the new tank) to the change described in part (a) when the connection is such that the tanks are (1) interacting, (2) non-interacting. (See Chap. 7.)

5.8. A mixing process may be described as follows: a stream with solute concentration C_i (pounds/volume) is fed to a perfectly stirred tank at a constant flow rate of q (volume/time). The perfectly mixed product is withdrawn from the tank, also at the flow rate q at the same concentration as the material in the tank, C. The total volume of solution in the tank is constant at V. Density may be considered to be independent of concentration. A trace of the tank concentration versus time appears as shown in Fig. P6.8.



(a) Plot on this same figure your best guess of the quantitative behavior of the inlet concentration versus time. Be sum to label the graph with quantitative information regarding times and magnitudes and any other data that will demonstrate your understanding of the situation.

(b) Write an equation for C_i as a function of time.

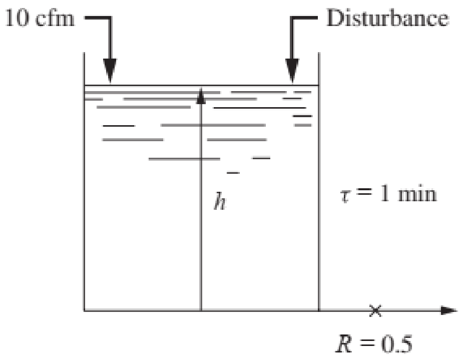
Data: Tank dimensions: 8 ft high by 5 ft diameter

Tank volume V: 700 gal

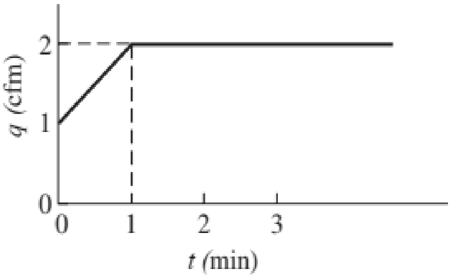
Flow rate q: 100 gal/min

Average density: 70 lb/ft3

5.9. The liquid-level process shown in the following figure is operating at steady state when the following disturbance occurs: at time t = 0, 1 ft3 water is added suddenly (unit impulse) to the tank; at t = 1, 2 ft^3 of water is added suddenly to the tank. Sketch the response of the level in the tank versus time and determine the level at t = 0.5, 1, and 1.5.



5.10. A tank having a cross-sectional area of 2 ft^2 and a linear resistance of R = 1 ft/cfm is operating at steady state with a flow rate of 1 cfm. At time zero, the flow varies as shown in the following figure.



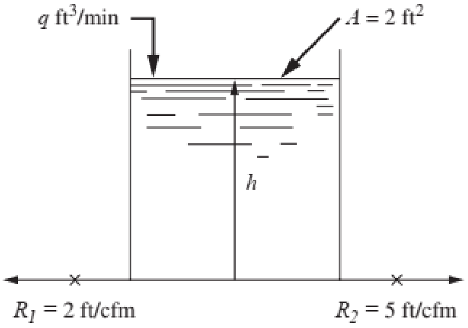
(a) Determine Q(t) and Q(s) by combining simple functions. Note that Q is the deviation in flow rate.

(b) Obtain an expression for H (t) where H is the deviation in level.

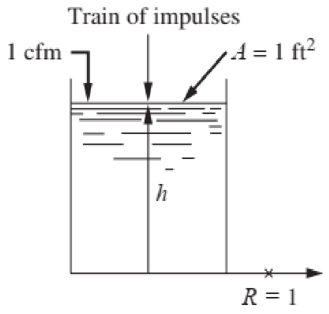
(c) Determine H(r) at r = 2 and t = ∞.

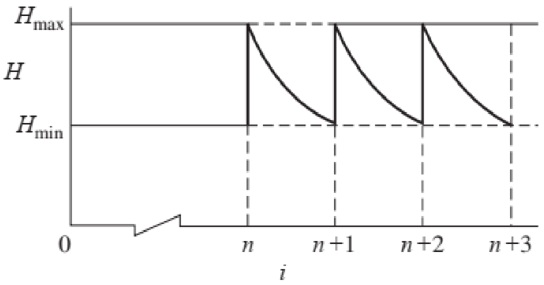
5.11. Determine Y (5) if $Y(s) = \frac{e^{-3s}}{s(7s+1)}$

5.12. Derive the transfer function H/Q for the liquid level system shown in the following figure. The resistances are linear. H and Q are deviation variables. Show clearly how you derived the transfer function. You are expected to give numerical values in the transfer function.

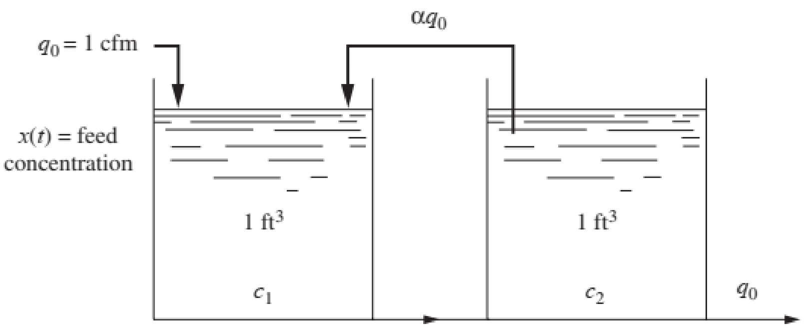


5.13. The liquid-level system shown in the following figure is initially at steady state with the inlet flow rate at 1 cfm. At time zero, one ft^3 of water is suddenly added to the tank; at t = 1, one ft^3 is added, etc. In other words, a train of unit impulses is applied to the tank at intervals of one minute. Ultimately the output wave train becomes periodic as shown in the sketch. Determine the maximum and minimum values of this output.





5.14. The two-tank mixing process shown in the following figure contains a recirculation loop that transfers solution from tank 2 to tank 1 at a flow rate of αq_o.

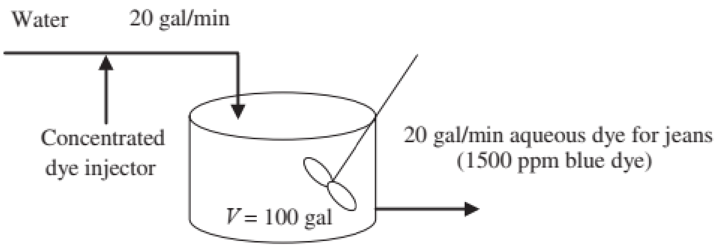


(a) Develop a transfer function that relates the concentration in tank 2, C_2, to the concentration in the feed, x; i.e. $C_2(s)/X(s)$ where C_2 and X are deviation variables. For convenience, assume that the initial concentrations are $x = C_1 = C_2 = 0$.

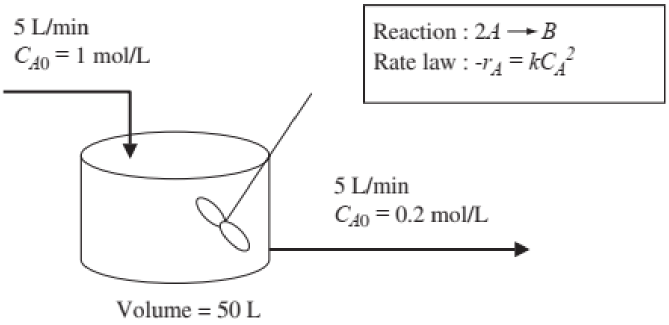
(b) If a unit-step change in x occurs, determine the time needed for C_2 to reach 60 percent of its ultimate value for the cases where $\alpha = 0, 1$, and ∞.

(c) Sketch the response for $\alpha = \infty$.

Assume that each tank has a constant holdup volume of 1 ft^3. Neglect transportation lag in the line connecting the tanks and the recirculation line. Try to answer parts (b) and (c) by intuition.



5.15. Dye for our new line of blue jeans is being blended in a mixing tank. The desired color of blue is produced using a concentration of 1500 ppm blue dye, with a minimum acceptable concentration of 1400 ppm. At 9 A.M. today the dye injector plugged, and the dye flow was interrupted for 10 min, until we realized the problem and unclogged the nozzle. For how many minutes was the flow leaving the mixer off-specification (< 1400 ppm)? How many gallons of off-spec dye were made? See the following figure.



5.16. For the reactor (CSTR) shown in the following figure, determine the transfer function that relates the exit concentration from the reactor to changes in the feed concentration. If we instantaneously double the feed concentration from 1 to 2 mol/L, what is the new exiting concentration 1 min later? What is the new steady-state reactor concentration? The rate constant is $K = \frac{2}{(\mathrm{mol/L})(\mathrm{min})}$ The reaction rate law is $-r_A = kC_A^2$, where r_A is the production rate of A in moles per liter per minute.

5.17. The Antoine equation for the vapor pressure of a liquid at a given temperature is given by

$$P^* = e^{A-B/(T+C)}$$

The constants for benzene are $A = 15.9008$ $B = 2788.51\ ℃$ $C = 220.8\ ℃$

for the vapor pressure in millimeters of mercury (mmHg). Linearize the equation about a temperature of 40°C. Compare the actual vapor pressure (from the Antoine equation) at 45 and 60°C. with the vapor pressure calculated from the linearized equation. What is the percent difference in each case? Comment on the suitability of the linearized equation.

5.18. Find the transfer function that relates the height in the vessel (figure below) to changes in the inlet flow rate.