Chapter Two: Response of First Order-Order Systems

Chapter Four in the textbook

This chapter and the three that follow describe in detail the behavior of several basic systems and show that a great variety of physical systems can be represented by a combination of these basic systems.

To summarize the procedure for determining the transfer function for a process:

Step 1: Write the appropriate balance equations (usually mass or energy balances

for a chemical process).

Step 2: Linearize terms if necessary (details on this step are given in the next chapters).

Step 3: Place balance equations in deviation variable form.

Step 4: Take Laplace-transform to the linear balance equations.

Step 5: Solve the resulting transformed equations for the transfer function, the output divided by the input.

Step 6: Select the appropriate change of the process.

Step 7: Take Laplace inverse to find the response of the system.

Standard Form for First-Order Transfer Functions

The general form for a first-order system is

$$\tau \frac{dy}{dt} + y = K_p x(t) \tag{1}$$

where y is the output variable and x (t) is the input forcing function. The initial conditions are

$$y(0) = y_s = K_p x_s$$

$$\tau \frac{dy_s}{dt} + y_s = K_p x_s \tag{2}$$

Subtracting Eq. (1) from Eq. (2):

$$\tau\frac{d(y-y_s)}{dt} + (y - y_s) = K_p(x - x_s) \tag{3}$$

Introducing deviation variables gives:

$$X = x - x_s$$

$$Y = y - y_s$$

Eq. (3) becomes:

$$\tau\frac{dY}{dt} + Y = K_pX(t) \tag{4}$$

Taking Laplace for Eq. (3):

$$\tau Y(s) + Y(s) = K_pX(s) \tag{5}$$

By rearranging, the standard first-order transfer function is:

$$\frac{Y(s)}{X(s)} = \frac{K_p}{\tau s+1} \tag{6}$$

Where K_p is the steady-state gain and τ is the time constant. The denominator must be of the form $\tau s + 1$.

Example 1:

Place the following transfer functions in standard first-order form, and identify the time constant and the steady state gain.

$\frac{Y(s)}{X(s)} = \frac{4}{2s+1}$ $K_p = 4$ and $\tau = 2$

$\frac{Y(s)}{X(s)} = \frac{8}{2s+4}$ divide by 4 $\rightarrow \frac{Y(s)}{X(s)} = \frac{2}{0.5s+1}$ $K_p = 2$ and $\tau = 0.5$

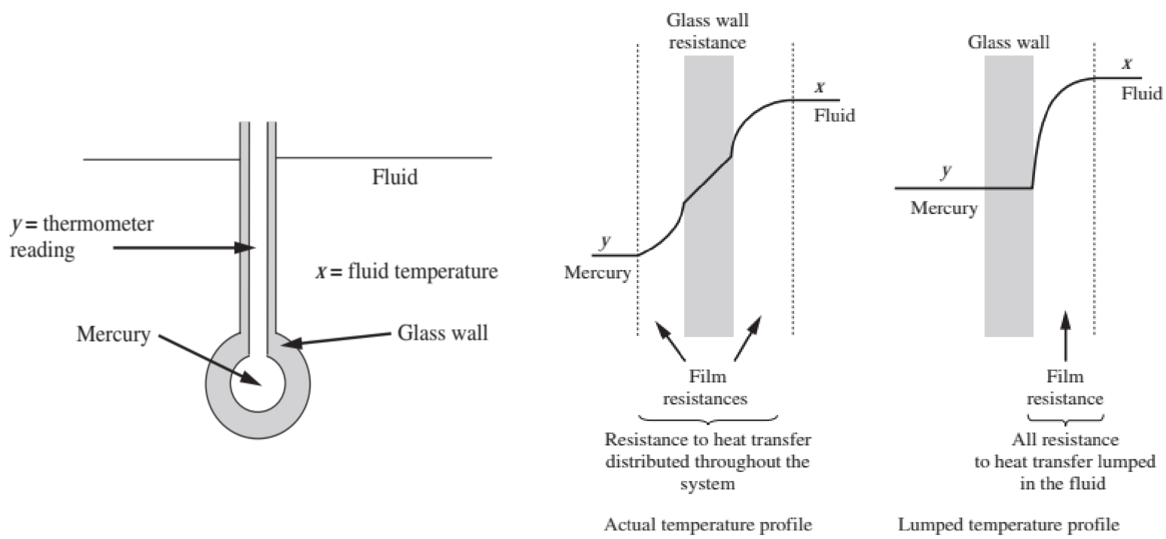
$\frac{Y(s)}{X(s)} = \frac{2}{s+\frac{1}{3}}$ multiple by 3 $\rightarrow \frac{Y(s)}{X(s)} = \frac{6}{3s+1}$ $K_p = 6$ and $\tau = 3$

Example 2:

It is a measuring device use to measure the temperature of a stream. Consider a mercury in glass thermometer to be located in a flowing stream of fluid for which the temperature x varies with time. The opject is to calculate the time variation of the thermometer reading y for a particular change of x. The following assumptions will be used in this analysis:-

1. All the resistance to heat transfer resides in the film surrounding the bulb (i.e., the resistance offered by the glass and mercury is neglected).
2. All the thermal capacity is in the mercury. Furthermore, at any instant the mercury assumes a uniform temperature throughout.
3. The glass wall containing the mercury does not expand or contract during the transient response.

It is assumed that the thermometer is initially at steady state. This means that, before time zero, there is no change in temperature with time. At time zero the thermometer will be subjected to some change in the surrounding temperature x(t). (i.e. at $t < 0$, $x(t) = y(t) =$ constant there is no change in temperature with time). At $t = 0$ there is a change in the surounding temperature x(t).



Unsteady state energy balance:

$$m\, C_p \frac{dy}{dt} = hA\,(x - y) - 0 = h\,A\,x - hA\,y \qquad (*)$$

1st order linear differential equation

Where A is the area of the bulb, Cp is the heat capacity of mercury, m is mass of mercury in the bulb, t is the time, h is the film heat transfer coefficient and depends on the flowrate and properties of the surounding fluid and the dimension of the bulb.

The dynamic behaviour must be defined by a deviation variables.

At steady state (s.s.) , t<0 , x(t) = constant = x_s , y(t) = constant = y_s ,

$$m\ cp\ \frac{dy_s}{dt} = h\ A\ (x_s - y_s s) = h\ Ax_s - h\ Ay_s \qquad (**)$$

Substract Eq. (*) from Eq. (**)

$$m\ cp\ \frac{d(y - y_s)}{dt} = h\ A\ (x - x_s) - h\ A\ (y - y_s s)$$

$Y = y - y_s =$ and $X = x - x_s$ at t=0 Y(0) = 0 and X(0) = 0

$$m\ cp\ \frac{dY}{dt} = h\ A\ X - h\ A\ Y \rightarrow \frac{m\ cp}{h\ A}\frac{dY}{dt} = X - Y$$

$\tau = \frac{m\ cp}{h\ A}$ = time constant and has time units

$\tau \frac{dY}{dt} + Y = X$ by taking laplace for the equation

$$\tau[sY(s) - Y(0)] + Y(s) = X(s) \rightarrow (\tau s + 1)Y(s) = X(s) \rightarrow \frac{Y(s)}{X(s)} = G(s) = \frac{1}{\tau s + 1}$$

$$T.F = \frac{Y(s)}{X(s)} = G(s) = \frac{\text{Lapalce teansform of the deviation in thermometer reading}}{\text{Lapalce teansform of the deviation in surounding Temperature}}$$

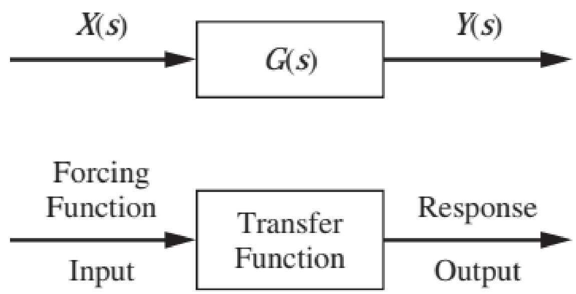
In terms of the example of the mercury thermometer, the surrounding temperature is the cause or input, whereas the thermometer reading is the effect or output.

Properties of Transfer Function

In general, a transfer function relates two variables in a physical process; one of these is the cause (forcing function or input variable), and the other is the effect (response or output variable). We may write

$$\text{Transfrt Function} = \frac{Y(s)}{X(s)} = G(s)$$

Where G(s) is the symbol for a transfer function, X(s) is the transform of forcing function or input in a deviation form and Y(s) is the transform of response or output in a deviation form. The transfer function completely describes the dynamic characteristics of the system. The functional relationship contained in a transfer function is often expressed by a block diagram representation, as shown in following figure: The arrow entering the box is the forcing function or input variable, and the arrow leaving the box is the response or output variable.



$$Y(s) = G(s)X(s)$$

$$Y(s) = \frac{K_p}{\tau s+1} X(s) \qquad (7)$$

$$\text{Initial value} = \lim_{t\to 0} Y(t) \quad or \quad \lim_{s\to\infty} sY(s)$$

$$\text{Final value} = \text{Ultimate value} = \lim_{t\to\infty} Y(t) \quad or \quad \lim_{s\to 0} sY(s)$$

$$\text{Maximum value} = \frac{d(Y(t))}{dt} \text{ at } t = 0$$

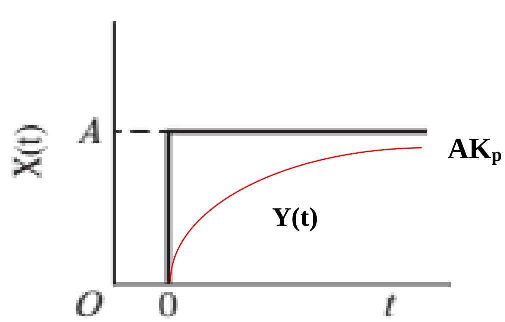
<u>Step Function</u>

$X(t) = A$ where A is the step value.

$X(s) = \frac{A}{s}$ Sub. in Eq. (7):

$$Y(s) = \frac{K_p}{\tau s + 1} X(s) \rightarrow \frac{A}{s} \frac{K_p}{\tau s + 1} \rightarrow \frac{AK_p}{s(\tau s + 1)}$$

By taking Laplace inverse using partial fractions:



$$Y(s) = \frac{AK_p}{s(\tau s + 1)} \rightarrow AK_p\left(\frac{1}{s} + \frac{1}{\tau s + 1}\right)$$

$$Y(t) = AK_p\left(1 - e^{-t/\tau}\right)$$

Where $Y(t)$ the response of the system in the deviation form, and t is the time.

$$Y(t) = y(t) - y_s$$

$$y(t) = AK_p\left(1 - e^{-t/\tau}\right) + y_s$$

$$\text{Initial value} = \lim_{s\to\infty} sY(s) \rightarrow s\frac{AK_p}{s(\tau s + 1)} = 0$$

$$\text{Final value} = \lim_{s\to 0} sY(s) \rightarrow \lim_{s\to 0} s\frac{AK_p}{s(\tau s + 1)} = AK_p$$

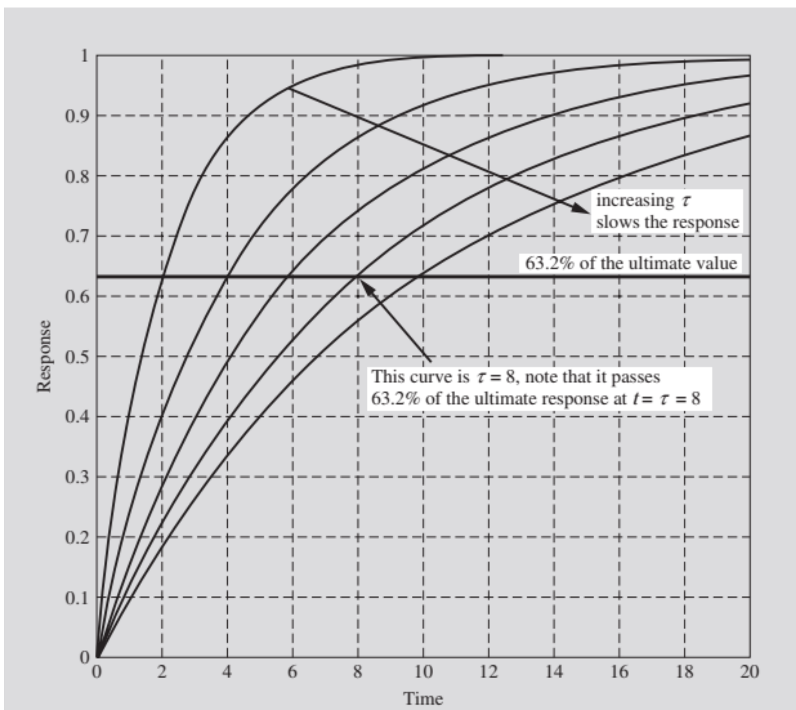
$$\text{Maximum value} = \frac{dY(t)}{dt} = \frac{d}{dt}\left(AK_p\left(1 - e^{-t/\tau}\right)\right) = AK_p$$

<u>Characteristics of step response</u>

A. The value of the output reaches 63.2% of its ultimate value after t= τ

B. If the initial rate of change is maintained the response will be completed after t= τ

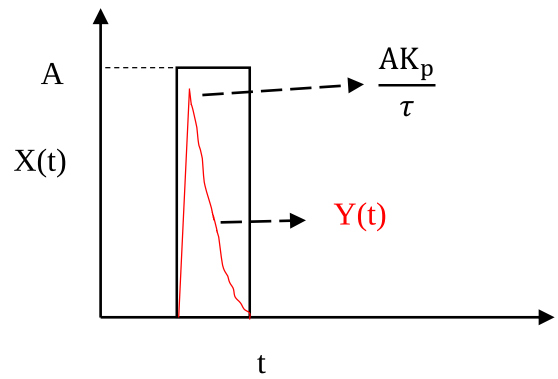
C. The speed of the response of a first-order system is determined by the time constant for the system. As t increases, it takes longer for the system to respond to the step disturbance.



D. When the time elapsed is 2 τ, 3 τ, and 4 τ, the percent response is 86.5%, 95%, and 98%, respectively.

E. The response is completed after t= 5 τ

Impulse Function



$X(t) = A\delta t$ where A is the impulse value.

$X(s) = A$ Sub. in Eq. (7):

$$Y(s) = \frac{AK_p}{\tau s + 1}$$

By taking Laplace inverse using partial fractions:

$$Y(t) = \frac{AK_p}{\tau} e^{-t/\tau}$$

$$y(t) = \frac{AK_p}{\tau} e^{-t/\tau} + y_s$$

$$\text{Initial value} = \lim_{s\to\infty} sY(s) \to \lim_{s\to 0} s\frac{AK_p}{(\tau s+1)} = 0$$

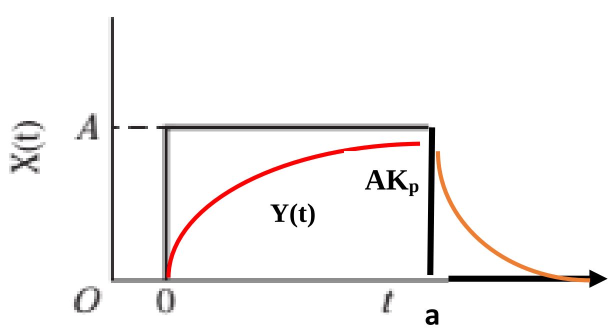
$$\text{Final value} = \lim_{s\to 0} sY(s) \to \lim_{s\to 0} s\frac{AK_p}{(\tau s+1)} = 0$$

$$\text{Maximum value} = \frac{dY(t)}{dt} = \frac{d}{dt}\left(\frac{AK_p}{\tau} e^{-t/\tau}\right) = \frac{AK_p}{\tau}$$

Pulse Function

$$X(t) = \begin{cases} A & 0 < t < a \\ 0 & t > a \end{cases}$$ where A is the step value.

$X(s) = \frac{A}{s} - \frac{A}{s}e^{-as}$ Sub. in Eq. (7):



$$Y(s) = \frac{K_p}{\tau s + 1}\left(\frac{A}{s} - \frac{A}{s}e^{-as}\right)$$

$$Y(s) = \frac{AK_p}{s(\tau s + 1)} - \frac{AK_p e^{-as}}{s(\tau s + 1)}$$

By taking Laplace inverse using partial fractions:

$$Y(t) = AK_p\left(1 - e^{-t/\tau}\right) - AK_p\left(1 - e^{-(t-a)/\tau}\right)$$

$$(t) = AK_p\left(1 - e^{-t/\tau}\right) \qquad 0 < t < a$$

$$Y(t) = AK_p\left(1 - e^{-t/\tau}\right) - AK_p\left(1 - e^{-(t-a)/\tau}\right) \qquad t > a$$

Where Y(t) the response of the system in the deviation form, and t is the time.

$$Y(t) = y(t) - y_s$$

$$y(t) = AK_p\left(1 - e^{-t/\tau}\right) - AK_p\left(1 - e^{-(t-a)/\tau}\right) + y_s$$

$$\text{Initial value} = \lim_{s\to\infty} sY(s) \rightarrow s\left(\frac{AK_p}{s(\tau s+1)} - \frac{AK_p e^{-as}}{s(\tau s+1)}\right) = 0$$

$$\text{Final value} = \lim_{s\to 0} sY(s) \rightarrow \lim_{s\to 0} s\left(\frac{AK_p}{s(\tau s+1)} - \frac{AK_p e^{-as}}{s(\tau s+1)}\right) = 0$$

$$\text{Maximum value} = \frac{dY(t)}{dt} = \frac{d}{dt}\left(AK_p\left(1 - e^{-t/\tau}\right) - AK_p\left(1 - e^{-(t-a)/\tau}\right)\right) = AK_p$$

Ramp Function

$X(t) = At$ where A is the slope value.

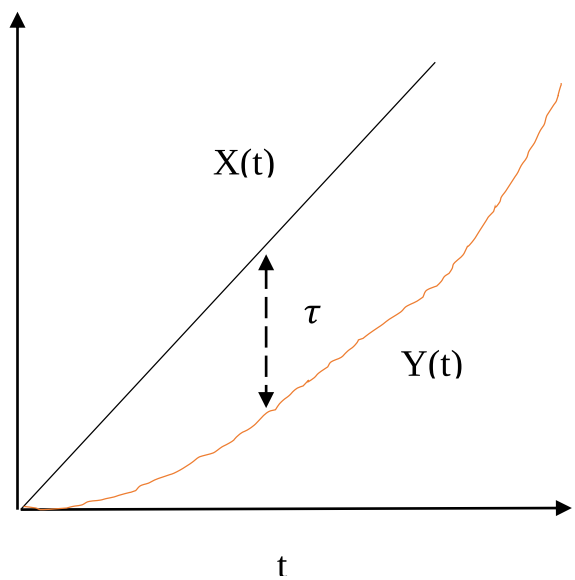
$X(s) = \frac{A}{s^2}$ Sub. in Eq. (7):

$$Y(s) = \frac{K_p}{\tau s+1}X(s) \rightarrow \frac{A}{s^2}\frac{K_p}{\tau s+1} \rightarrow \frac{AK_p}{s^2(\tau s+1)}$$

By taking Laplace inverse using partial fractions:

$$Y(s) = \frac{AK_p}{s^2(\tau s+1)} \rightarrow AK_p\left(\frac{1}{s^2} - \frac{\tau}{s} + \frac{\tau^2}{\tau s+1}\right)$$

$$Y(t) = AK_p\left(t - \tau\left(1 - e^{-t/\tau}\right)\right)$$



Where Y(t) the response of the system in the deviation form, and t is the time.

$\mathrm{Y(t)} = \mathrm{y(t)} - y_s$

$\mathrm{y(t)} = \mathrm{AK_p}\left(\mathrm{t} - \tau\,(1 - \mathrm{e}^{-t/\tau})\right) + y_s$

$$\text{Initial value} = \lim_{s\to\infty} \mathrm{sY(s)} \rightarrow \mathrm{s}\frac{\mathrm{AK_p}}{s^2(\tau s + 1)} = 0$$

$$\text{Final value} = \lim_{s\to 0} \mathrm{sY(s)} \rightarrow \lim_{s\to 0} \mathrm{s}\frac{\mathrm{AK_p}}{s^2(\tau s + 1)} = \infty$$

$$\text{Maximum value} = \frac{\mathrm{dY(t)}}{dt} = \frac{\mathrm{d}}{dt}\left(\mathrm{AK_p}\left(\mathrm{t} - \tau\,(1 - \mathrm{e}^{-t/\tau})\right)\right) = \infty$$

<u>Sinusoidal Function</u>

$\mathrm{x(t)} = x_s + \mathrm{A}\sin\omega t$

$\mathrm{X(t)} = \mathrm{A}\sin\omega t$

$\mathrm{X(s)} = \frac{A\omega}{s^2+\omega^2}$ Sub. in Eq. (7):

$$\mathrm{Y(s)} = \frac{\mathrm{K_p}}{\tau s + 1}\frac{\alpha\omega}{s^2 + \omega^2}$$

This equation can be solved for Y(t) by means of a partial fraction expansion, as described. The result is:

$$\mathrm{Y(t)} = \frac{\mathrm{A}\omega\tau \mathrm{e}^{-t/\tau}}{\tau^2\omega^2+1} - \frac{\mathrm{A}\omega\tau}{\tau^2\omega^2+1}cos\omega t + \frac{\mathrm{A}}{\tau^2\omega^2+1}sin\omega t \qquad (*)$$

The equation above can be written in another form by using the trigonometric identity:

$p\ cosB + q\ sin\ B = r\sin(B + \theta)$

$\mathrm{r} = \sqrt{p^2 + q^2}$ $\tan\theta = \frac{p}{q}$

Where Y(t) the response of the system in the deviation form, and t is the time.

$$Y(t) = \frac{A\omega\tau}{\tau^2\omega^2 + 1} e^{-t/\tau} + \frac{A}{\sqrt{\tau^2\omega^2 + 1}} \, sin(\omega t + \emptyset)$$

$$\emptyset = \tan^{-1}(-\omega t)$$

As $t \rightarrow \infty$, the first term on the right side of equation above vanishes and leaves only the ultimate periodic solution, which is sometimes called the steady-state solution.

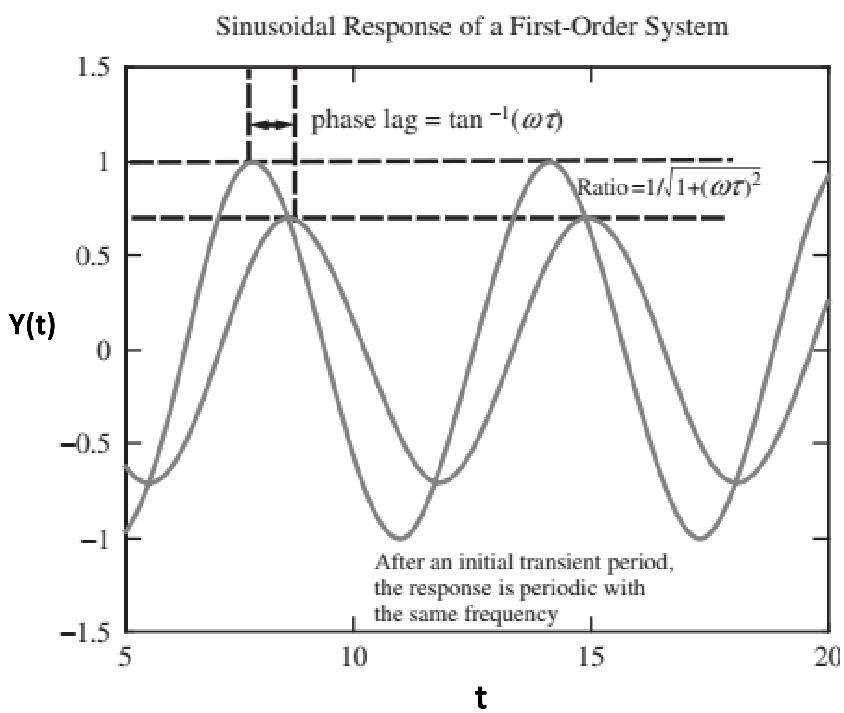
$$Y(t)\,|_s = \frac{A}{\sqrt{\tau^2\omega^2+1}} \, sin(\omega t + \emptyset) \qquad (**)$$

By comparing Eq. (*) for the input forcing function with Eq. (**) for the ultimate periodic response, we see that

1. The output is a sine wave with a frequency ω equal to that of the input signal.
2. The ratio of output amplitude to input amplitude is $\frac{1}{\sqrt{\tau^2\omega^2+1}}$. This ratio is always smaller than 1. We often state this by saying that the signal is attenuated.
3. The output lags behind the input by an angle $\emptyset$. It is clear that lag occurs, for the sign of $\emptyset$ is always negative.

$$\emptyset < 0 \qquad phase\ lag$$

$$\emptyset > 0 \qquad phase\ lead$$



Example 3: A mercury thermometer having a time constant of 0.1 min is placed in a temperature bath at 100°F and allowed to come to equilibrium with the bath. At time $t = 0$, the temperature of the bath begins to vary sinusoidally about its average temperature of 100°F with an amplitude of 2°F. If the frequency of oscillation is $10/\pi$ cycles/min, plot the ultimate response of the thermometer reading as a function of time. What is the phase lag?

In terms of the symbols used in this chapter

$$\tau = 0.1 \text{ min}$$

$$x_s = 100°\text{F}$$

$$A = 2°\text{F}$$

$$f = \frac{10}{\pi} \text{ cycles/min}$$

$$\omega = 2\pi f = 2\pi \frac{10}{\pi} = 20 \text{ rad/min}$$

From Eq. (4.28), the amplitude of the response and the phase angle are calculated; thus

$$\frac{A}{\sqrt{\tau^2\omega^2 + 1}} = \frac{2}{\sqrt{4 + 1}} = 0.896°\text{F}$$

$$\phi = -\tan^{-1} 2 = -63.5° = -1.11 \text{ rad}$$

or

$$\text{Phase lag} = 63.5°$$

The response of the thermometer is therefore

$$\mathbf{Y(t) = 0.896 \sin (20t - 1.11)}$$

or

$$y(t) = 100 + 0.896 \sin (20t - 1.11)$$

To obtain the lag in terms of time rather than angle, we proceed as follows: A frequency of $10/\pi$ cycles/min means that a complete cycle (peak to peak) occurs in $(10/\pi)^{-1}$ min. Since one cycle is equivalent to 360° and the lag is 63.5°, the time corresponding to this lag is

$$\text{Lag time} = \frac{63.5}{360} \times (\text{time for 1 cycle})$$

or

$$\text{Lag time} = \left(\frac{63.5}{360}\right)\left(\frac{\pi}{10}\right) = 0.0555 \text{ min}$$

thus,

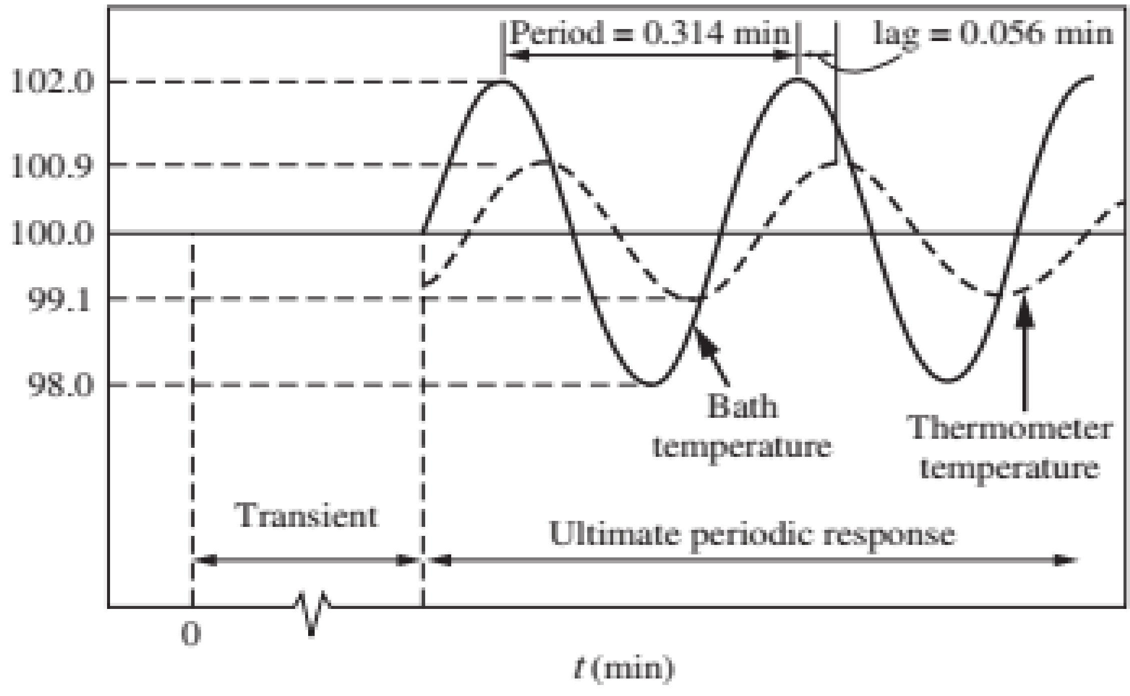
$$y(t) = 100 + 0.896 \sin[20(t - 0.0555 \text{ min})]$$

In general, the lag in units of time is given by

$$\text{Lag time} = \frac{|\phi|}{360 f}$$

when ϕ is expressed in degrees.

The response of the thermometer reading and the variation in bath temperature are shown in Fig. 4–18. Note that the response shown in this figure holds only after sufficient time has elapsed for the nonperiodic term of Eq. (4.27) to become negligible. For all practical purposes this term becomes negligible after a time equal to about 3τ. If the response were desired beginning from the time the bath temperature begins to oscillate, it would be necessary to plot the complete response as given by Eq. (4.27).



Problems

4.1. A thermometer having a time constant of 0.2 min is placed in a temperature bath, and after the thermometer comes to equilibrium with the bath, the temperature of the bath is increased linearly with time at a rate of l°/min. Find the difference between the indicated temperature and the bath temperature.

(a) 0.1 min after the change in temperature begins,

(b) 1.0 min after the change in temperature begins?

(c) What is the maximum deviation between indicated temperature and bath temperature, and when does it occur?

(d) Plot the forcing function and response on the same graph. After a long enough time, by how many minutes does the response lag the input?

4.2. A mercury thermometer bulb is $\frac{1}{2}$ in. long by $\frac{1}{8}$ in. diameter. The glass envelope is very thin. Calculate the time constant in water flowing at 10 ft/sec at a temperature of 100°F. In your solution, give a summary which includes:

(a) Assumptions used

(b) Source of data

(c) Results

4.3. Given a system with the transfer function $\frac{Y(s)}{X(s)} = \frac{\tau_1 s+1}{\tau_2 s+1}$. Find Y(t) if X(t) is a unit-step function. If $\frac{\tau_1}{\tau_2} = 5$, sketch Y(t) versus $\frac{t}{\tau_2}$. Show the numerical values of minimum, maximum, and ultimate values that may occur during the transient. Check these using the initial-value and final-value theorems of App. A3.

4.4. A thermometer having first-order dynamics with a time constant of 1 min is placed in a temperature bath at 100 °F. After the thermometer reaches steady state, it is suddenly placed in a bath at 110 °F at t = 0 and left there for 1 min, after which it is immediately returned to the bath at 100 °F.

(a) Draw a sketch showing the variation of the thermometer reading with time.

(b) Calculate the thermometer reading at t = 0.5 min and at t = 2.0 min.

4.5. Repeat Prob. 4.4 if the thermometer is in the 110 °F bath for only 10 sec.

4.6. A mercury thermometer, which has been on a table for some time, is registering the room temperature, 75 °F. Suddenly, it is placed in a 400°F oil bath. The following data are obtained for the response of the thermometer.

Time, sec	0	1	2.5	5	8	10	15	30
Thermometer reading, ^{o}F	75	107	140	205	244	282	328	385

4.7. Rewrite the sinusoidal response of a first-order system [Eq. (4.27)] in terms of a cosine wave. Re-express the forcing function [Eq. (4.22)] as a cosine wave, and compute the phase difference between input and output cosine waves.

4.8. The mercury thermometer of Prob. 4.6 is again allowed to come to equilibrium in the room air at 75°F. Then it is placed in the 400^{o}F oil bath for a length of time less than 1 sec, and quickly removed from the bath and re-exposed to the 75°F ambient conditions. It may be estimated that the heat-transfer coefficient to the thermometer in air is one-fifth that in the oil bath. If 10 sec after the thermometer is removed from the bath it reads 98^{o}F, estimate the length of time that the thermometer was in the bath.

4.9. A thermometer having a time constant of 1 min is initially at 50°C. It is immersed in a bath maintained at 100°C at t = 0. Determine the temperature reading at t = 1.2 min.

4.10. In problem 4.9, if at t = 1.5 min, the thermometer is removed from the bath and put in a bath at 75ºC, determine the maximum temperature indicated by the thermometer. What will be the indicated temperature at t = 20 min?

4.11. A process of unknown transfer function is subjected to a unit-impulse input. The output of the process is measured accurately and is found to be represented by the function $Y(t) = te^{-t}$. Determine the unit-step response of this process.

4.12. The temperature of an oven being heated using a pulsed resistance heater varies as

$$T = 120 + \cos(25t + 30^o)$$

where t is the time in seconds. The temperature of the oven is being measured with a thermocouple having a time constant of 5 sec.

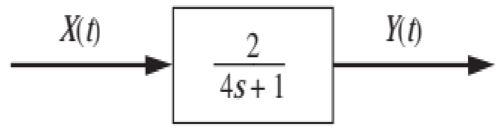
a) What are the maximum and minimum temperatures indicated by the thermocouple?
b) What is the maximum difference between the actual temperature and the indicated temperature?
c) What is the time lag between the actual temperature and the indicated temperature?

4.13. The temperature of an experimental heated enclosure is being ramped up from 80 to 450°F at the rate of 20°F/min. A thermocouple, embedded in a thermowell for protection, is being used to monitor the oven temperature. The thermocouple has a time constant of 6 sec.

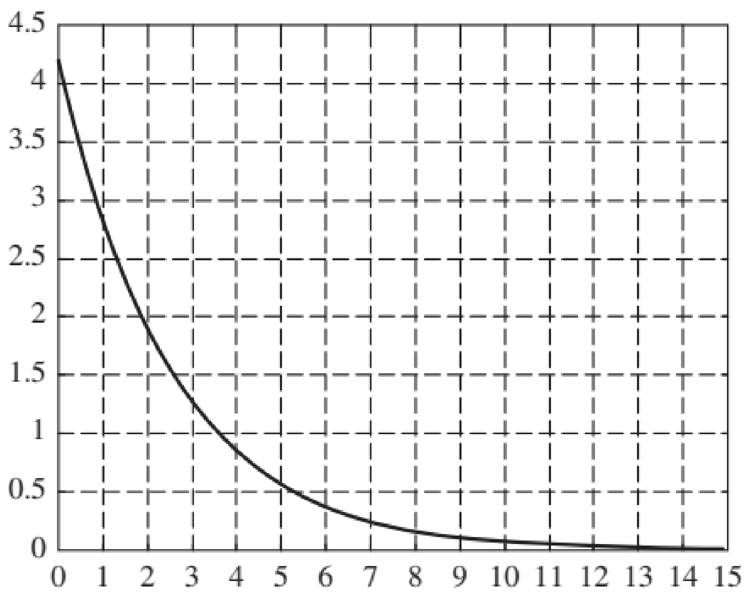
a) At t = 10 min, what is the difference between the actual temperature and the temperature indicated by the thermocouple? What is it at 60 min?

b) When the thermocouple indicates 450°F, the heater will begin to modulate and maintain the temperature at the desired 450°F. What is the actual oven temperature when the thermocouple first indicates 450°F?

4.14. For the transfer function in following figure, the response Y(t) is sinusoidal. The amplitude of the output wave is 0.6 and it lags behind the input by 1.5 min. Find X (t). Note: the time constant in the transfer function is in minutes.

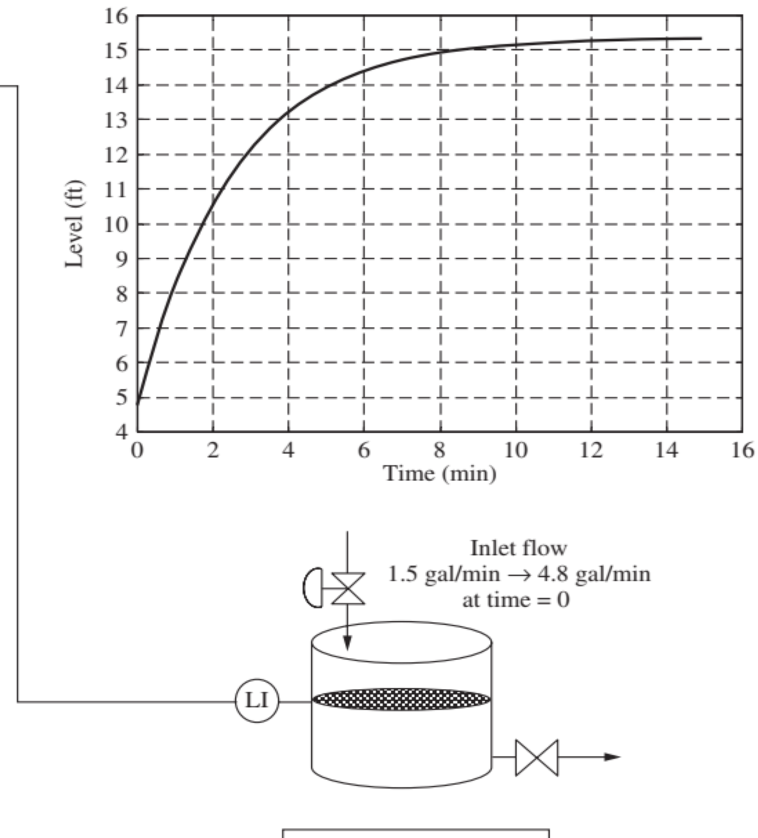


4.15. The graph in following figure is the response of a suspected first-order process to an impulse function of magnitude 3. Determine the transfer function G(s) of the unknown process.



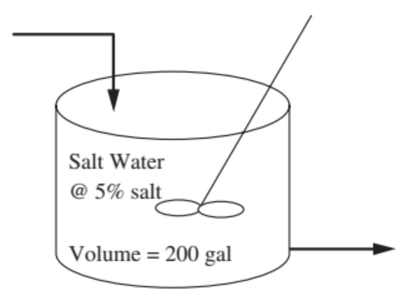
4.16. The level in a tank responds as a first-order system with changes in the inlet flow. Given the following level versus time data that were gathered (figure below) after the inlet flow was increased quickly from 1.5 to 4.8 gal/min, determine the transfer function that relates the height in the tank to the inlet flow. Be sure to use deviation variables and include units on the steady-state gain and the time constant.

Time (min)	Level (ft)
0	4.8
0.138	5.3673
0.2761	5.9041
0.4141	6.412
0.5521	6.8927
0.6902	7.3475
0.8282	7.7779
0.9663	8.1852
1.1043	8.5706
1.2423	8.9354
1.3804	9.2805
1.5184	9.6071
1.6564	9.9161
1.7945	10.2085
1.9325	10.4853
2.0705	10.7471
2.2086	10.9949
2.3466	11.2294
2.4847	11.4513
2.6227	11.6612
2.7607	11.8599
....	....
14.3558	15.3261
14.4938	15.328
14.6319	15.3297
14.7699	15.3313



Note: LI = level indicator

4.17. A simple mixing process follows first-order behavior. A 200-gal mixing tank process, initially at steady state, is shown in following figure. At time t = 0, the inlet flow is switched from 5% salt to freshwater. What does the inlet flow rate need to be to reduce the exit concentration to less than 0.5% in 30 min?

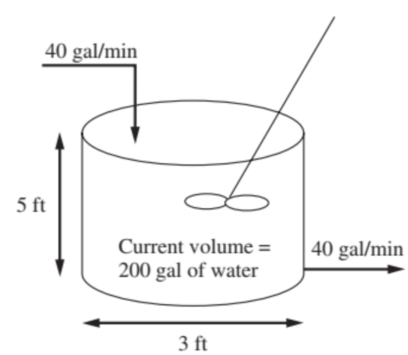


4.18. Joe, the maintenance man, dumps the contents of a 55-gal drum of water into the tank process shown below.

a) Will the tank overflow?

b) Plot the height as f (t), starting at t = 0, the time of the dump.

c) Plot the output flow as f (t), starting at t = 0, the time of the dump.



NOTE: The output flow is proportional to the height of fluid in the tank.

Multi-Choice Questions

1. Laplace transform of a ramp input of slope K is:

   a) $\frac{K}{s}$ b) $\frac{K}{s^2}$ c) $\frac{1}{s}$ d) $\frac{K}{s^3}$

2. Laplace transform of time lag of L time unit is:

   a) e^{-Ls} b) e^{-Lt} c) e^{Ls} d) e^{Lt}

3. The time constant of a first order system is defined as time for the system to reach following a step input change ………% of its final value.

   a) 63.2 b) 99.8 c) 85.4 d) 18.8