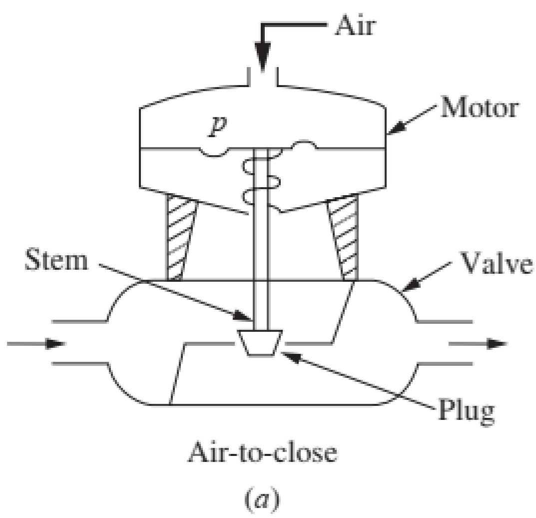
Chapter Two: Controllers and Final Control Elements

Chapter Nine in the textbook

This chapter will focus attention on the controller and final control element and will discuss the dynamic characteristics of some of these components that are in common use. the input signal to the controller is the error, and the output signal of the controller is fed to the final control element. In many process control systems, this output signal is an air pressure, and the final control element is a pneumatic valve that opens and closes as the air pressure on the diaphragm changes as shown in Figure 1.



P = 3	–	15 psig		P = 3	–	15 psig
Open		Close		Close		Open
Max. flow		flow=0		flow=0		Max. flow

Figure 1: Pneumatic control valves. (*a*) Air to close; (*b*) air to open.

The control hardware required to control the temperature of a stream leaving a heat exchanger is shown in Figure 2. This hardware consists of the following components listed here along with their respective conversions:

Transducer (temperature-to-current)

Computer/Controller (current-to-current)

Converter (current-to-pressure)

Control valve (pressure-to-flow rate)

A thermocouple is used to measure the temperature; the signal from the thermocouple is sent to a transducer, which produces a current output in the range of 4 to 20 mA, which is a linear function of the input. The output of the transducer enters the controller where it is compared to the set point to produce an error signal. The computer/controller converts the error to an output signal in the range of 4 to 20 mA by the computer control algorithm. The output of the computer/controller enters the converter, which produces an output in the range of 3 to 15 psig, as a linear function of the input. Finally, the air pressure output of the converter is sent to the top of the control valve, which adjusts the flow of steam to the heat exchanger. the valve is linear and is the air-to-open type. The external power (120 V) is needed for each component and Electricity is needed for the transducer, computer/controller, and converter. A source of 20 psig air is also needed for the converter.

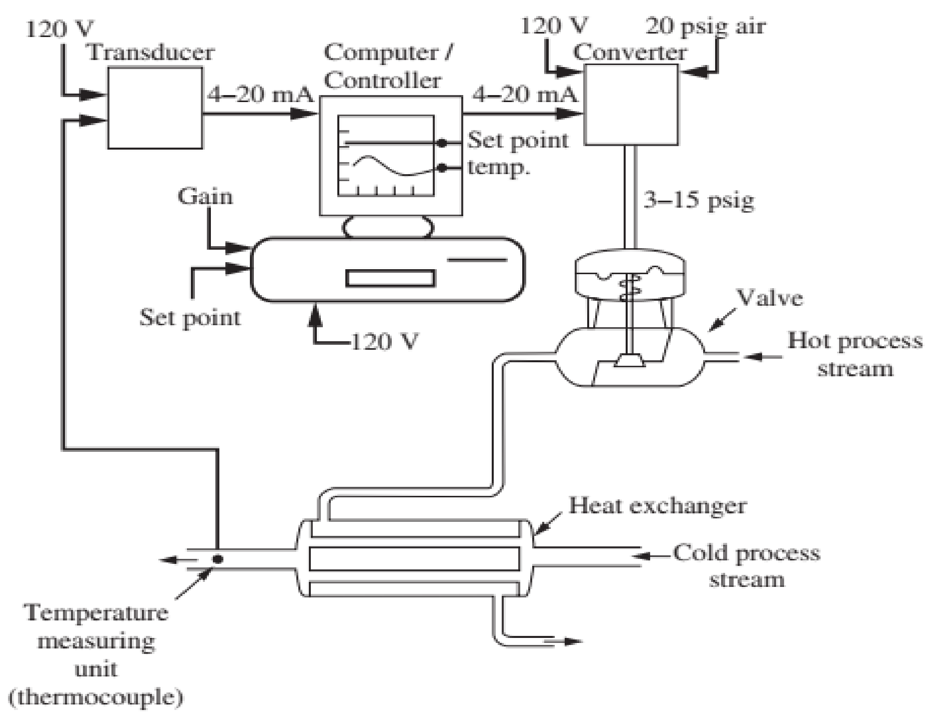


Figure 2: Schematic diagram of control system.

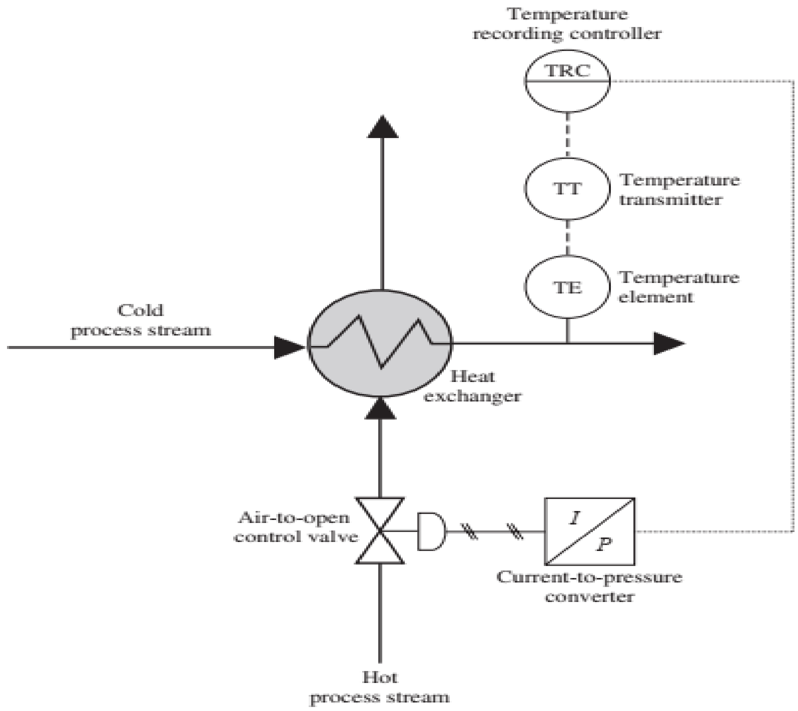


Figure 3: Piping and instrumentation diagram for the control system of Figure 2.

Instrument Line Symbols

	Instrument supply or piping connection to process (capillary)
	Pneumatic
	Electrical
	Software signal

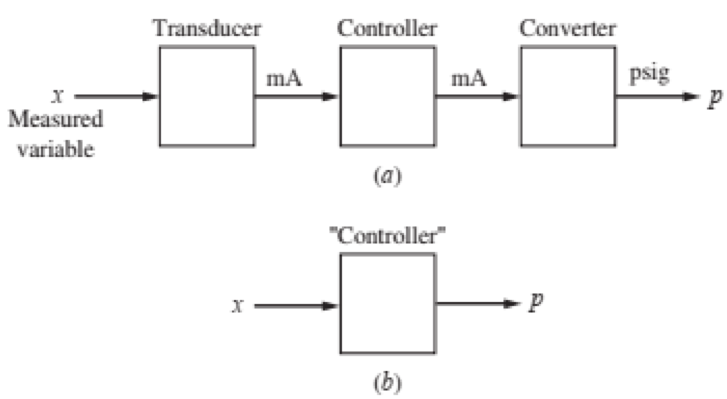


Figure 4: Equivalent block for transducer, controller, and converter.

Control Valve

A pneumatic valve always has some dynamic lag, which means that the stem position does not respond instantaneously to a change in the applied pressure from the controller. the relationship between flow and valve-top pressure for a linear valve can often be represented by a first-order transfer function as follows:

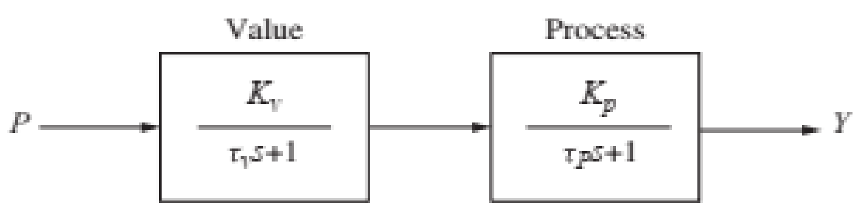
$$\text{Control valve first-order transfer function} \qquad \frac{Q(s)}{P(s)} = \frac{K_v}{\tau_v s + 1}$$

where K_v is the steady-state gain, and τ_v is the time constant of the valve.

In many practical systems, the time constant of the valve is very small when compared with the time constants of other components of the control system, and the transfer function of the valve can be approximated by a constant.

$$\text{Control valve (fast dynamics) transfer function} \qquad \frac{Q(s)}{P(s)} = K_v$$

Consider a first-order valve and a first-order process connected in series as shown below:



$$\frac{Y(s)}{P(s)} = \frac{K_v K_P}{(\tau_v s + 1)(\tau_P s + 1)}$$

For a unit-step change in the valve-top pressure P,

$$Y = \frac{1}{s} \frac{K_v K_P}{(\tau_v s + 1)(\tau_P s + 1)}$$

Taking Laplace inverse,

$$Y(t) = K_v K_P \left[1 - \frac{\tau_v \tau_P}{\tau_v - \tau_P} \left(\frac{1}{\tau_P} e^{-t/\tau_v} - \frac{1}{\tau_v} e^{-t/\tau_p} \right) \right]$$

If $\tau_v \ll \tau_p$, this equation is approximately,

$$\frac{Y(s)}{P(s)} = K_v \frac{K_P}{\tau_P s + 1}$$

Taking Laplace inverse,

$$Y(t) = K_v K_P \left(1 - e^{-t/\tau_p} \right)$$

A typical pneumatic valve has a time constant of the order of 1 s. Many industrial processes behave as first-order systems or as a series of first-order systems having time constants that may range from a minute to an hour. For these systems, the lag of the valve is negligible.

Controller

1. **Proportional Control (P Control)**

$$p = K_c E + p_s$$

$$G_c = \frac{P(s)}{E(s)} = K_c \rightarrow P(s) = K_c \times \frac{A}{s}$$

A step change in E(s)

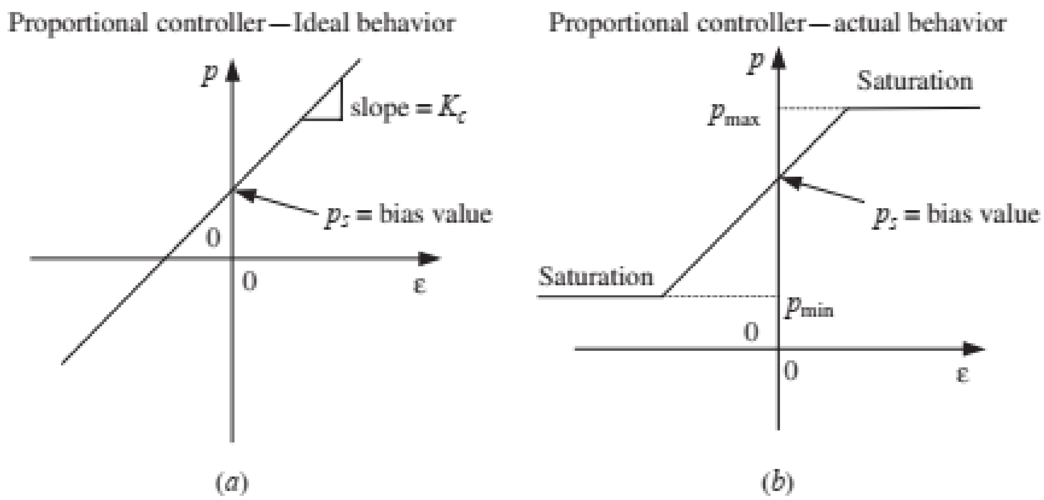
Taking Laplace inverse, $P(t) = AK_c$

Where p = output signal from controller, psig or mA

K_c = proportional gain of conductivity

E = error = set point - measured variable

p_s = a constant, a steady-state output from controller



Example 1:

A pneumatic proportional controller is used in the process shown in Figure 2 to control the cold stream outlet temperature within the range of 60 to 120 °F. The controller gain is adjusted so that the output pressure goes from 3 psig (valve fully closed) to 15 psig (valve fully open) as the measured temperature goes from 71 to 75 °F with the set point held constant. Find the controller gain Kc.

Solution:

$$\text{Gain} = \frac{\Delta p}{\Delta \varepsilon} = \frac{15\text{ psig} - 3\text{ psig}}{75°\text{F} - 71°\text{F}} = 3\text{ psi/°F}$$

Now, assume that the gain of the controller is changed to 0.4 psi/°F. Find the error in temperature that will cause the control valve to go from fully closed to fully open.

$$\Delta T = \frac{\Delta p}{\text{gain}} = \frac{12\text{ psi}}{0.4\text{ psi/°F}} = 30°\text{F}$$

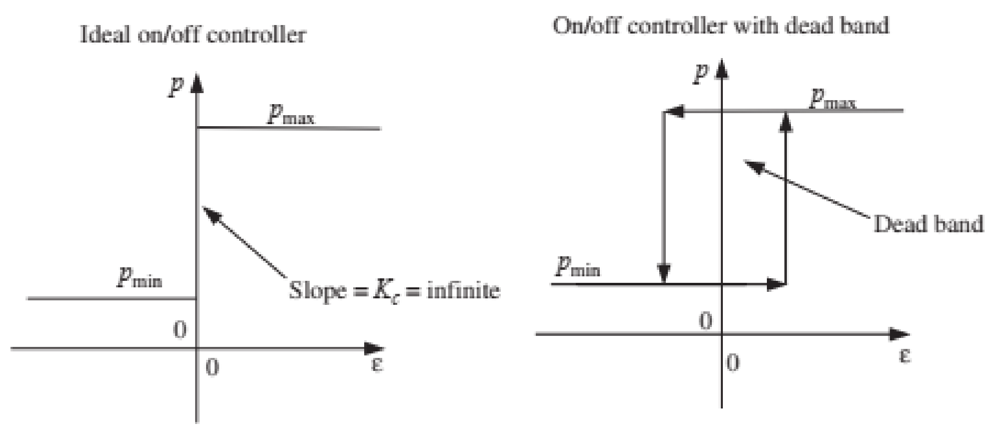
At this level of gain, the valve will be fully open if the error signal reaches 30 °F. The gain Kc has the units of psi per unit of the measured variable. [Regarding the units on the controller gain, if the actual controller of the figure above is considered, both the input and the output units are in milliamperes. In this case, the gain will be dimensionless (i.e., mA/mA).]

Proportional Band (Band Width): Is defined as the error (expressed as a percentage of the range of measured variables) required to make the valve from fully close to fully open.

$$\mathrm{P.B.\%} = \frac{1}{\mathrm{K_c}} \times 100$$

On/Off Control

A special case of proportional control is on/off control. If the gain Kc is made very high, the valve will move from one extreme position to the other. This very sensitive action is called the on/off action because the valve is either fully open (on) or fully closed (off); i.e., the valve acts as a switch. This is a very simple controller and is exemplified by the thermostat used in a home-heating system. The P.B. of the on-off controller reaches a zero because the gain is very high $P.B. \cong 0$.



(a) (b)

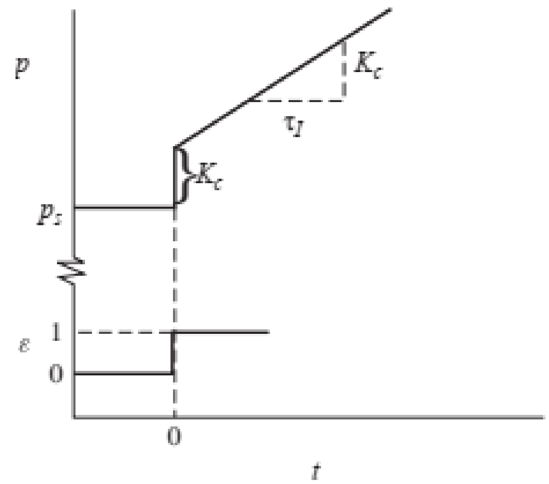
2. **Proportional-Integral Control (PI Control)**

$$p = K_c E + \frac{K_c}{\tau_I}\int_0^t E dt \; + \; p_s$$

$$G_c = \frac{P(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s}\right) \rightarrow P(s) = K_c\left(1 + \frac{1}{\tau_I s}\right) \times \frac{A}{s}$$

Taking Laplace inverse, $P(t) = A\left(K_c + \frac{K_c}{\tau_I} t\right)$

Where τ_I = integral time, min



3. Proportional-Derivative Control (PD Control)

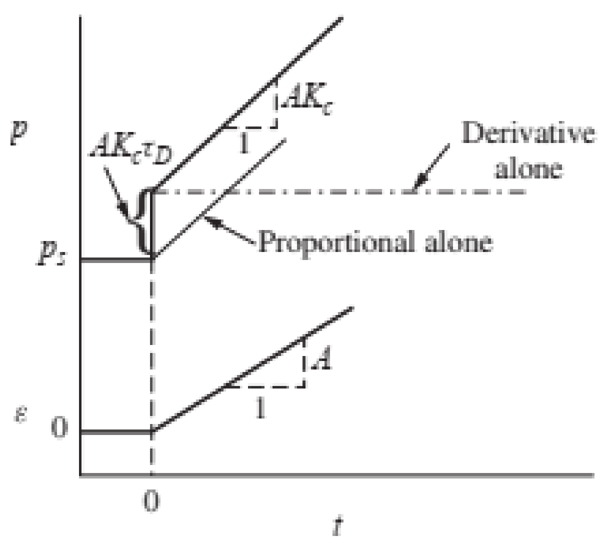
$$p = K_c E + K_c \tau_D \frac{dE}{dt} + p_s$$

$$G_c = \frac{P(s)}{E(s)} = K_c(1 + \tau_D s) \ \rightarrow P(s) = K_c(1 + \tau_D s) \times \frac{A}{s^2}$$

A ramp change in E(s)

Taking Laplace inverse, $P(t) = A(K_c t + K_c \tau_D)$

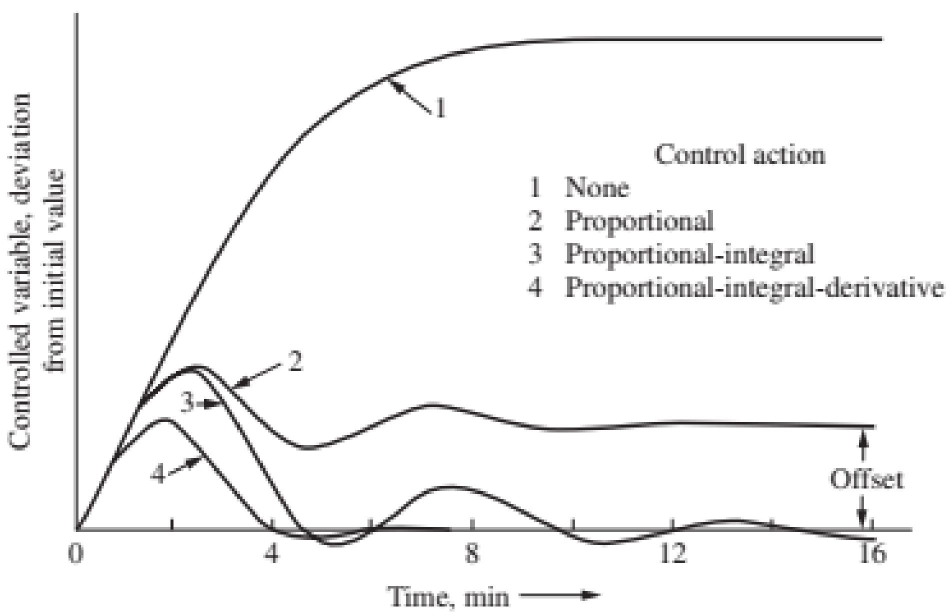
Where τ_D = derivative time, min



4. Proportional-Integral-Derivative Control (PID Control)

$$p = K_c E + \frac{K_c}{\tau_I}\int_0^t E dt + K_c \tau_D \frac{dE}{dt} + p_s$$

$$G_c = \frac{P(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s} + +\tau_D s\right)$$



Problems

9.1. A pneumatic PI temperature controller has an output pressure of 10 psig when the set point and process temperature coincide. The set point is suddenly increased by 10°F (i.e., a step change in error is introduced), and the following data are obtained:

Time, s	psig
0−	10
0+	8
20	7
60	5
90	3.5

Determine the actual gain (psig per degree Fahrenheit) and the integral time.

9.2. A unit-step change in error is introduced into a PID controller. If $K_c = 10$, $\tau_I = 1$, and $\tau_D = 0.5$, plot the response of the controller $P(t)$.

9.3. An ideal PD controller has the transfer function

$$\frac{P}{\varepsilon} = K_c(\tau_D s + 1)$$

An actual PD controller had the transfer function

$$\frac{P}{\varepsilon} = K_c \frac{\tau_D s + 1}{(\tau_D/\beta)s + 1}$$

where β is a large constant in an industrial controller.

If a unit-step change in error is introduced into a controller having the second transfer function, show that

$$P(t) = K_c\left(1 + Ae^{-\beta t/\tau_D}\right)$$

where A is a function of β which you are to determine. For $\beta = 5$ and $K_c = 0.5$, plot $P(t)$ versus t/τ_D. As $\beta \to \infty$, show that the unit-step response approaches that for the ideal controller.

9.4. A PID temperature controller is at steady state with an output pressure of 9 psig. The set point and process temperature are initially the same. At time $t = 0$, the set point is increased at the rate of 0.5°F/min. The motion of the set point is in the direction of *lower* temperatures. If the current settings are

$$K_c = 2 \text{ psig/°F}$$
$$\tau_i = 1.25 \text{ min}$$
$$\tau_D = 0.4 \text{ min}$$

plot the output pressure versus time.

9.5. The input ε to a PI controller is shown in Fig. P9–5. Plot the output of the controller if $K_c = 2$ and $\tau_I = 0.50$ min.

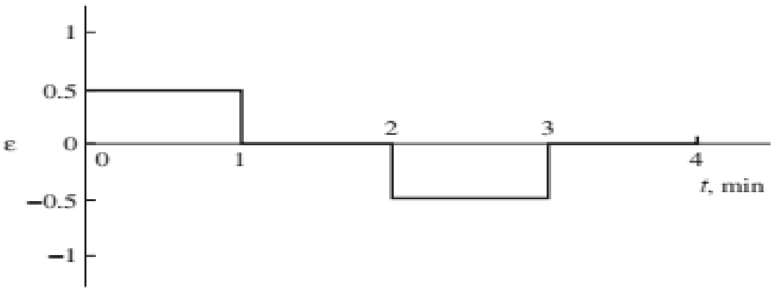


FIGURE P9–5

9.6. A PI controller has the transfer function

$$G_c = \frac{5s + 10}{s}$$

Determine the values of K_c and τ_I.

9.7. Dye for our new line of blue jeans is being blended in a mixing tank. The desired color of blue is produced using a concentration of 1500 ppm blue dye, with a minimum acceptable concentration of 1400 ppm. At 9 A.M. today the dye injector plugged, and the dye flow was interrupted for 10 min, until we realized the problem and unclogged the nozzle. see Fig. P9–7.

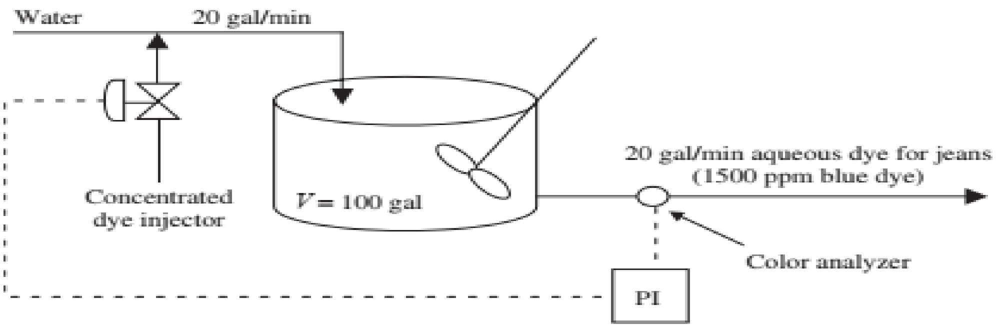


FIGURE P9–7

Plot the controller ouput from 9 A.M. to 9:10 A.M. The steady-state controller output (the bias value) is 8 psig. Does the controller output saturate (output range is 3 to 15 psig)? If so, at what time does it occur? The controller is a PI controller with $K_c = 0.001$ psig/ppm and $\tau_I = 1$ min.